# RECEIVER CANCELLATION OF NONLINEAR POWER AMPLIFIER DISTORTION IN SDMA-OFDM SYSTEMS

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### ABSTRACT

Space Division Multiple Access in conjunction with OFDM results in a robust system that increases spectral efficiency and system capacity. This combination is positioning as one of the most promising candidates for future WLAN implementations. One drawback of OFDM systems is the high peak to average power ratio, which imposes severe requirements for the linearity of the power amplifier (PA). Such linearity requirements translate into high back-off resulting in low power efficiency. In order to improve power efficiency, *power amplifier nonlinearity cancellation* is introduced. This technique reduces the nonlinear distortion over the recovered symbols. The performance of the novel structure is evaluated with simulations giving power efficiency improvements in the order of 30 %. Also simulation results shows the robustness of the cancellation technique at channel estimation accuracy.

# 1. INTRODUCTION

High bandwidth efficiency and multipath mitigation capabilities provided by Space-Division-Multiple-Access (SDMA)-OFDM systems position it as an excellent candidate for future WLAN implementations. SDMA is an interesting option that can help to increase spectral efficiency through bandwidth reuse in wireless communication systems. On the other hand, OFDM provides robustness to multipath fading channels.

The main drawback of OFDM signals is the high dynamic range associated to the peak to average power ratio (PAPR) [1]. This high dynamic range requires linear power amplifiers. Such linearity requirements translate into high back-off resulting in low power efficiency. A trade-off between distortion level and power efficiency must be considered in order to define the most useful region where the power amplifier must work. PAPR reduction techniques using mapping or coding, linear scaling, low PAPR implementations [2], allocation methods minimizing intermodulation products (IMP) [3] and beamforming design adding PAPR constrains [4], are some frequently used tools to combat nonlinearities in the transmitter. Predistortion techniques are also potential methods to obtain a power efficient amplifier with reduced out-band emission and low distortion [5].

One interesting approach is the application of multiuser detection and clipping noise mitigation techniques in the receiver [6]. In these techniques, the symbols are initially detected and decoded. Then, assuming that the power amplifier model is known, the distortion term is estimated. In this way the distortion can be removed from the received signal and a new symbol estimation is obtained. This technique can be applied iteratively to achieve the desired performance in two or three iterations.

In this article we propose a *Power Amplifier Nonlinearity Cancellation* (PANC) technique for multiuser SDMA with application in wireless systems. The proposed technique is inspired in [6], used previously in wired channels.

The paper is organized as follows. PANC for a single-user receiver is described in Section 2. Section 3 introduces multiuser OFDM, the multiuser version for the proposed technique and channel estimation concepts. Section 4 presents simulation results. Conclusions are drawn in Section 5.

## 2. PANC SINGLE-USER DETECTOR

An OFDM signal can be represented by

 $x(t) = \frac{1}{\sqrt{N_c}} \sum_{k=0}^{N_c-1} X_k(t) e^{(j\frac{2\pi}{N_c}kt)}$  where  $N_c$  is the number of carriers and  $X_k$  is the QAM symbol associated to carrier k. The multicarrier signal x(t) (note that indices [n,k] will be omitted when convenient) is sampled at the Nyquist rate x(n) and driven to the power amplifier. The output of a nonlinear power amplifier is formed by two terms

$$\boldsymbol{x}_g = K_L \, \boldsymbol{x} + \boldsymbol{d}[\boldsymbol{X}, g] \tag{1}$$

where x is the discrete input signal vector (time domain) and  $K_L$ is the gain of the linear part. The nonlinear distortion is given by d[X,g] which is a function of the input QAM vector X (the PAPR distribution of the input signal is a function of the QAM frequency domain symbols) and the power amplifier transfer function g. The value of  $K_L$  approaches to 1 for levels of clipping higher than 8 dB [5]. This means that the scaling of the constellation is insignificant. On the other hand, assuming high levels of clipping, the output of the power amplifier can be reduced to  $x_g = x + d[X,g]$ .

