PEAK REDUCTION OF MULTI-CARRIER SYSTEMS BY CONTROLLED SPECTRAL OUTGROWTH

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ABSTRACT

One of the major drawbacks with multi-carrier (MC) modulations is the large Peak-to-Average Power Ratio (PAPR) of the transmitted signal which significantly reduces the power amplifier efficiency and the system performance. Several PAPR reducing methods which do not decrease the system performance have been proposed. In this paper, we present a new low complexity PAPR reduction technique based on the controlled spectral outgrowth (CSO) of the nearby subcarriers which neither requires the transmission of additional information nor reduces the system performance and is independent of the mapping scheme. The proposed method can be easily combined with most of the methods in the literature to provide a further peak reduction in any MC or MC Spread Spectrum (MC-SS) system. An analog PAPR reduction about 2.5 dB at 10^{-4} symbol clip probability and a power increase about 0.12 dB are obtained for a 256-subcarrier OFDM transmission.

1. INTRODUCTION

MC modulations have become popular in most of the wireless communication systems due to their high spectral efficiency and robustness against multipath fading channels. One of the major drawbacks with MC modulations is that when the sinusoidal signals of the N subcarriers add constructively the peak envelope power is as much as N times the mean envelope power. The ratio between the instantaneous power of these peaks and the average power of the signal (PAPR) is too large and requires the use of power amplifiers (PA) with a large linear margin working at a back-off approximately equal to the PAPR for distortionless transmissions, additional back-off is necessary for nonlinear PA. PAPR is specially critical for very large distance HF communications where the received signal-to-noise ratio is very low and the interference level is usually higher than the level of the desired signal [1]. In this environment, increasing the average transmitted power by reducing the peak values and setting the PA to lower back-offs becomes a design key, particularly when expensive linear high power amplifiers are used.

Several PAPR reducing methods can be found in the literature. Recently, some methods based on tone reservation and non-bijective constellations have shown low complexity with no performance degradation and no transmission of additional information. Tone reservation methods [2]-[3] use reserved tones to introduce a peak canceling signal that reduces the peak power of the MC symbols. Since extra tones are added, symbol power increments up to 1 dB may be found for a 5% bandwidth increase [2]. Non-bijective constellations based methods reduce the peak power by modifying the constellation points. Each symbol is not mapped to a certain point but to a set of constellation points. A proper definition of those can assure no performance degradation. Two methods of this type can be found, tone injection [2] and active constellation extension [4].

In this paper a new PAPR reduction technique is presented which is based on the controlled spectral outgrowth of the nearby subcarriers. The peak reduction is achieved by applying a more severe clipping to the one required to achieve the target PAPR and subsequently restoring the original data subcarriers and limiting the spectral outgrowth. The whole procedure is iterated to achieve the maximum peak reduction. A first approach to this idea was introduced in [5] by the authors of this paper.

2. DESCRIPTION OF THE CSO METHOD

Let $\mathbf{A}$ be a length-M OFDM symbol vector, then the L times oversampled symbol vector $\mathbf{X}$ is created by inserting $(L-1)M$ zeros at the $\lfloor M/2 \rfloor$-th position of $\mathbf{A}$; $\forall\, L \geq 1$ such that $N = LM \in \mathbb{N}$. By applying the IFFT and clipping the resulting signal $\mathbf{x}$, the time domain clipped symbol is obtained,

$$\mathbf{x}_c[n] = \begin{cases} \mathbf{x}[n] & \text{if } \alpha_n \leq A \\ Ae^{j\theta_n} & \text{otherwise} \end{cases} \quad (1)$$

where A is the clipping threshold and $\mathbf{x}[n] = \alpha_n e^{j\theta_n}$. From $\mathbf{x}$ and $\mathbf{x}_c$ we can compute the peak reducing signal as $\mathbf{c} = \mathbf{x}_c - \mathbf{x}$

