A LOW-COMPLEXITY COMPANDING TRANSFORM FOR PEAK-TO-AVERAGE POWER RATIO REDUCTION IN OFDM SYSTEMS

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ABSTRACT

The high peak-to-average power ratio (PAPR) feature is one main drawback associated with orthogonal frequency division multiplexing (OFDM) systems. To overcome this problem, many approaches have been presented in the literature, such as the selected mapping technique, the partial transmit sequence technique, the linear nonsymmetrical transform (LNST), and the exponential companding transform (Exp-CT). However, most of them involve high computational complexity. In this paper, we propose a new low-complexity companding transform scheme for PAPR reduction. From computer simulations, the proposed low-complexity scheme can successfully reduce the PAPR value with a lower bit error rate than the Exp-CT scheme and smaller spectrum side-lobe generation than the LNST and Exp-CT schemes. It is useful for physical realization of an OFDM system.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a promising technique for high-bit-rate transmission since its robustness to multipath fading and high spectral utilization efficiency. It has been adopted in many standards of wireline/wireless applications, such as very-high-rate digital subscriber lines (VDSL) [1], terrestrial digital video broadcasting (DVB) [2], and the IEEE 802.11a wireless local area network [3]. One major drawback associated with an OFDM system is that the transmitter's output signal may have a high peak-to-average ratio (PAPR). The high PAPR feature will cause poor efficiency of power consumption, in-band distortion, and spectral spreading when an OFDM signal is passed through a nonlinear power amplifier.

To solve the high PAPR problem of OFDM systems, many methods have been presented in the literature, which can be categorized as two groups. One group is to reduce the probability of generating high PAPR signals before doing multicarrier modulation, such as coding [4], [5], selective mapping (SLM) [6]-[8], and partial transmit sequence (PTS) [9]-[11]. For the coding schemes, the original data sequences are replaced by a subset of longer sequences with low peak power. However, such coding schemes need large look-up tables for coding/decoding and exhaustive search for a good code. It is only feasible for systems with a small number of subcarriers. In the SLM schemes, a set of candidate signals that represent the same information are generated and the signal with the smallest PAPR value is selected for transmission. For the PTS methods, the original data sequence is partitioned into a number of disjoint subblocks; the inverse fast Fourier transform (IFFT) of each subblock is multiplied by a selected phase factor and then combined together to form an OFDM signal with low PAPR for transmission. Both the SLM and PTS methods can effectively reduce the PAPR value, but they usually have high computational complexity and need to send side information to the receiver.

The other group is to deal with the signals after multicarrier modulation, such as clipping [12], [13], and the companding transform (CT) [14]-[17]. Clipping is the simplest method that limits the amplitude of an OFDM signal to some desired threshold value. It is a nonlinear process and may cause in-band distortion and out-of-band radiation. A CT scheme uses a compander to reduce the amount of clipped signals as small as possible. Such approaches can effectively reduce the PAPR with less computational complexity than the SLM and PTS schemes. However, they will increase the average power of the companded OFDM signal and the out-of-band radiation. Moreover, the corresponding transmitter and receiver need a compander and an expander respectively, which of course increases the hardware cost.

In this paper, we propose a new companding transform scheme with low complexity for PAPR reduction of OFDM signals. The proposed method can balance the tradeoff among the bit error rate (BER), the spectral efficiency, and the PAPR reduction performance. It is useful for physical realization of an OFDM system.

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2. BACKGROUND

Fig.1 illustrates the basic block diagram of an OFDM system using the CT technology for PAPR reduction, where a CT block follows the IFFT block, immediately. The IFFT output, x_n, composes of a sum of N orthogonal subcarriers modulated by the input data symbols, X_k, and can be represented as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp(\frac{j2\pi nk}{N}),\ 0 \le n \le N-1 \qquad (1)$$

Assume that the input data symbols are statistically independent and identically distributed. From the central limit theorem it states that for large values of N (≥ 64), the real and imaginary parts of x_n individually become Gaussian distributed with a mean of zero and a variance of $E[|X_k|^2]/2$, where E[.] denotes the expected value. Hence, the amplitudes of OFDM signals have the Rayleigh distribution while the power distribution becomes a central chi-square distribution with two degrees of freedom and zero mean. [18]

The PAPR of the transmitted OFDM signals during a symbol period is defined as the ratio of the maximum to the average power of the signals. It can be represented as

$$\text{PAPR}=\frac{\max |x_n|^2}{E[|x_n|^2]} \qquad (2)$$

The peak power will happen when the N modulated symbols are added with the same phase and PAPR $\le N$.