The amplified signal is transmitted over a wireless channel and the received signal after the FFT is

$$Y = H(X + D[X,g]) + N$$
<sup>(2)</sup>

where  $D[\mathbf{X}, g]$  is given by  $FFT[d[\mathbf{X}, g]]$ ,  $\mathbf{H}$  is the channel frequency response and  $\mathbf{N}$  is the AWGN noise. If the cyclic prefix is larger than the channel impulse response, which means zero ISI, it is possible to consider that each distortion sample affects only a single multicarrier symbol. Each carrier k can be written

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<sup>&</sup>lt;sup>1</sup>Notation: Bold letters are used for vectors and matrix representation. Uppercase denotes frequency domain signal and lowercase is used to represent time domain signal.



Fig. 1. Power Amplifier Nonlinearity Cancellation (PANC).

$$Y_k = H_k(X_k + D_k[\boldsymbol{X}, g]) + N_k \tag{3}$$

where  $D_k[\mathbf{X}, g]$ ) is the frequency domain distortion term associated to subcarrier k of the vector  $D[\mathbf{X}, g]$ ). Assuming an ISI free system, only one tap per carrier is needed for equalization. In that case FEQ tap is given by  $F_k = 1/H_k$ . The recovered signal is

$$\hat{X}_k = F_k Y_k = X_k + D_k [\boldsymbol{X}, g] + \frac{N_k}{H_k}$$
(4)

With this signal, the original vector can be recovered by using the Maximum likelihood (ML) receiver [6]. The complexity of the ML receiver is directly proportional to the number of users, the size of the antenna array and the constellation dimensions. As a consequence, the implementation of ML receivers is in general too complex from a practical point of view. One option to reduce receiver complexity is to consider the distortion term as AWGN and to assign the constellation considering the original noise plus the distortion term. Of course, this results in a reduced system capacity [6].

A better option is to compute the distortion term  $D_k[X, g]$  from the received vector Y under the assumption that the receiver knows the transmit nonlinear function. Using an initial estimate of vector  $X^{i-1}$  at iteration i - 1, it is possible to obtain  $D[X^{i-1}, g]$ . The distortion vector is then cancelled from the original received vector and a new estimate of  $X^i$  can be obtained, which is further used to *re-estimate* D[X, g]. This process is repeated iteratively. A block diagram is presented in Figure 1 and the iterative process is summarized in Table 1 [6].

Table 1. Basic PANC algorithm

For i=1 to number of iterations	
Demapping	$X_k^i = \left(\frac{Y_k}{H_k} - D_k[\boldsymbol{X}^{i-1}, g]\right)$
Temporal domain	$\boldsymbol{x}^i = IFFT[\boldsymbol{X}^i]$
Distortion Term	$d[oldsymbol{X}^i,g]=g(oldsymbol{x}^i)-oldsymbol{x}^i$
Distortion Term in frequency domain	$D[\mathbf{X}^i, g] = FFT[d[\mathbf{X}^i, g]]$
end	

#### 2.1. Power Amplifier models

Two simple power amplifier models will be discussed in this subsection. We represent the input signal in polar coordinates as  $x = \rho e^{j\phi}$ . The output of the PA is expressed as  $g(x) = M[\rho]e^{j(\phi+P[\rho])}$  where  $M[\rho]$  represents the AM/AM and  $P[\rho]$  the AM/PM conversion characteristics of the power amplifier. Several models have been developed for nonlinear power amplifiers. The most commonly used are: 1) Limiter

Transfer characteristics of a Limiter (clipping) amplifier are expressed as

$$M_L(\rho) = \begin{cases} \rho \ |\rho| < A\\ A \ |x| > A \end{cases}$$
(5)

where A is defined as the clipping level. The clipping ratio CR is the relation between the clipping level A to the root mean square value of the OFDM signal. For example, with a CR = 1, the signal is limited at the root mean square value of the OFDM signal. AM/PM conversion is not considered in this model.



#### 2) Solid-State Power Amplifier (SSPA)

The transfer characteristics is modeled by

$$M_{SSPA}(\rho) = \frac{\rho}{\left[1 + (\rho/A)^{2p}\right]^{1/2p}}$$
(6)

where the parameter p adjusts the smoothness of the transition from the linear region to the saturation region.