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and transform it to the frequency domain by means of the FFT to finally get $\mathbf{C}$. Let $\mathcal{S}_C$ be the set of subcarrier indexes of the nearby subcarriers

$$\begin{aligned}\mathcal{S}_C &= (\lfloor M/2 \rfloor, \cdots, \lfloor M/2 \rfloor + \lfloor \eta M/2 \rfloor - 1, \\ &\quad N - \lceil M/2 \rceil - \lceil \eta M/2 \rceil, \cdots, N - \lceil M/2 \rceil - 1)\end{aligned}$$

where η is the spectral outgrowth factor satisfying $\eta M \in \mathbb{N}$, so an integer number of out-of-band subcarriers are added. Furthermore $\eta \leq \frac{N}{M} - 1$ to assure Nyquist sample rate. Next we define the peak reducing signal with controlled spectral outgrowth in the frequency domain as

$$\widetilde{\mathbf{C}}[k] = \begin{cases} \mathbf{C}[k] & \forall k \in \mathcal{S}_C \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

and apply an IFFT to obtain its time domain representation $\widetilde{\mathbf{c}}$. Finally the peak reduced OFDM symbol can be computed as

$$\widetilde{\mathbf{x}} = \mathbf{x} + \widetilde{\mathbf{c}} \tag{3}$$

Let $\mathcal{S}_X$ be the indexes of the nonzero subcarriers in $\mathbf{X}$, since $\mathcal{S}_X$ and $\mathcal{S}_C$ are disjoint sets and all subcarriers are orthogonal no loss of performance is introduced, i.e. bit error rate (BER) is not increased.

2.1. Iterative implementation

The previously described method can be applied iteratively to achieve larger peak reductions. The proposed algorithm is as follows:

1. Compute $\mathbf{X}$ by oversampling $\mathbf{A}$ and apply an IFFT to get $\mathbf{x}$. Define $\widetilde{\mathbf{x}}^{(0)} = \mathbf{x}$ and set $i = 0$.
2. Clip $\widetilde{\mathbf{x}}^{(i)}$ to get $\mathbf{x}_c^{(i)}$.
3. Compute $\mathbf{c}^{(i)} = \mathbf{x}_c^{(i)} - \widetilde{\mathbf{x}}^{(i)}$ and $\mathbf{C}^{(i)} = \mathrm{FFT}(\mathbf{c}^{(i)})$.
4. Obtain $\widetilde{\mathbf{C}}^{(i)}$ as described in (2). If $\widetilde{\mathbf{C}}^{(i)} = \mathbf{0}$ return $\widetilde{\mathbf{x}}^{(i)}$ and terminate. Otherwise, apply an IFFT to get $\widetilde{\mathbf{c}}^{(i)}$.
5. Compute $\widetilde{\mathbf{x}}^{(i+1)} = \widetilde{\mathbf{x}}^{(i)} + \widetilde{\mathbf{c}}^{(i)}$.
6. Increment i, go to step 2 if the maximum number of iterations has not been reached and the target PAPR not been achieved. Otherwise, return $\widetilde{\mathbf{x}}^{(i)}$ and terminate.

Since $\widetilde{\mathbf{c}}^{(i)}$ is obtained by filtering $\mathbf{c}^{(i)}$ in the frequency domain as stated in (2), it can not be assured that $|\widetilde{\mathbf{x}}^{(i)} + \widetilde{\mathbf{c}}^{(i)}|_\infty \leq A$. Nevertheless, $\widetilde{\mathbf{c}}^{(i)}$ can be rescaled by a scaling factor μ in order to assure that $|\widetilde{\mathbf{x}}^{(i)} + \mu\widetilde{\mathbf{c}}^{(i)}|_\infty$ is minimized. This will result in a much faster approach to the maximum achievable PAPR reduction, i.e. fewer iterations will be required. The optimum scaling factor at each iteration is

$$\mu_{opt}^{(i)} = \arg\min_{\mu} ||\widetilde{\mathbf{x}}^{(i)} + \mu\widetilde{\mathbf{c}}^{(i)}||_\infty \tag{4}$$

Finding this optimum scaling factor requires a large computing complexity since methods such as successive approximation have to be used. A reduced complexity approach known as smart gradient-project (SGP) is proposed in [4] by assuming that at the optimal μ two different samples will have the same magnitude. One of those is considered to be the largest magnitude sample in $\widetilde{\mathbf{x}}^{(i)}$, denoted as n_{max}, and the other is obtained by finding the minimum μ that balances the magnitude of the tested sample with the magnitude of n_{max}. This minimum μ is the reduced complexity approach of μ_{opt}.