Recently, many companding transform (CT) schemes have been proposed for reducing the PAPR (see, for example, [14]-[17]). In the work of [16], the authors presented four CT schemes. Among them, linear nonsymmetrical transform (LNST) has the best performance when the word length of D/A converter is larger than 12 bits. For a real input signal a, the transformed signal b by the LNST scheme of [16] can be expressed as:

$$b = \begin{cases} \dfrac{a}{u}, & |a| \le v_0 \\ u \cdot a, & |a| > v_0 \end{cases} \qquad (3)$$

where $0<u \le 1$ and $0 \le v_0 \le A$ are the transform parameters and A is the maximum input value. A transfer curve of (3) with u=0.625 and v_0=1 is shown in Fig. 2. This transform can be also applied to the real or imaginary parts of a complex signal. By (3), the input signal a will be enlarged by a factor of $1/u$ if its absolute value $|a|$ is less than or equal to the threshold value v_0, and compressed by a factor of u if $|a|$ is larger than v_0. By increasing the average power and lowering the maximum peak value, the LNST scheme effectively reduces the PAPR. However, there are two drawbacks for the LNST scheme. First, the transformed signals are not one-to-one mapping to the input signals in the range of $uv_0 < b \le v_0/u$, as shown in Fig. 2. This will cause confusion when the receiver executes the inverse companding transform. In the work of [16], it is assumed that the receiver uses some index messages to resolve this confusion. It means that the transmitter has to send the side information to the receiver. Second, the transformed signal b will have abrupt jump if the input signal passes across v_0. This jump will generate unwanted high frequency components in the transformed signals.

In the work of [17], the authors proposed exponential companding transform (Exp-CT) to reduce the PAPR by compressing the peak power value of OFDM signals with a constant average power level. However, it needs more complex computation and circuitry than the LNST scheme of [16]. In the next section, we will introduce a new linear CT method which can solve the two drawbacks of the LNST scheme with lower computational and circuitry complexity than the Exp-CT scheme.

3. PROPOSED NEW COMPANDING TRANSFORM

In OFDM systems, the signals are presented by the fixed-point format before the D/A converter. Assume that both the real and imaginary parts of the IFFT outputs are represented by the two's complement format. The multiplication or division by a power of two can be simply realized by doing bit shift-left or shift-right operation for the operand data. Based on the idea, we propose a new linear CT scheme. Let a be the magnitude of the real or imaginary part of an OFDM signal. The transformed output, b, is defined as

$$b = \begin{cases} u_1 \cdot a, & v_1 \ge |a| \\ \text{sgn}(a)\cdot(u_2\cdot(|a|-v_1)+u_1\cdot v_1), & v_1 < |a| \le v_2 \\ \text{sgn}(a)\cdot(u_3\cdot(|a|-v_2)+u_2\cdot(v_2-v_1)+u_1\cdot v_1), & v_2 < |a| \end{cases} \qquad (4)$$

where sgn(.) denotes the sign function, u_1, u_2, $u_3 > 0$, and $0 \le v_1 \le v_2 \le A$ (A is the maximum input value). To reduce the PAPR value by enlarging the small signals and compressing the maximum peak value, the relation among u_1, u_2 and u_3 should be $u_1 > u_2 > u_3$ with $u_1 > 1$ and $u_3 < 1$.