### 3. PANC IN MULTIUSER SDMA-OFDM SYSTEMS

#### 3.1. Multiuser SDMA-OFDM systems

An OFDM system with L single-antenna users and a P-antenna array receiver at the base station is considered in this analysis. The (complex vector) signal, Y[n, k], received by the P-antenna array (related to the k-th subcarrier of the n-th OFDM symbol) is formed by the superposition of the independently faded signals associated with the L users sharing the same space-frequency resource. The received signal is corrupted by additive white Gaussian noise (AWGN) at the array elements. This signal at subcarrier k can be written as

$$Y = HX + N \tag{7}$$

where  $\boldsymbol{Y} = [Y_1, \ldots, Y_p]^T$  is the received signal vector,  $\boldsymbol{X} = [X_1, \ldots, X_L]^T$  is the transmitted signal vector and  $\boldsymbol{N}$  is the array noise vector. The frequency domain channel transfer factor matrix  $\boldsymbol{H}$  is formed by the set of channel transfer factor vectors  $\boldsymbol{H}_l$  with  $l = 1, \ldots, L$  of the L users  $\boldsymbol{H} = [\boldsymbol{H}_1, \boldsymbol{H}_2, \ldots, \boldsymbol{H}_l]$ , each of which hosts the frequency domain channel transfer factors between the single transmitter antenna associated with a particular user l and the reception antenna elements  $p = 1, \ldots, P$ . A diagram of the OFDM-SDMA multiuser system is illustrated in Figure 2.

In a linear multiuser detector [7] an estimate  $\hat{X}$  of transmitted signal X of the L simultaneous users is generated by linearly combining the signals received by the P-antenna array elements with the aid of the weight matrix W. This can be written as  $\hat{X} = W^H Y$ . The LS error combiner attempts to recover X transmitted by the L different users, regardless of the signal quality quantified in terms of the signal to noise ratio SNR at the receiver antenna array. The weight matrix is given by  $W_{LS} = (H^H H)^{-1} H^H$  [7]. One drawback of the LS structure is noise amplification. Other option that minimize the noise amplification is the Minimum mean-squareerror (MMSE) combiner. The optimum weight matrix  $W_{MMSE}$ obtained minimizing the mean square estimation error can be written as  $W_{MMSE} = (HP_{SNR}H^H + \sigma_n^2 I)^{-1}HP_{SNR}$  where  $P_{SNR}$  is a diagonal matrix with the SNR of the different users at the receiver antenna array.

#### 3.2. PANC for multiuser systems

The approach presented in the previous section can be extended to the multiuser case using SDMA-OFDM. Assuming that channels are uncorrelated, the nonlinearity compensation technique can be applied to each user in the receiver after the combining process. We will focus on linear receiver structures, i.e., LS and MMSE.

PANC iterative decoder can be applied directly in a LS receiver. The weight matrix is calculated to remove the multiuser interference (MUI). The distortion cancellation process can be applied as in a single user system. In linear receivers, the estimation of signal transmitted by user *l* is given by  $\hat{X}_l = (Hcol^l \{(H^H H)^{-1}\})^H Y$ , where  $col^l \{.\}$  denotes the *l*-th column vector of the matrix product  $(H^H H)^{-1}$ .

The original transmitted constellation  $X_l$  is estimated from  $\hat{X}_l$ applying hard decoding, or other demapping if coding was included in the transmitter. This process is carried out in all active carriers. With the recovered symbols the time domain signal can be recovered by  $x = IFFT[X_l]$ . Assuming the nonlinear model of the power amplifier is known in the receiver, the distortion term can be calculated by  $d[X^i, g] = g(x^i) - x^i$ , and transformed to the frequency domain as  $D = FFT[g(X^i) - x^i]$ . After the cancellation iteration the transmitted constellation is re-estimated, starting again the decoding/distortion removing process. Following a similar procedure, PANC can be applied to the MMSE receiver resulting in a reduced noise amplification.