2.2. Parameters choice

Three parameters describe the behavior of the CSO method: the clipping threshold, the number of iterations and the spectral outgrowth (η). Optimum parameters must be determined by simulations and depend on the number of subcarriers and the used mapping scheme (assuming OFDM). In this paper, a 256-subcarrier OFDM system employing QPSK, 16-QAM and 64-QAM has been evaluated. Maximum peak reductions have been achieved with $\eta = 0.375$, four iterations (using the SGP approach) and a clipping threshold of 4.8 dB, 5 dB and 5.3 dB for QPSK, 16-QAM and 64-QAM, respectively. However, very close results to the maximum achievable can be obtained with $\eta = 0.25$ and 3 iterations.

3. RESULTS

The PAPR reduction is evaluated in terms of its complementary cumulative density function (CCDF), that is, the probability that the PAPR exceeds a given threshold. The PAPR is computed as stated in (5) assuming the nomenclature from Section 2, this assures that the peak reduction of the OFDM symbols does not affect its average power computation.

$$\mathrm{PAPR}(\widetilde{\mathbf{x}}) = \frac{||\widetilde{\mathbf{x}}||_\infty^2}{E[||\mathbf{x}||^2]/N} \tag{5}$$

A total oversampling $L = 8$ has been used to better approximate the analog PAPR, which is of great interest since it represents the PAPR of the transmitted signal at the input of the PA and, hence, determines its input back-off if distortionless transmissions are desired. In order to reduce the computational complexity an oversampling of $L = 4$ has been used in the CSO method and the obtained peak reduced signal has subsequently been oversampled to $L = 8$. Lower oversampling factors in the CSO method computation lead to poorer peak power reductions. Simulation results have been obtained by using $5 \cdot 10^6$ randomly generated OFDM symbols.

3.1. Effect of the number of iterations

Figure 1 shows the peak power reduction results of the CSO method at different iterations over an OFDM system with 256 subcarriers and QPSK mapping. A peak reduction of 2.31 dB is obtained at a symbol clip probability of 10^{-4}, the maximum peak reduction is not obtained until the 9-th iteration. Figure 2 shows the PAPR reduction of the CSO method at different iterations when the peak reducing signal is scaled by the SGP approach of μ. There are two advantages with respect to the non-SGP implementation: larger peak reductions are achieved and only 4 iterations are needed to achieve the maximum peak reduction. Furthermore, it is also interesting to notice that a 2.2 dB PAPR reduction at 10^{-4} symbol clip probability is achieved with just one iteration.

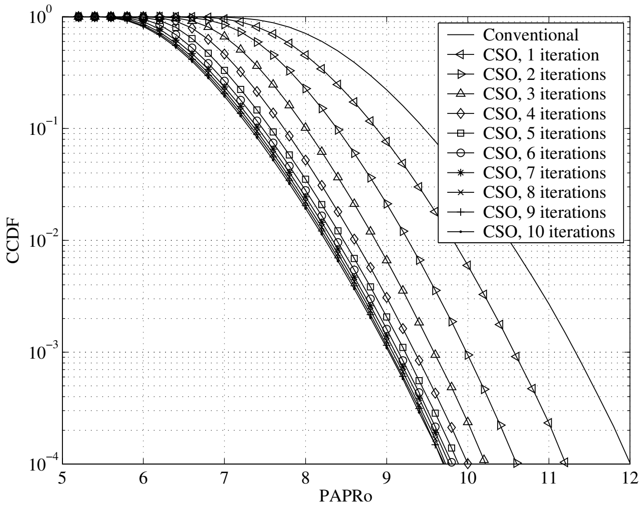

Fig. 1. Iterations effect on the PAPR of the CSO method applied to a QPSK-mapped 256-subcarrier OFDM system.

