EMBEDDED SYNCHRONIZATION/PILOT SEQUENCE CREATION USING POCS

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ABSTRACT

In this paper we build on the orthogonal frequency division multiplexing (OFDM) peak-to-average power ratio (PAR) reduction work by Chen and Zhou. It has been demonstrated that pilot sequences that are constant modulus in the time domain can lead to an ensemble PAR-reduction across all data realizations. However, the problem of creating constant modulus sequences from arbitrary frequency domain power profiles has never been addressed. Often, it is desirable to have a some freedom of choice in how pilot and, possibly synchronization, energy is allocated in the frequency domain. In this paper we present a projection on to convex sets (POCS) method for creating low-PAR synchronization/pilot (S/P) sequences with arbitrary frequency-domain power profiles.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has become a popular modulation method in high-speed wireless networks. By partitioning a wideband fading channel into flat narrowband channels, OFDM is able to mitigate the detrimental effects of multipath fading using a simple one-tap equalizer. However, one drawback of OFDM is that OFDM signals exhibit large peak-to-average power ratios (PARs). Additionally, OFDM signals are very sensitive to timing offset (TO) and carrier frequency offset (CFO) [1].

One method for estimating the channel and combatting TO and CFO is to send a preamble sequence, $\{s_{pre}[n]\}_{n=0}^{N-1}$ prior to the information-bearing OFDM symbols. The receiver can perform a conjugate correlation of the received preamble and the expected preamble to extract an estimate for both the TO and the CFO with excellent probability of miss and false detection performance [2].

The received signal, after being corrupted by additive white Gaussian noise $\eta[n]$, a multipath channel $h[n]$, a TO n_o and a CFO ε, can be written as

$$w[n] = s_{pre}[n-n_o]e^{j2\pi n\varepsilon/N} * h[n] + \eta[n]. \tag{1}$$

Then the conjugate correlator output is

$$R[\tau] = \left(\sum_{l=0}^{N/2-1} s^*_{pre}[l]w[l-\tau]\right)\left(\sum_{l=N/2}^{N-1} s^*_{pre}[l]w[l-\tau]\right)^* \tag{2}$$

and the TO and CFO estimates are given, respectively, as $\hat{\tau} = \max_\tau |R[\tau]|$ and $\hat{\varepsilon} = \arg R[\hat{\tau}]$.

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After TO and CFO correction, the channel can be estimated by dividing the frequency-domain version of the received signal, $W[k]$ by the frequency-domain version of the preamble, $S_{pre}[k]$. That is, $\hat{H}[k] = W[k]/S_{pre}[k]$.

In [3] it was suggested that embedded pilots can improve channel tracking performance in time-varying channels. This idea was furthered in [4] where it was shown that it is possible to forego the preamble sequence completely in favor of an embedded synchronization/pilot (S/P) sequence that is not completely orthogonal to the OFDM data. The authors of [4] argued that by linearly combining the S/P subsequence with the information-bearing OFDM sequence, a spectral efficiency improvement could be realized over the preamble synchronization approach.

In that model, the OFDM baseband frequency-domain data sequence is labelled $X[k]$, which is linearly combined with the S/P sequence, $S[k]$, so that the proportion of signal power in $S[k]$ is

$$\rho = \frac{\sum_{k=0}^{N-1}|S[k]|^2}{\sum_{k=0}^{N-1}|S[k]+X[k]|^2}. \tag{3}$$

If $S[k]$ and $X[k]$ have equal power, then the combined signal, $Y[k]$, is

$$Y[k] = \sqrt{1-\rho}X[k] + \sqrt{\rho}S[k]. \tag{4}$$

The discrete-time version of $Y[k]$ is

$$y[n] = \frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} Y[k]\, e^{j\frac{2\pi kn}{N}}, \qquad 0 \le n \le N-1. \tag{5}$$

In a practical system, a cyclic prefix is appended to $y[n]$, $y[n]$ is windowed, filtered, converted to an analog signal and up converted to the passband prior to transmission. In this paper we are interested in peak-to-average power ratio (PAR). Accordingly, we can ignore the cyclic prefix as it does not contribute to the PAR. Also, the up conversion is known to increase the PAR by a constant of 3dB [5], so it too can be ignored in PAR considerations. Furthermore, the PAR of a signal before and after windowing, filtering and analog conversion is highly correlated, which means the affect on PAR of these processes is minimal [5]. Despite being beyond the scope of this paper, it may be of interest to extend this work by taking windowing, filtering and analog conversion effects into account.