At the receiver, the proposed inverse CT function for the received OFDM signals is:

$$a' = \begin{cases} \dfrac{b'}{u_1}, & v_1' \ge |b'| \\ \text{sgn}(b')\cdot(\dfrac{1}{u_2}\cdot(|b'|-v_1')+\dfrac{v_1'}{u_1}), & v_1' < |b'| \le v_2' \\ \text{sgn}(b')\cdot(\dfrac{1}{u_3}\cdot(|b'|-v_2')+\dfrac{1}{u_2}\cdot(v_2'-v_1')+\dfrac{v_1'}{u_1}\cdot v_1'), & v_2' < |b'| \end{cases} \qquad (5)$$

where b' is the magnitude of the real or imaginary part of the received OFDM signals, which consist of the original OFDM signals, the channel noises, and the quantization noises. v_1' and v_2' are the transformed values of v_1 and v_2 by

(4). By modifying u_1, u_2, u_3, v_1 and v_2, the PAPR value and the average power of the companded signals by (4) can be adjusted. If u_1, u_2 and u_3 are chosen to be one or a combination of powers of two, then the multiplication and division operations in (4) and (5) can be easily implemented by using bit shift-left or shift-right operation for the operand data. This can significantly reduce the complexity of computation and circuitry. A transfer curve of (4) is shown in Fig. 2 with u_1=1.125, u_2=1, u_3=0.5, v_1 =0.5 and v_2=1. It illustrates that the proposed CT has one-to-one mapping between the input and the transformed signals, and the transformed signals have no abrupt jump when the input signals move across the threshold values v_1 and v_2. It can successfully resolve the two drawbacks of the LNST scheme in [16]. And the value of 1.125 can be regarded as a combination of two powers of two, i.e., 2^0+2^{-3}. A signal multiplied by 1.125 can be easily realized by shifting three bits right for the signal, and then adding the shifted value with the original one.

4. SIMULATION RESULTS

In this section, we will verify the proposed linear CT scheme and compare its performance with the LNST scheme of [16] and the Exp-CT scheme of [17]. The OFDM system used for simulation has 64 subcarriers and QPSK modulation format. The oversampling factor is set to be four so that the length of IFFT and FFT is 256. The input signals, X_k, are generated by the random number generator and the channel is assumed to be the AWGN channel. The transform parameters for the computer simulation are u =0.625 and v_0= 1 for the LNST scheme, d=2 for the Exp-CT scheme, and u_1=1.125, u_2=1, u_3=0.5, v_1=0.5 and v_2= 1 for the proposed scheme. It is assumed that the transmitter and the receiver are perfectly synchronized and the receiver knows the transform parameters for each CT. The nonlinear effect of the power amplifier is also ignored.

The complementary cumulative distribution function (CCDF) of the reduced PAPR is defined as the probability of PAPR larger than a given value *PAPR0*. Fig. 3, Fig. 4 and Fig. 5 show the CCDF of PAPR, the BER, and the power spectral density (PSD) for the original OFDM signals and the three different CT schemes. These figures illustrate that the Exp-CT scheme has the best PAPR reduction performance, but with the worst BER performance and the most complicated computation/circuitry complexity. The worst BER performance results in the flatness of the transfer curve when the input signals are larger than 2.0, as shown in Fig. 2. It will increase the error probability when the inverse CT is executed. The bad BER performance can be improved by increasing the average power of the companded signals, at cost of more power consumption.

On the other hand, the LNST scheme has better BER performance than the other two schemes, but it has the worst PSD performance since there is an abrupt jump in the transfer curve. It also needs to transmit index information to the receiver, which will degrade the system throughput performance. As compared with the previous two schemes, the proposed linear CT scheme has better PSD and moderate BER performance. Its PAPR reduction can reach about 3.1 dB when the CCDF is 10^{-4}. In the meantime, its CT block can be easily implemented by bit-shifted circuit and adder if the transform parameters u_1, u_2, and u_3 are properly selected. It has lower computational and circuitry complexity than the Exp-CT scheme.

5. CONCLUSION

In this paper, we have proposed a new low-complexity linear companding transform scheme to overcome the high PAPR problem of OFDM systems. As compared with the previous companding transform approaches in [16] and [17], the proposed one can effectively reduce the PAPR with lower complexity, better PSD, and moderate BER performance. These features make it useful for practical OFDM applications.

6. REFERENCES

[1] VDSL Alliance SDMT VDSL Draft Standard Proposal, ETSI STC/TM6, April 1998.

[2] *Digital Video Broadcasting: framing structure, channel coding, and modulation for digital terrestrial television*, ETSI Standard, EN 300 744 v1.4.1, 2000.

