A LOW COMPLEXITY TREE ALGORITHM FOR PTS-BASED PAPR REDUCTION IN WIRELESS OFDM

Byung Moo Lee and Rui J.P. de Figueiredo

Laboratory for Intelligent Signal Processing and Communications
The Henry Samueli School of Engineering
University of California, Irvine CA 92697-2625
E-mail:rui@uci.edu

ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) has several attributes which make it a preferred modulation scheme for high speed wireless communications. However, its high Peak-to-Average-Power Ratio (PAPR) causes nonlinear distortion thus limiting the efficiency of the transmitter's High Power Amplifier. Among the approaches proposed for PAPR mitigation, the Partial Transmit Sequence (PTS) technique is very promising since it does not generate any signal distortion. However, its high complexity makes it difficult for use in high speed communication systems. We present a new low-complexity tree algorithm to implement the PTS approach, which seeks the best trade-off between performance and complexity. Simulation results show that this new technique's performance is similar to that of the optimum case, however with significantly lower complexity.

1. INTRODUCTION

The explosive growth of mobile wireless communications is producing the demand for high-speed, efficient, and reliable communication over the hostile wireless medium. As a modulation scheme for such applications, Orthogonal Frequency Division Multiplexing (OFDM) possesses several desirable attributes, such as high immunity to inter-symbol interference, robustness with respect to multi-path fading, and ability for high data rates, all of which are making OFDM to be incorporated in wireless standards like IEEE 802.11a/g/n WLAN and ETSI terrestrial broadcasting. However one of the major problems posed by OFDM is its high Peak-to-Average-Power Ratio (PAPR), which seriously limits the power efficiency of the transmitter's High Power Amplifier (HPA). This is because PAPR forces the HPA to operate beyond its linear range with a consequent nonlinear distortion in the transmitted signal. This distortion is viewed as a major impediment to progress by the RF system design community. Clever signal processing techniques are necessary to deal with this problem. As shown in our previous work, Digital Pre-Distortion techniques belong to this category [1] [2]. However, these pre-distortion techniques only work in the limited range that extends up to the saturation region of the HPA. In order to mitigate this problem, there is considerable interest in further developing and combining the existing techniques to mitigate nonlinear distortion in HPA under ever increasing throughput demands from the system. Among various PAPR reduction techniques, the Partial Transmit Sequence (PTS) technique which was developed by Muller and Huber [3] constitutes a promising approach to reduce PAPR in OFDM communication systems. However, a major problem of PTS is its very high complexity which makes it difficult for use in high speed mobile communications. Cimini and Sollenberger proposed a low complexity algorithm for PTS [4], but still there is a significant performance gap between ordinary PTS technique and their algorithm. In this paper, we present a novel sub-optimum algorithm for PTS to reduce PAPR in OFDM systems. In this technique, we use a tree-based algorithm to find a best-compromise-based sub-optimum solution.

2. OFDM AND PEAK-TO-AVERAGE POWER RATIO

An OFDM signal on N subcarriers can be represented as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi f_k t}, \; 0 \leq t \leq T_s \tag{1}$$

where T_S is duration of the OFDM signal and $f_k = \frac{k}{T_s}$.
The high PAPR of OFDM signal arises from the summation in the above IDFT expression. The PAPR of OFDM signal in analog domain can be represented as

$$PAPR_c = \frac{\max_{0 \leq t \leq T_s} |x(t)|^2}{E(|x(t)|^2)} \tag{2}$$

Nonlinear distortion in HPA occurs in the analog domain, but most of the signal processing for PAPR reduction occurs in the digital domain. The PAPR of digital domain is not necessarily the same as the PAPR in the analog domain. However, in some literature [5] , it is shown that one can closely approximate PAPR in the analog domain by oversampling the

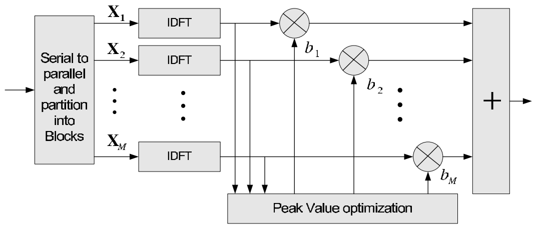

Fig. 1. Block diagram of the PTS scheme

signal in the digital domain. Usually, an oversampling factor $L = 4$ is sufficient to satisfactorily approximate the PAPR in the analog domain. For these reasons, in this paper, we express PAPR of the OFDM signal as follows.

$$PAPR = \frac{\max_{0 \le n \le LN} |x(n)|^2}{E(|x(n)|^2)} \tag{3}$$

3. EXISTING PTS TECHNIQUES

The PTS technique is a powerful PAPR reduction technique first proposed in [3]. Thereafter various related papers were published. In this section, we show two representative PTS techniques, the original PTS technique and Cimini and Sollenberger's iterative flipping technique.