2. S/P SEQUENCES

There are several considerations in designing S/P sequences. First, the authors of [6] were able to prove that, in order to minimize BER, the pilot tones should be equally spaced in the frequency domain and each assigned equal power. Second, for embedded synchronization applications, the authors of [4] relying partially on the work from [2] pointed out that synchronization sequences should have good

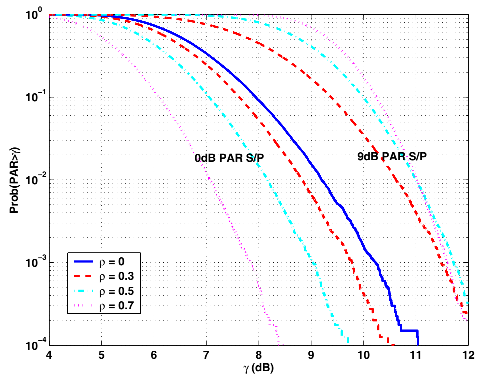

Fig. 1. CCDFs of ODFM with a constant modulus embedded S/P sequence for different embedding factors and the CCDFs for a 9dB PAR embedded S/P sequence for different embedding factors.

conjugate correlation properties. As illustrated in [2], statistically independent PN-preamble sequences have excellent conjugate correlation properties. Third, in most OFDM applications, the S/P sequence will need to meet a spectral mask that includes null edge subcarriers. Fourth, in multiuser or multi-antenna systems it is desirable to have a large number of S/P sequences to choose from so that different users or antennaes can be distinguished. Finally, as we will explain in the next paragraph, the S/P sequences should have a low time-domain PAR.

Recall that $y[n]$ is a linear combination of $x[n]$ and $s[n]$, where $x[n]$ and $s[n]$ are the discrete time-domain representations of $X[k]$ and $S[k]$. Note that according to the central limit theorem, $x[n]$ is approximately complex-Gaussian distributed. We would like to create $s[n]$ so that the ensemble PAR of $y[n]$, across all realizations of $x[n]$, is minimum. The authors of [7] were able to experimentally show that a constant modulus $s[n]$, when linearly combined with $x[n]$, creates a $y[n]$ with a significantly lower PAR than when using a time-domain impulse train $s[n]$. In Figure 1 the CCDF of the PAR for two different embedding sequences with various embedding factors is plotted. The lower-PAR set of CCDF curves correspond to a S/P sequence that is constant modulus, while the higher-PAR curves correspond to a time-domain impulse train S/P sequence with a 9dB PAR. From the plot, it is obvious that the constant-modulus curves produce a significant PAR reduction.

In [8], the authors showed that chirp time domain sequences of the form $s[n] = e^{j\pi 2^v k^2/N}$ are equally-spaced (with spacing v), equally-power pilot sequences in the frequency domain. Here we would like to extend that result to more arbitrary frequency-domain sequences. Specifically, we are interested in being able to create $S[k]$ sequences that have null edge subcarriers, arbitrarily placed pilots, and, possibly, non-zero entries at the rest of the subcarriers as illustrated in Figure 2. Additionally, the time-domain versions of $S[k]$ should be low PAR and have excellent conjugate correlation properties.

Stated mathematically, the problem is

$$\begin{aligned} &\text{minimize} \quad \text{PAR}\{s[n]\}, \\ &\text{subject to } |S[k]|^2 = P[k], \end{aligned}$$

where $P[k]$ is the desired frequency-domain power profile. Notice that both $s[n]$ and $S[k]$ are N-length sequences, but $P[k]$ may contain zeros (at the null subcarriers).

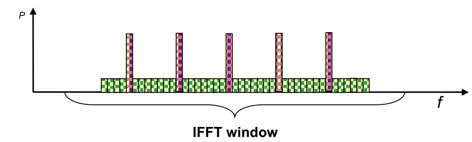

Fig. 2. One possible example of a frequency-domain power profile.

The problem of finding the minimum-PAR sequence for a certain power profile can be solved using quadratically constrained quadratic programming (QCQP). However, using QCQP will only give one sequence per power profile without any guarantee of good self-correlation characteristics, whereas we want to be able to create many sequences with good self- and cross-correlation characteristics.