[3] IEEE, "part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications," IEEE Std. 802.11, Aug. 1999.

[4] T. A. Wilkinson and A. E. Jones, "Minimisation of the peak to mean envelope power ratio of multicarrier transmission schemes by block coding," in *Proc. 1995 IEEE Veh. Technol. Conf. (VTC '95)*, Chicago, July 1995, pp. 825-829.

[5] R. van Nee, "OFDM codes for peak-to-average power reduction and error correction," in *Proc. 1996 IEEE Global Telecommun. Conf.* (*GLOBECOM '96*), London, vol. 1, 1996, pp. 740-744.

[6] R. W. Bauml, R. F. H. Fisher and J. B. Huber, "Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping," *Electron Lett.*, vol. 32, pp. 2056-2057, Oct. 1996.

[7] C. L. Wang, M. Y. Hsu, and Y. Ouyang, "A low-complexity peak-to-average power ratio reduction technique for OFDM systems," in *Proc. 2003 IEEE Global Telecommun. Conf.* (*GLOBECOM '03*), San Francisco, CA, Dec. 2003, pp. 2375-2379.

[8] D. W. Lim, J. S. No, C. W. Lim, and H. Chung, "A new SLM OFDM scheme with low complexity for PAPR reduction," *IEEE Signal Processing Lett.*, vol. 12, pp. 93-96, Feb. 2005.

[9] S. H. Müller and J. B. Huber, "OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences," *Electron Lett.*, vol. 33, pp. 368-369, Feb. 1997.

[10] S. G. Kang, J. G. Kim, and E. K. Joo, "A novel subblock partition scheme for partial transmit sequence OFDM," *IEEE Trans. Broadbasting*, vol. 45, pp. 333-338, Sept. 1999.

[11] S. H. Han and J. H. Lee, "PAPR reduction of OFDM signals using a reduced complexity PTS technique," *IEEE Signal Processing Lett.*, vol. 11, pp. 887-890, Nov. 2004.

[12] X. Li and L. J. Cimini, "Effects of clipping and filtering on the performance of OFDM," *IEEE Commun. Lett.*, vol. 2, pp. 131-133, May 1998.

[13] D. Wulich and L. Goldfeld., "Reduction of peak factor in orthogonal multicarrier modulation by amplitude limiting and coding," *IEEE Trans. Commun.*, vol. 47, pp. 18-21, Jan. 1999.

[14] X. Wang, T.T. Tjhung, and C. S. Ng, "Reduction of peak-to-average power ratio of OFDM system using a companding technique," *IEEE Trans. Broadbasting*, vol. 45, pp. 303-307, Sept. 1999.

[15] T. Jiang and G. Zhu, "Nonlinear companding transform for reducing peak-to-average power ratio of OFDM signals," *IEEE Trans. Broadbasting*, vol. 50, pp. 342-346, Sept. 2004.

[16] X. Huang, J. Lu, J. Zheng, and J. Gu., "Companding transform for reduction in peak-to-average power ratio of OFDM signals," *IEEE Trans. Wireless Commun.*, vol. 03, pp. 2030-2039, Nov. 2004.

[17] T. Jiang, Y. Yang, and Y. H. Song, "Exponential companding technique for PAPR reduction in OFDM systems," *IEEE Trans. Broadbasting*, vol. 51, pp. 244-248, June. 2005.

[18] R. van Nee and R. Prasad, OFDM for Wireless Multimedia Communications. Boston: Artech House, 2000.

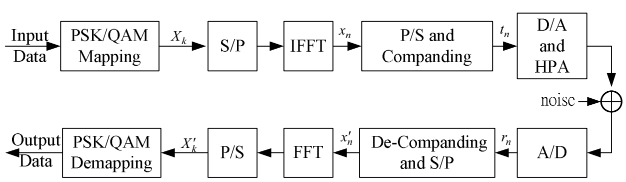

Fig. 1. The basic block diagram of an OFDM system with the companding transform.

