PEAK-TO-AVERAGE POWER REDUCTION OF OFDM SIGNALS USING ADAPTIVE DIGITAL FILTER

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ABSTRACT

The high peak-to-average power ratio (PAPR) of the transmitted signal is a major drawback in an OFDM system. In this paper, we propose a novel method to reduce the PAPR of the OFDM signals. The presented method adaptively filters the modulated data (M-PSK and QAM) to obtain OFDM symbols with a minimized PAPR. With optimally selected digital filter coefficients, the PAPR of the OFDM symbols can be minimized. The PAPR reduction of OFDM signals varies with the order of the digital filter. In a receiver, the modulated data may be recovered by a reversed procedure. Simulation study demonstrates that our method is more superior than the PCR method for the PAPR reduction.

1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) technique has been included in IEEE 802.11a and 802.16a standards for satisfying the high data rate requirements. It is a very effective technique to provide reliable data transmission over fading channels and multi-paths. A major problem of using the OFDM technique in a wireless communication system is that OFDM signal has a large dynamic range or high peak to average power ratio (PAPR), which requires a very large linear dynamic range for RF power amplifiers to avoid distortions and spectral spreading. Therefore, how to reduce the PAPR in an OFDM system has been becoming a research interest since the 1990's.

There are many methods to reduce the PAPR of the OFDM signal. The simplest method is to clip the high amplitude peaks [1], which leads to a significant distortion of the desired signal and an increase in the bit error rate. Other methods include coding [4, 5], selective mapping [3], redundant carriers and amplitude redisctribution [6, 7], and companding [2], etc. In a coding method, block codes are used to change the construction of the QPSK and M-PSK so that coded system has a lower PAPR than the un-coded system. However, the coding will reduce the bit rate and increase computational cost when the number of carriers increases. Selective mapping methods use a large set of phase rotations for the same information, and select the one with minimized PAPR. The potential problem of this method is the de-mapping in a receiver. Redundant carrier methods use additional carriers to reduce the PAPR. For example, G. Freiman [6] used two redundant tones to reduce the PAPR about 2.5dB in a BPSK-modulated OFDM system. Companding methods compress the dynamic range but increase the average signal power.

In this paper, we present a novel method that changes both amplitude and phase of the digital modulated data by an adaptive digital filter to minimize the peak-to-average power ratio of OFDM signal.

2. METHOD

2.1. PAPR of an OFDM signal

In the transmitter of an OFDM system, the binary information sequence is first encoded by a digital modulation constellation, such as BPSK, QPSK, and QAM, and then an IDFT operation is performed to convert the encoded data into the

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time-domain signals called OFDM symbols. Let $\mathbf{X}=[X_0,X_1,......X_{N_d-1}]\in C^{N_d}$ be M-ary PSK or QAM constellation encoded data and $x=[x_0,x_1,......x_{NL-1}]\in C^{NL}$ be the corresponding time-domain signal by taking the IDFT to $\boldsymbol{X}$ with L-times oversampling, where N_d is the number of the subcarriers which carry the data and N is the total number of the subcarriers. Then the PAPR of an OFDM symbol is defined as:

$$PAPR=\max_{i=0,1,...NL-1}\frac{\|x_i\|^2}{\frac{1}{NL}\|x\|^2}=\max_{i=0,1,...NL-1}\frac{\|x_i\|^2}{\frac{1}{N_d}\|X\|^2} \quad (1)$$

Equation (1) shows that the PAPR of an OFDM symbol highly depends on amplitude and phase of the encoded data. As an example, the PAPR of QPSK symbols with $N_d=48$ and $L=4$ using equation (1) is shown by the top line in figure 1. As shown in the figure, the dynamic range of the PAPR for the original QPSK constellation is from 4 to 11 dB, which is a wide range depending on data.