The QCQP problem can be solved suboptimally by using a method known as projection on to convex sets (POCS). POCS is an active field with many applications like filter design, array signal processing, electron microscopy, speckle interferometry, topography, spectral estimation and neural networks [9]. Because POCS is an iterative search method, it may converge to a solution that is not the global minimum. However, it has the nice property that independent initial conditions cause the POCS algorithm to converge to independent solutions. We will exploit this property to allow for the creation of many independent near-optimal sequences.

For the problem at hand, we have two convex sets. The first is the power profile in the frequency domain and the second is the power profile in the time domain (i.e. low PAR). The 'projection' between the sets can easily be performed with the Fourier and inverse Fourier transforms. The procedure is listed below

1. Initialize the algorithm with a random-phase constant-modulus sequence $\bar{s}[n] = e^{j\theta_n}$, where $\theta_n \sim U[0, 2\pi)$.
2. Generate $\hat{S}[k] = \text{FFT}\{\bar{s}[n]\}$.
3. Using the phase of $\hat{S}[k]$, generate $\bar{S}[k] = \sqrt{P[k]}e^{j\angle \hat{S}[k]}$.
4. Generate $\hat{s}[n] = \text{IFFT}\{\bar{S}[k]\}$.
5. If the maximum number of iterations has been reached, exit the loop; else, go to step two.

After the algorithm finishes, there are two pairs of time/frequency sequences to choose from, $\{\bar{s}[n], \hat{S}[k]\}$ and $\{\hat{s}[n], \bar{S}[k]\}$. For the first pair, $\{\bar{s}[n], \hat{S}[k]\}$, the time-domain sequence has zero decibels of PAR, while the the frequency-domain sequence is not an exact match the desired power-profile. Instead, only the approximation, $|\hat{S}[k]|^2 \approx P[k]$, holds. Conversely, the pair $\{\hat{s}[n], \bar{S}[k]\}$ exactly meets the frequency-domain power-profile specification, but $\hat{s}[n]$ has a non-zero PAR. The question is, which pair should be used? Generally, it is desirable to choose $\{\hat{s}[n], \bar{S}[k]\}$ so that the frequency-domain power profile is perfectly matched to the desired profile. This choice ensures that the null subcarriers are actually nulls and that the pilots all have the same power.

3. CONVERGENCE PROPERTIES

As was pointed out in the last section, the properties of a S/P sequence depend on the initial conditions of the POCS algorithm. In practice, the S/P sequences are found offline so the speed of convergence is not a limiting factor. We are interested in finding the number of trials as well as the number of different initial conditions that must be used to find a suitably low-PAR sequence.

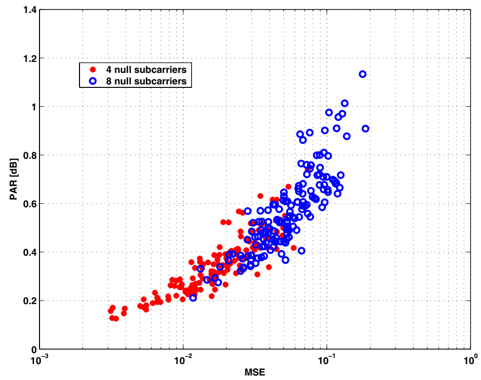

Fig. 3. Plot of MSE versus PAR of 150 different initial conditions after 600 iterations.

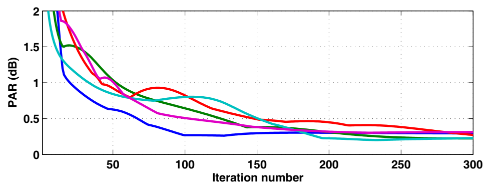

Fig. 4. Plot of PAR for 5 low-PAR initial conditions through 300 iterations.

Figure 3 is a scatter plot of the PAR and MSE after 600 POCS iterations for 150 different initial conditions. The plot parameters are $N = 64$, there are 7 pilots and the amount of power in the non-pilot subcarriers is 20 percent of the power in the pilot subcarriers. In this plot, the MSE refers to the mean squared difference between $\hat{S}[k]$ and $\sqrt{P[k]}$ and the PAR is the PAR of $\hat{s}[n]$. As was expected, the PAR and MSE are highly dependent on the initial conditions of the POCS algorithm.