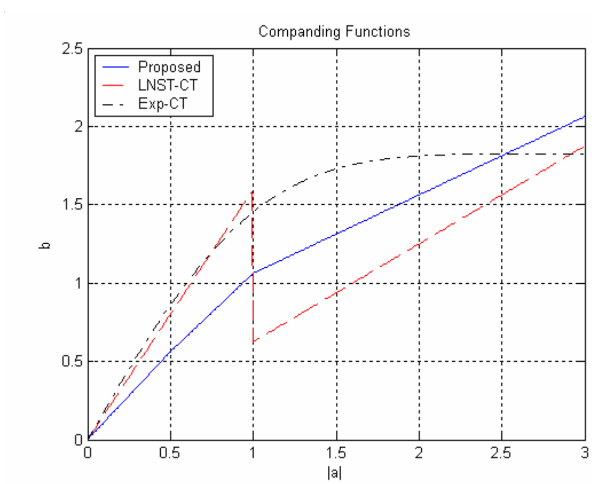

Fig.2. The transfer curves of three different companding transform schemes.

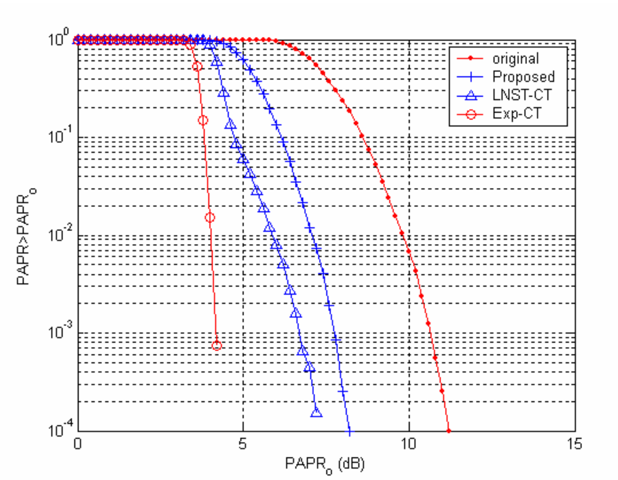

Fig. 3. Comparison of PAPR reduction performance of three different companding transform schemes.

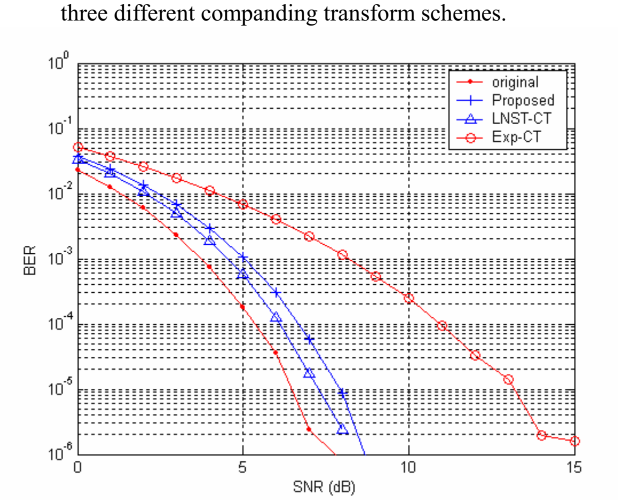

Fig. 4. Comparison of BER performance of three different companding transform schemes.

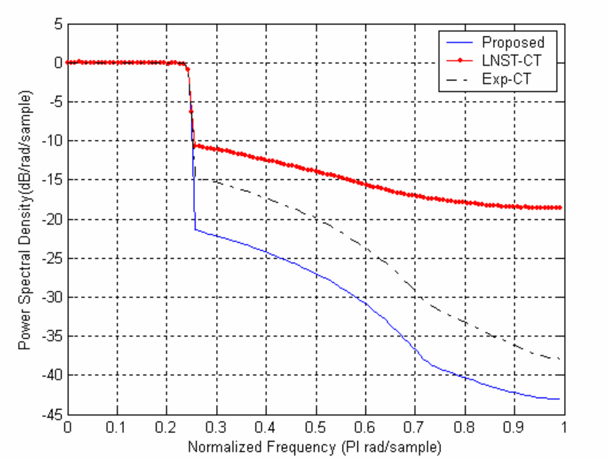

Fig. 5. Comparison of PSDs of three different companding transform schemes.