Because the technique is based on a decision-driven strategy [8], the application of PANC is useful only when the number of detection errors is relatively low. The PA model is assumed to be known at the base station. In the limiter model only one parameter must be known (clipping level), and for the SSPA model clipping level and the smoothness factor are required. These parameters can be transmitted in the initialization process before the initial channel estimation.

The introduction of the cancellation technique in a receiver must consider the level of out-band interference allowed in the communication system. The level of clipping and power back-off are usually selected in order to fulfill the out of band interference limits, maximizing power efficiency. As a general rule, the level of clipping is defined to maintain the interference level as large as possible adjusting the power back-off to verify bit error rate requirements.

#### 3.3. Channel estimation

The quasi-stationary characteristics of the WLAN channel enables the channel estimation to be carried out only once in the initialization.

Power amplifier nonlinearities affects severally the channel estimation process as shown in [9]. Training symbols must not be affected by nonlinearities in order to obtain a good channel estimation. One option, applied in HIPERLAN II, is to design the training symbols with low PAPR such that power amplifier nonlinearities have little impact over the channel estimation process.

Considering a multiuser SDMA-OFDM system with N nonzero data carriers (with spectral shaping purposes some carriers are not used) and U users, a maximum of T = N/U subcarriers can be allocated to each user. That means, all users transmit their training symbols over non-overlapping T subcarriers. The advantage of this method is that each user has active only a maximum of N/U subcarriers during the training period. This results in a reduced PAPR. Furthermore, the power amplifier will operate in the linear region generating a reduced distortion. If a subset T < N of X are known symbols (training sequence), the channel frequency response coefficients  $\hat{H}_c$  can be estimated over these carriers. The whole channel

**Table 2.**  $E_b/N_0$  level in function of back-off ( $BER = 10^{-4}$ )

	LA	IBO=0dB	IBO=1dB	IBO=2dB	IBO=3dB
$(E_b/N_0)_1$	11.8 dB	17 dB	14.1 dB	13.5 dB	12.3 dB
$(E_b/N_0)_2$	18 dB	22.3 dB	20.6 dB	19.8 dB	18.9 dB

frequency coefficients can be obtained using the initial estimates as

[10]

$$\tilde{\boldsymbol{H}} = \boldsymbol{Q} (\boldsymbol{Q} \boldsymbol{h}_T^H \boldsymbol{Q} \boldsymbol{h}_T)^{-1} \boldsymbol{Q} \boldsymbol{h}_T^H \hat{\boldsymbol{H}}_c$$
(8)

where  $Qh_T$  are the *T* columns of the DFT matrix Q associated with the initially estimated subcarriers and the  $L_h$  rows associated with the nonzero impulse response terms. It is important to mention that the number of subcarriers *T* must be chosen considering the level of nonlinearity and the desired channel estimation accuracy. With a linear amplifier, *T* is chosen equal to N/U (maximum available), in the case of nonlinear amplifier, the number *T* is reduced obtaining a reasonable accuracy in the channel estimation.

#### 4. SIMULATION RESULTS

The BER evaluation using simulations gives us a good insight about the performance of the system. BER is evaluated as a function of power amplifier model and back-off for a system with a nonlinear power amplifier (NLA) when PANC is applied. The results are compared with a system using a linear amplifier (LA) in order to have a BER lower bound. Also power efficiency  $\eta$  with and without PANC is evaluated.  $\eta$  is derived from the back-off value in a direct way. In our case, Class A power amplifier, typical for nonconstant envelope signals, is considered [11].

A multiuser SDMA-OFDM system is simulated with a fourelement antenna-array at the base station, and four mobiles with a single transmit antenna equipped with solid state power amplifier (SSPA) or Limiter PA. The OFDM system is implemented using 512 carrier QPSK with cyclic prefix of length 8. A four-path Rayleigh fading channel with independent propagation paths is implemented. The carrier frequency is 5 GHz with a bandwidth of 20 MHz and a mobile speed of 5 km/h. The paths are generated using Jake's Doppler spectrum. In addition, to evaluate a more realistic context related to channel parameters we consider simulation using not only a perfect channel knowledge but also a channel estimation technique with the PANC. A set of 16 subcarriers were used in the channel estimation process.