Figure 4 is a plot of the convergence characteristics of the PAR for the five lowest-PAR sequences out of 150 different initial conditions. The plot parameters are $N = 64$, there are 7 pilots, a total of 4 null edge subcarriers, and the amount of power in the non-pilot subcarriers is 20 percent of the power in the pilot subcarriers. From the plot, we can see that the PAR is well below 1 dB for almost any initial conditions after 300 iterations.

4. CORRELATION PROPERTIES

The correlation properties of the S/P sequences are very important for both TO and CFO estimation. In a multiuser system it may also be possible to use different S/P sequences for different users or for different antennas in a multi-antenna system. For the multi S/P sequence applications, it is be possible to distinguish users or antenna in addition to estimation the TO and CFO. We are also exploring the possibility of using a selected mapping [10] approach with different S/P sequences to realize large PAR reductions. However, in order for any this to be feasible, the S/P sequences must have good conjugate correlation properties. In the present context, 'good' means that the non-peak self conjugate correlation values have to be small so that the TO can be correctly estimated. Additionally, for multi S/P sequence application it is also desirable to have all conjugate cross correlation values between different S/P sequences be small.

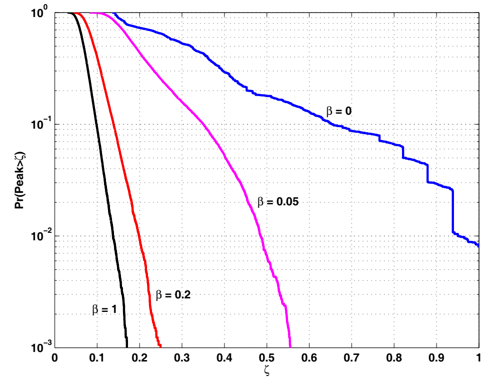

Fig. 5. CCDF of the maximum non-peak conjugate correlation outputs for different amounts of non-pilot energy.

Figure 5 shows the CCDF of the non-peak maximum values of the conjugate correlation output with $N = 64$, 8 edge null subcarriers and 7 pilot subcarriers. Here we define $\beta = \frac{P[k_s]}{P[k_p]}$, where $P[k_s]$ is the power in the sync subcarriers that will be superimposed onto the information-bearing OFDM subcarriers and $P[k_p]$ is the power in the pilot subcarriers. This plot illustrates how sensitive the conjugate correlation operation is to β. As beta increases, the maximum non-peak conjugate correlation decreases. We note that, however, a larger β will increase the interference level that the S/P sequence contributes to the data-bearing OFDM subcarriers. More precisely, the ratio of the data-conveying energy to the noise energy is

$$SNR_e = \frac{(1-\rho)\sigma_Y^2}{\sigma_\eta^2 + \frac{\beta K_s}{K_p+\beta K_s}\rho\sigma_Y^2}, \tag{6}$$

assuming that the S/P signal is Gausssian and independent of the data signal, where σ_Y^2 is the power of the transmitted signal, σ_η^2 is the noise power, K_s is the number of sync subcarriers and K_p is the number of pilot subcarriers.

In choosing β and ρ, several parameters come into consideration. Among others the channel response, the maximum possible CFO, the amount of noise in the channel, and the size of N all affect the choice of β and ρ. Determining β and ρ in terms of those parameters is beyond the scope of this paper. However, in Figure 6 we illustrate the effect of β and ρ on the maximum non-peak conjugate correlation output. The plot is of the CCDF of the maximum peak conjugate correlation output for different combinations of β and ρ with $N = 64$, 8 edge null subcarriers and 7 pilot subcarriers. The value of interest is $\zeta = 1$ as this is where the height of a non-peak output and a peak output are equal and thus indistinguishable. The distinguishability of these two cases is necessary for signal detection, so the probability level where $\zeta \geq 1$ is the probability that a S/P sequence for the given parameters will fail on average. For example, from Figure 6, we can see that for $\beta = 0.05$ and $\rho = 0.2$ the CCDF curve crosses $\zeta = 1$ with probability 0.005. So we can count on a S/P sequence created by the POCS method with those parameters to not be dependable for synchronization purposes five times in 1,000.