3.1. Ordinary PTS technique

The block diagram of the PTS technique is shown in Fig.1. The algorithm of the original PTS technique can be explained as follows. First, the subcarrier vector is partitioned into M disjoint subblocks which can be represented as

$$\mathbf{X_m} = [X_{m,0},\ X_{m,1},\ \cdots,\ X_{m,N-1}]^T,\ m = 1,\ 2,\ \cdots,\ M \tag{4}$$

All the subcarrier positions which are presented in another block must be zero so that the sum of all the subblocks constitute the original signal, i.e,

$$\sum_{m=1}^{M} \mathbf{X_m} = \mathbf{X} \tag{5}$$

Each subblock is converted through IDFT into an OFDM signal $\mathbf{x_m}$ with oversampling, which can be represented as

$$\mathbf{x_m} = [x_{m,0},\ x_{m,1},\ \cdots,\ x_{m,NL-1}]^T,\ m = 1,\ 2,\ \cdots,\ M \tag{6}$$

where L is the oversampling factor. After that, each subblock is multiplied by a different phase factor b_m to reduce PAPR of the OFDM signal. The phase set can be represented as

$$P = \{e^{j2\pi w/W} | w = 0,\ 1,\ \cdots,\ W-1\} \tag{7}$$

Because of the high complexity of the PTS technique, one generally uses only a few phase factors. The choice $b_m \in \{\pm 1,\ \pm j\}$ is very interesting since actually no multiplication is performed to rotate the phase [6]. The peak value optimization block in Fig.1 iteratively searches the optimum phase sequence which shows minimum PAPR. Finding the optimum phase sequence is usually needs a lot of computational power. After finding the optimum phase sequence which minimize PAPR of the OFDM signal, all the subblocks are summed as in the last block of Fig.1 with multiplication of the optimum phase sequence. Then, the transmit sequence can be represented as

$$\begin{aligned}\mathbf{x}'(\mathbf{b}) &= [\mathbf{x_1},\ \mathbf{x_2},\ \cdots,\ \mathbf{x_M}] \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_M \end{bmatrix} \\ &= \sum_{m=1}^{M} b_m \cdot \mathbf{x_m}\end{aligned} \tag{8}$$

3.2. Iterative flipping PTS technique

Cimini and Sollenberger's iterative flipping technique is developed as a sub-optimum technique for PTS. In their original paper [4], they only use binary weighting factors. The algorithm is as follows. After dividing the data block into M disjoint subblocks, one assumes that $b_m = 1,\ (m = 1, 2, \cdots M)$ for all of subblocks and calculate PAPR of the signal. Then change the sign of the first subblock phase factor from 1 to -1 ($b_1 = -1$), and calculate the PAPR of the signal again. If PAPR of the previously calculated signal is larger than that of the current signal, keep $b_1 = -1$. Otherwise, revert to the previous phase factor, $b_1 = 1$. One performs this procedure iteratively until the end of subblocks (M^{th} subblock and phase factor b_M). A similar technique was also proposed by Jayalath and Tellambura [7]. The difference in the Jayalath and Tellambura's technique and that of Cimini and Sollenberger is that, in the former, the flipping procedure does not necessarily go to the end of subblocks (M^{th} block). To reduce complexity, the flipping can be stopped at the middle of the procedure if one gets the desired PAPR signal during the procedure.

4. THE NEW PTS TECHNIQUE BASED ON A TREE ALGORITHM

In this section, we introduce a novel PTS technique using a tree algorithm. In Cimini and Sollenberger's iterative flipping algorithm, even if we choose a phase factor which shows minimum PAPR in the first subbolck, that is not necessarily true when we reach the second or M^{th} subblock. That is, if we choose another phase factor in the first subblock which does not have minimum PAPR, rather than choose a phase factor which shows minimum PAPR, it is also possible that the PAPR of the signal would be smaller in the second or M^{th} subblock. For this reason, we should keep more information rather than discard the information which does not

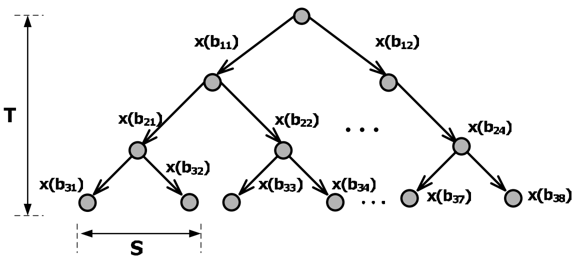

Fig. 2. Tree Structure of the proposed algorithm

show minimum PAPR at the current subblock. If we keep all of information until the end subblock, that would be the case of the ordinary PTS. If we only keep the one which shows minimum PAPR at each subblcok, that would be the case of Cimini and Sollenberger's iterative flipping algorithm. The important point to note is that the more information we keep, the better performance we get but with higher complexity. As a compromise between these two extremes, we introduce two new adjustable parameters, which we call S and T. Our algorithm is based on the following.