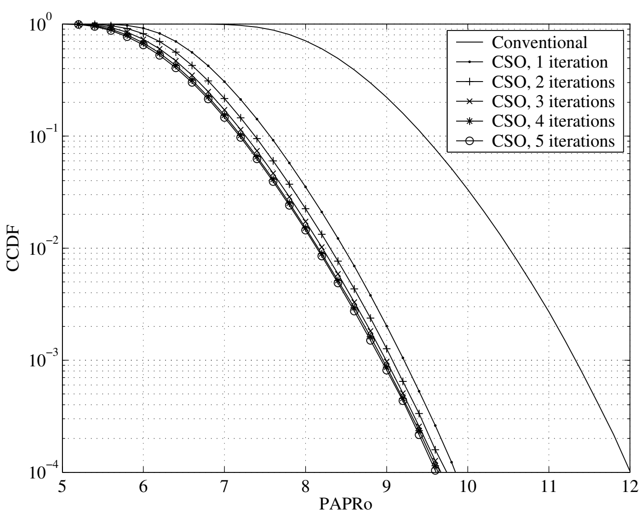

Fig. 2. Iterations effect on the PAPR of the CSO-SGP method applied to a QPSK-mapped 256-subcarrier OFDM system.

3.2. Effect of the spectral outgrowth

The effect of η over the PAPR is shown in Figure 3. As it might be expected, the PAPR reduction increases as η increases. However, there is no need to increase η beyond 0.375 since the optimum reduction is achieved for that value. In fact, $\eta = 0.25$ almost reaches this maximum peak reduction.

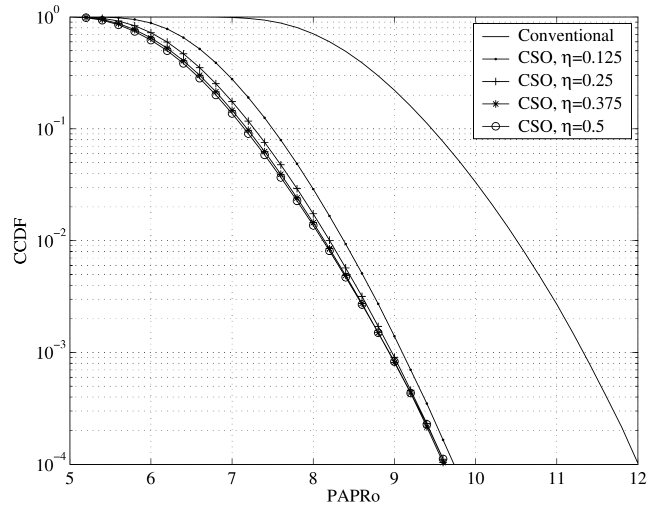

Fig. 3. η effect on the PAPR results of the CSO method applied to a QPSK-mapped 256-subcarrier OFDM system.

Figure 4 shows the power spectral density (PSD) of a CSO based peak reduced OFDM system with 256-subcarrier employing QPSK. Several simulations are shown at different values of η, for $\eta = 0.375$ the PSD of the described OFDM system with 16-QAM and 64-QAM mapping is also shown. Two statements shall be made: (i) the larger the spectral outgrowth is, the smaller the amplitude of the out-of-band subcarriers will be, and (ii) lower power increase occur when larger constellation sizes are used (assuming the same spectral outgrowth and peak power reduction). It is interesting to notice that low values of η in CSO lead to an addition of extra tones with an average power close to the one of the data tones. This suggests that the proposed method could be used to compute the added tones in the tone reservation method.

Power increase (dB)	QPSK	16-QAM	64-QAM
η = 0.125	0.238	0.227	0.202
η = 0.25	0.172	0.159	0.133
η = 0.375	0.136	0.124	0.101
η = 0.5	0.111	0.101	0.080

Table 1. Power increase in dB of a 256-subcarrier OFDM system due to the peak power reduction by CSO.