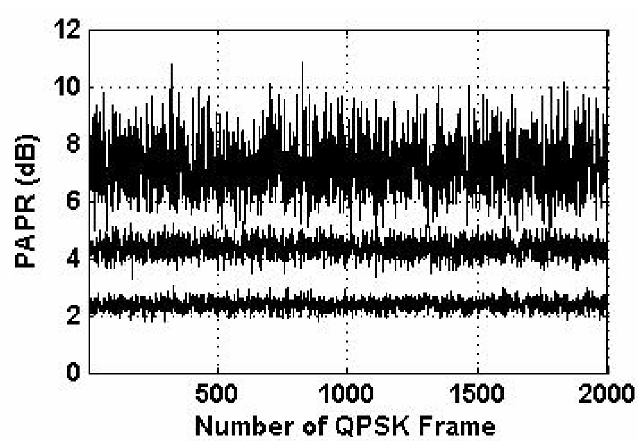

Figure 1 PAPR of the OFDM symbols for a QPSK constellation

2.2. PAPR reduction by adaptive FIR filter

Our approach to reduce the PAPR is to redistribute the amplitude and phase of the OFDM symbols for minimizing the PAPR. It is done by designing an adaptive digital filter for modulated data before the IDFT takes place. The filter coefficients are adaptively changed to minimize the PAPR. The filter coefficients are sent to a receiver by the free subcarriers that do not carry the data. For example, if we use a FIR filter as the adaptive digital filter, for an OFDM system with 64-point IFFT and 48 data subcarriers, the order of the FIR filter can be up to 16 with the 16 free subcarriers.

Let $\mathbf{X}=[X_0,X_1,......X_{N_d-1}]\in C^{N_d}$ be an M-ary PSK or QAM constellation and $A=[a_0,a_1,......a_{N_o}]^T\in C^{N_o+1}$ be the coefficients of the digital FIR filter, the output of the filter, $Y=[Y_0,Y_1,......Y_{N_d-1}]^T\in C^{N_d}$, can be represented as $Y=M_XA$, where M_X is a matrix with the dimensions of $N_d\times(N_o+1)$ given by

$$M_X=\begin{pmatrix} X_0 & 0 & ...... & 0 \\ X_1 & X_0 & ...... & 0 \\ & .... & & \\ X_{N_d-1} & X_{N_d-2} & ...... & X_{N_d-N_o-1} \end{pmatrix} \quad (2)$$

where N_o is the order of the digital FIR filter.

Before performing the IDFT to Y, we add A to $\boldsymbol{Y}$, and get a new vector $Y_1\in C^{N_d+N_o+1}$. The positions of $a_i\,(i=0,1,...N_o)$ in $\boldsymbol{Y}_1$ depend on the positions of the free subcarriers. Taking N-point IDFT with an oversampling length L to $\boldsymbol{Y}_1$, the resulted OFDM symbol is $Z=[Z_0,Z_1,......Z_{NL-1}]\in C^{NL}$, where Z_i is a function of $\boldsymbol{A}$ and $\boldsymbol{X}$. Therefore the PAPR of the OFDM symbols is a function of $\boldsymbol{A}$ and $\boldsymbol{X}$:

$$F(A,X)=\max_{i=0,1,...NL-1}\frac{\|Z_i\|^2}{\frac{1}{NL}\|Z\|^2} \quad (3)$$

In (3), $\boldsymbol{X}$ consists of the data, and $\boldsymbol{A}$ is the coefficient vector of the adaptive digital FIR filter. For different $\boldsymbol{X}$, we can select $\boldsymbol{A}$ to minimize the $F(A,X)$. Therefore, we form an optimization problem as

$$PAPR = \min_{A}\{F(A,X)\} = \min_{A}\left\{ \max_{i=0,1,\ldots NL-1} \frac{\|Z_i\|^2}{\frac{1}{NL}\|Z\|^2} \right\} \quad (4)$$

The above equation can be solved for desired digital filter coefficients ***A***. To find the solution of equation (4), we use an initial vector of ***A***, which can be the worst case, to filter the original data ***X*** and calculate the F(A,X) in equation (3), and then change the ***A*** values within a given range until a minimum F(A,X) obtained. This adaptive procedure can be done by a sequential quadratic programming method [8]. This method includes three main steps: updating of the Hessian matrix of the Lagrangian function, quadratic programming problem solution, and calculating line search and merit function. This can be done in Matlab by a function named “fminimax’.

3. SIMULATION AND RESULTS

Our method has been studied with simulation. We present simulation results for an IEEE802.11a compliant OFDM system in which $N_d = 48$ of 64 subcarriers carry data. The positions of the 16 free subcarriers without carrying data are set to (1, 5, 9, 13, 17, 21, 25, 29, 34, 38, 43, 47, 51, 56, 60, and 64). The optimization problem (4) is solved using Matlab function ***fminimax***. In order to keep the average power level of the filter output as the same as that of the original data, the ranges of the filter coefficients are limited for different modulations. In this simulation, the range of the coefficients is from 1 to –1.