1. Instead of keeping all of PAPRs and phase information, we keep S of PAPRs and phase information in each subblock where $1 \leq S \leq W$.
2. Instead of keeping information until at the end of the subblock, we keep until T^{th} subblocks and continue iteratively where $1 \leq T \leq M$.

Each node has its own signal $\mathbf{x}(\mathbf{b}_{ij})$ and corresponding PAPR as in Fig.2. After the depth T, we choose the signal which has minimum PAPR and back to their $(T-1)^{th}$ parent node. Then discard unrelated nodes and continue until the end of subblocks. The detailed steps of the new algorithm are as follows.

1. After signal mapping and change of the data from serial to parallel, partition the one OFDM block into M disjoint subblocks.
2. Perform zero padding with a factor of L and IDFT.
3. Set initial phase sequence as $\mathbf{b} = \{1,\ 1,\ \cdots,\ 1\}$.
4. Multiply different W phase factors to the first/ current data subblocks and measure PAPRs of each signal.
5. Choose S phase factors which show low PAPRs of signals among W phase factors and discard the rest.
6. Using each S phase factors, from each chosen node, iterate from step 4 to 5 and do this procedure until reach T^{th} subblock.
7. At the depth of T, choose the minimum PAPR signal and phase sequence.
8. Back to the $(T-1)^{th}$ parent node, make a final decision for the $(T-1)^{th}$ parent node, and keep corresponding child nodes, and discard the others.
9. Set chosen node as a new root node and repeat the procedure until at the end of subblock.

We used similar tree structure to improve the efficiency of the V-BLAST detection algorithm at the receiver [8] [9]. However, the advantage of using this flexible low complexity tree algorithm at the transmitter, as done here, is that we can change parameters depending on input signals. That is, it is difficult to get the exact original signal at the receiver because of noise or fading effects. This limits flexibility of tree algorithm usage at the receiver. However, at the transmitter, we always know the original input signal, and also the PAPR of the signal. Depending on the variation of PAPR, we can adjust parameters so that we get high performance with low complexity, as proposed in this paper.

5. SIMULATION RESULTS AND DISCUSSIONS

In this section, we present simulation results as well as related discussion about the results. We use 16 QAM with $N = 64$ subcarriers and choose $W = 4$ $(b_m \in \{\pm 1,\ \pm j\})$. We divide one OFDM block as $M = 4$ disjoint subblocks with each subblock has adjacent N/M subcarriers for simplicity. The signal was oversampled by a factor of $L = 4$ for estimate of the PAPR of analog domain. Fig.3 shows performance of the proposed algorithm with increasing S and fixing $T = 2$. Without any PAPR reduction technique, the PAPR of OFDM signal exceeds 10.5 dB at the 0.1% of complementary cumulative distribution function (or clipping probability, $Pr(PAPR > PAPR_0)$). From the simulation result,

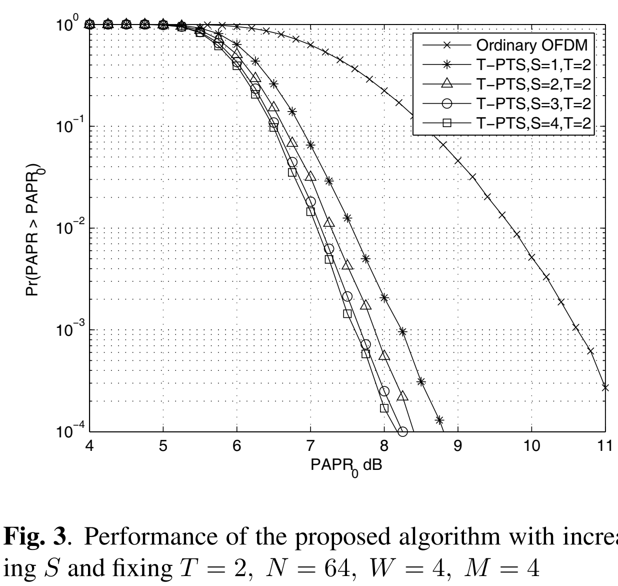

Fig. 3. Performance of the proposed algorithm with increasing S and fixing $T = 2,\ N = 64,\ W = 4,\ M = 4$