Table 1 shows the power increase of the transmitted signal when the peak power is reduced by CSO using several values

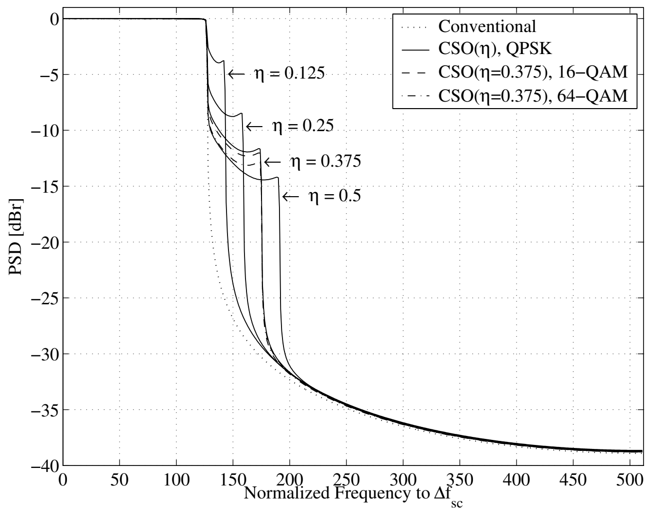

Fig. 4. PSD results of the CSO method applied to a 256-subcarrier OFDM system.

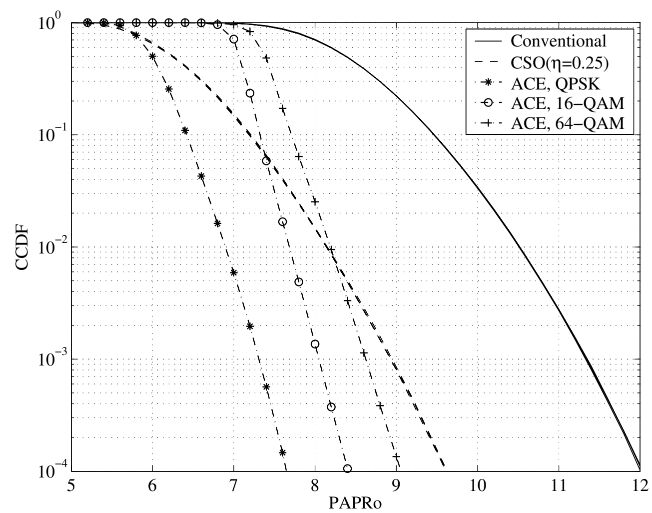

Fig. 5. PAPR results of the CSO and ACE methods applied to a 256-subcarrier OFDM system.

of η and different mapping schemes. It can be appreciated that a lower increase of the signal power is obtained at larger values of η. Therefore, the choice of the appropriate η will strongly depend on the wireless system scenario.

3.3. Effect of the mapping scheme in OFDM

Figure 5 shows the CCDF of the PAPR in a 256-subcarrier OFDM system employing no PAPR reduction, CSO method ($\eta = 0.25$ and the clipping thresholds in Section 2.2) and active constellation extension (ACE) method with the parameters defined in [4]. QPSK, 16-QAM and 64-QAM mapping schemes are shown. As it can be appreciated the peak power reduction of the CSO method is independent of the modulation scheme. ACE achieves better peak reductions if large PAPR can be accepted (over 5.8 dB, 7.38 dB and 8.24 dB in QPSK, 16-QAM and 64-QAM respectively), CSO offers better results below this values. It must be taken into account that the PAPR reduction capabilities of the ACE method will decrease if some of the subcarriers are used as pilots (no modification of their amplitude and phase is allowed). Furthermore, the PAPR reduction by ACE depends on the modulation scheme and the symbol power increase is greater than in CSO. ACE's power increase is 0.907 dB in QPSK, 0.472 dB in 16-QAM and 0.428 dB in 64-QAM.

4. CONCLUSIONS

In this paper a new PAPR reduction technique based on the controlled spectral outgrowth of the nearby subcarriers has been presented. Peak reductions similar to the ones achieved by other methods in the literature are achieved with low complexity, no performance degradation, no transmission of additional information and by scarcely decreasing the power efficiency. An analog PAPR reduction about 2.5 dB at 10^{-4} symbol clip probability and a power increase about 0.12 dB are obtained for a 256-subcarrier OFDM transmission. The proposed method can be easily combined with most of the methods in the literature to provide further peak power reductions in any MC or MC-SS system. Appropriate parameters should be chosen to fit specific scenarios.

5. REFERENCES

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