Table 2 resumes the simulation results for different values of IBO without PANC with Limiter PA with clipping ratio CR = 1 dB. The level of  $E_b/N_0$  for 1 and 2 active users in order to achieve BER=  $10^{-4}$  are indicated. Also levels of  $E_b/N_0$  are indicated for a linear amplifier. For IBO= -3dB, BER curves are very close to linear case (0.5 dB). If back-off is not applied, there is a degradation of more than 4 dB in  $E_b/N_0$  to achieve BER=  $10^{-4}$  for 1 active user case. The application of IBO= -3dB reduce the power efficiency to 18% ( See power efficiency vs back-off curve in Figure 5). Without back-off levels of 35% can be reached with Class A amplifier. The situation is even worse with larger constellation sizes because high levels of back-off must be used to obtain suitable BER.

Comparison using LA and NLA, with and without PANC, both with MMSE receiver is illustrated in Figure 3 without back-off with PA limiter model. In this case, the implementation of PANC gives more than 2.5 dB of  $E_b/N_0$  gain for 1 user and 2.2 dB for 2 active users. In order to obtain a BER of  $10^{-4}$  with PANC and without back-off (maximum power efficiency  $\eta = 40\%$ ), a  $E_b/N_0 = 15.5$  dB is needed. Without PANC, for equal level of  $E_b/N_0$  and BER, the amplifier must work with a back-off of -2 dB, that results in  $\eta = 25\%$ . That means that a significant relative power efficiency enhancement (more than 37 %) is obtained applying the proposed



**Fig. 3.** BER Limiter with *CR*=1 dB-MMSE receiver with PANC (CK-Perfect channel knowledge, CE-Channel estimation)



Fig. 4. BER SSPA with CR=1 dB-MMSE receiver with PANC (CK- Perfect channel knowledge , CE-Channel estimation)

PANC. Similar comparison but for SSPA model with smoothness factor p = 5 is illustrated in figure 4 giving a  $E_b/N_0$  gain of 2.1 dB and 2.4 dB for 1 and 2 users respectively.

A good measure that gives a balance between level of degradation and power efficiency is designated as Total Degradation (TD). TD is defined as  $TD_{dB} = (E_b/N_0)_{dB}^{NL} - (E_b/N_0)_{dB}^{L} + IBO_{dB}$ where  $(E_b/N_0)_{dB}^{NL}$  is the level of  $E_b/N_0$  required to obtain a BER = $10^{-4}$  in presence of power amplifier nonlinearities with a fixed IBO and  $(E_b/N_0)_{dB}^{L}$  expresses  $E_b/N_0$  required in case of linear amplifier. Figure 5 shows TD curves for 1 and 2 active users applying PANC. From these curves is possible to conclude that the best operation points are IBO = 2 dB ( $\eta = 26\%$ ) with TD=2.35 dB for 1 active user and IBO = 1 dB ( $\eta = 30\%$ ) with TD=2.6 dB for 2 active users without PANC and 0 dB ( $\eta = 35\%$ ) with TD=0.8 dB and TD=1.2 dB for 1 and 2 users respectivaly with PANC.

#### 5. CONCLUSIONS

A novel *Power Amplifier Nonlinearity Cancellation* technique for multiuser SDMA-OFDM is proposed. Simulation results show an excellent performance under severe nonlinearities (low back-off and low clipping levels). Levels of power efficiency improvements in the order of 30 % with a reduced SNR degradation are obtained.

An attractive feature of PANC is that the process is carried out



Fig. 5. Total Degradation as a function of back-off values

at the base station where more resources are available in comparison with other techniques that must be applied in the mobiles.

A PANC can be added at the nonlinear PIC (Parallel Interference Canceler) structure to improve the performance of the SDMA-OFDM system. In this implementation the iterative decoder is included after the demodulation process. The introduction of error correction techniques can improve the performance of the method because more reliable bits can be obtained. Both topics and an implementation complexity analysis are currently under research.

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