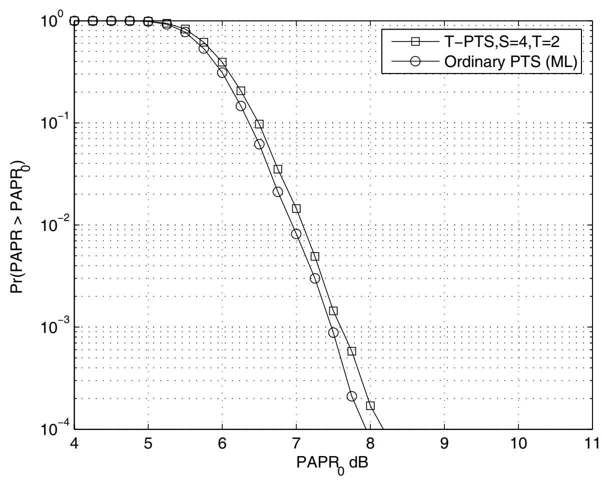

Fig. 4. Performance comparison between the proposed algorithm ($S = 4, T = 2$) and ordinary PTS.

Table 1. Comparison of the number of iterations between ordinary PTS and T-PTS with increase S and fixing T, when $M = 4, W = 4$

	Number of iterations
T-PTS, $S = 1, T = 2$	13
T-PTS, $S = 2, T = 2$	22
T-PTS, $S = 3, T = 2$	31
T-PTS, $S = 4, T = 2$	40
Ordinary PTS	64

we can see that our reduced complexity tree algorithm can improve PAPR of OFDM signal around 3 dB at the 0.1% of clipping probability with $M = 4$. Fig.4. presents the performance difference between ordinary PTS and proposed Tree(T)-PTS with $S = 4$, $T = 2$. As we can see in this simulation result, there is only around 0.2 dB difference at 0.1% of clipping probability.

Table 1 provides comparison of complexity. In this paper, we assume complexity is only dependent on the number of iterations. Compared with ordinary PTS technique, we get very close performance (within 0.2dB to 0.8 dB difference) with around 20% to 60% less computational complexity. This complexity reduction will be greatly improved if we increase the number of subblocks. More detailed and complete simulation results / analysis will be provided in a journal paper [10].

6. CONCLUSION

In this paper, we proposed a low complexity PTS technique using a new tree algorithm. This T-PTS (Tree-PTS) technique has some adjustable parameters which one can choose depending on circumstances. That is, its flexibility and low complexity makes it easy to adapt to any kind of environment. Simulation results show that the proposed technique gives very good performance with significantly low complexity.

7. REFERENCES

[1] Byung Moo Lee and R. J.P. de Figueiredo, "Adaptive Pre-Distorters for Linearization of High Power Amplifiers in OFDM Wireless Communications", *(Accepted), to appear in Circuits, Systems & Signal Processing*.

[2] Rui J. P. de Figueiredo and Byung Moo Lee, "A New Pre-Distortion Approach to TWTA Compensation for Wireless OFDM Systems". *2nd. IEEE International Conference on Circuits and Systems for Communications, ICCSC-2004*, Moscow, Russia, no.130, June 30 - July 2, 2004. (Invited Plenary Lecture)

[3] S. H. Muller and H. B. Huber, "OFDM with reduced peak-to-mean power ratio by optimum combination of partial transmit sequences", *Electronics Letters*, vol.33, pp.368-369, Feb. 1997.

[4] L. J. Cimini, Jr. and N.R. Sollenberger, "Peak-to-average power ration reduction of an OFDM signal using partial transmit sequences", *IEEE Communication Letters*, vol.4, pp.86-88, Mar.2000.

[5] R. O'Neil and L. N. Lopes, "Envelope variations and spectral splatter in clipped multicarrier signals", *in Proc. of PIMRC'95*, pp.71-75, Sept. 1995.

[6] S. H. Muller and J. B. Huber, "A Comparison of Peak Power Reduction Schemes for OFDM", *IEEE GLOBECOM'97*, Phoenix, Arizona, pp.1-5, Nov. 1997.

[7] A.D.S. Jayalath and C. Tellambura, "Adaptive PTS approach for reduction of peak-to-average power ratio of OFDM signal", *Electronics Letters*, vol.36, no.14, pp.1226-1228, 6th July 2000.

[8] Ashish Bhargave, Rui J.P. de Figueiredo, and Torbjorn Eltoft, "A Detection Algorithm for the V-BLAST System", *GLOBECOM, 01 IEEE*, vol.01, pp.494-498, 2001.

[9] Byung Moo Lee and Rui J.P. de Figueiredo, "Enhanced V-BLAST Performance in MIMO Wireless Communication Systems", *2nd. IEEE International Conference on Circuits and Systems for Communications (ICCSC-2004)*, no.129, Moscow, Russia, June 30 - July 2, 2004.

[10] Byung Moo Lee and Rui J.P. de Figueiredo, "A Low Complexity Tree-PTS Technique for PAPR Reduction in OFDM Communication Systems", *to be submitted for publication*.