As was pointed out before, we are interested in searching through many different POCS initial conditions to find a low-PAR S/P sequence or, possibility, a set of low-PAR S/P sequences. But, in addition to finding low-PAR sequences, it is also necessary to find sequences with low non-peak correlation outputs so that the S/P se-

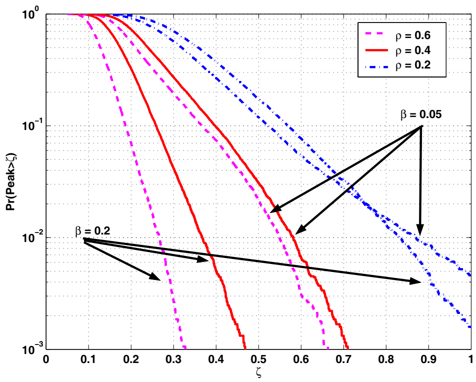

Fig. 6. CCDF of the normalized maximum non-peak conjugate correlation outputs for various combinations of β and ρ.

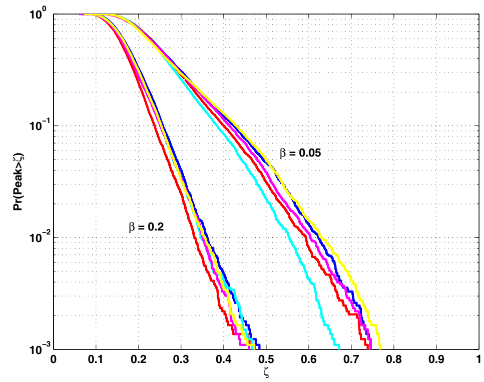

Fig. 7. CCDF of the normalized maximum non-peak conjugate correlation outputs for the five lowest-PAR S/P sequences.

quence can be used in synchronization. Figure 7 is a plot of the normalized CCDF of the conjugate correlation output for the five lowest-PAR S/P sequences found out of 150 initial conditions with $N = 64$, 8 edge null subcarriers and 7 pilot subcarriers. To make the plot, 10,000 data sequences were created and combined in proportion with ρ for each of the five S/P sequences tested. The plot shows that there are not any drastic differences in conjugate correlation performance from one S/P sequence to the next for $\beta = 0.2$. On the other hand, the for $\beta = 0.05$, there are marked differences between the different S/P sequences. So depending on β and the application, correlation performance may or may not be worth examining when selecting S/P sequences.

Finally, Figure 8 is a plot of the probability of of a false detection (P_f) versus the probability of a miss detection (P_m) for two minimum-PAR S/P sequences generated from 300 iterations and 150 initial conditions, one with $\beta = 1$ and the other with $\beta = 1/7$ as well as a complex m-sequence created using a feedback shift register. For all three sequences $N = 64$, $\rho = 1$ and the $SNR = -7$dB.

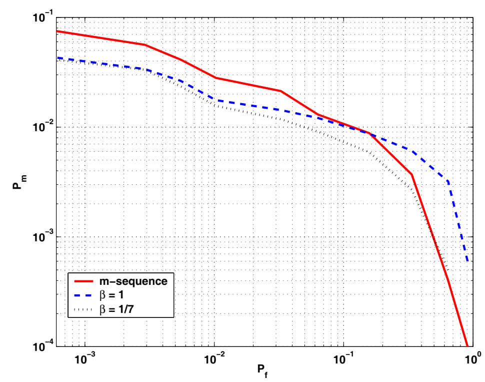

Fig. 8. P_f versus P_m for two POCS-created S/P sequences compared with an m-sequence.

5. CONCLUSIONS

In this paper we have presented a novel method for creating low-PAR OFDM sychronization/pilot sequences with arbitrary frequency-domain power profiles. By examining the convergence and correlation properties of the S/P sequences created by our scheme we have shown that only several hundred initial conditions and POCS iterations are necessary to find excellent S/P sequences. Through simulation, we were also able to show that for reasonable embedding factors, POCS-created S/P sequences can be used for embedded synchronization of OFDM signals that have P_m/P_f detection rates comparable to those of m-sequences. In future work, we will show how POCS-created S/P sequences can be integrated into a selected mapping PAR-reduction scheme to create even lower PAR embedded synchronization OFDM signals. [1]

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