As shown in figure 1, the median and bottom lines represent the PAPRs of the data processed by an adaptive digital filter with order of 6 and 16 with L=4, respectively. The PAPR of the original data is shown by the top line in the figure.

Table I shows the simulation results. We simulate 2000 OFDM symbols for each modulation. The order of the digital FIR filter is 8 for all cases and L=8. The coefficients are sent to a receiver through the 8 of 16 free subcarriers, and the rest of free subcarriers are set to zero.

For the different modulation schemes, the reduction of the mean PAPR is from 3 to 3.5 dB using the adaptive digital filter with the order of 8. The standard deviation of PAPR by the new method is three times less than that of the original PAPR. On the other hand, the mean PAPR can be further reduced by increasing the range of coefficients of the filter. However, the larger coefficients will increase the average power level.

Table I

Modulation	PAPR without digital filter		PAPR with digital filter	
	Mean (dB)	STD (dB)	Mean (dB)	STD (dB)
BPSK	6.78	1.05	3.83	0.35
QPSK	7.27	0.94	3.83	0.26
16-QAM	7.24	0.93	3.81	0.27
64-QAM	7.25	0.90	3.83	0.27

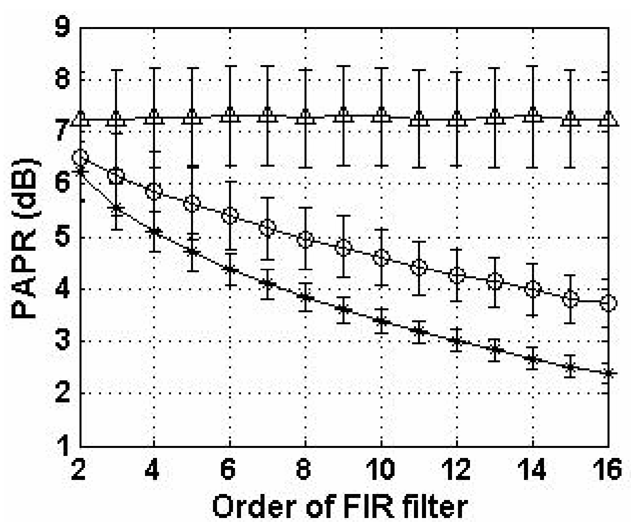

Figure 2 Mean PAPR verses Order of digital FIR filter. The top line is the original PAPR, The median line is the PAPR by PCR method [7], and the bottom line is the PAPR by our proposed method

Figure 2 shows the comparison between our method and the PCR method [7]. The figure also shows the PAPR verses the order of adaptive digital filter. We simulate 2000 OFDM symbols for QPSK modulation. Each OFDM symbol is obtained by taking 64-point IDFT with L = 8 of the data. In figure 2, the top line shows that the average PAPR and its standard deviation of the original data in which set the 16 free subcarriers are set with zeros and then perform 64-point IDFT with L = 8. The bottom line shows the average PAPR and the standard deviation of OFDM signals using our new method. The median line shows the results of the peak reduction carriers (PRC) method proposed in [7]. In PRC method, we also use Matlab function ***fminimax*** to find the optimal amplitude of the additional peak reduction carriers (PRC). In order to compare with the PRC method, the number of the additional PRCs is the same as the order of our new method, and the range of the amplitude is the same as the range of the coefficients in our method.

The simulation result shows that the mean value and standard deviation of the PAPRs by our method are much lower than that of original data and PRC method. Because the range of the coefficients in our method is limited, our method dose not increase the transmission power. Table II shows the average powers (AP) for different methods. The average power in table II is normalized by the average power of the original data. As we can see, by limiting the range of the FIR filter coefficients within (-1, 1), our method provides a lower average power than PCR method. If without the limitation, the ideal mean PAPR will approach to 0dB. However, this will request a very high average transmission power.

Table II

Modulation	AP (dB) Original data	AP (dB) New Method	AP (dB) PCR
BPSK	0	-2.57	0.81
QPSK	0	-2.50	0.51
16-QAM	0	-2.33	0.17
64-QAM	0	-2.52	0.04

The effect of the order of digital filter on the PAPR can be varied. With the increased order of the filter, the net PAPR improvement increases and the standard deviation of PAPR reduces.

4. CONCLUSION

Adaptive digital FIR filter method can significantly reduce the mean PAPR of the OFDM symbols for different modulation schemes. It is found that a high order digital FIR filter can improve the performance compared with a low order filter. It is also found that the proposed method does not increase the cost of average transmission power by limiting the range of the filter coefficients.

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