PEAK-TO-AVERAGE POWER RATIO REDUCTION IN OFDM: BLIND SELECTED MAPPING FOR PSK INPUTS

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ABSTRACT

Orthogonal Frequency Division Multiplexing (OFDM) is a modern transmission format that has gained much popularity in the past decade. However, a serious drawback of OFDM is the high peak-to-average power ratio (PAR) of its time domain waveforms. Selected mapping (SLM) is a distortionless technique that has good PAR reduction capability; the biggest limitation of SLM however, is the need to transmit side information. In this paper, we assume constant modulus (i.e., phase shift keying - PSK) inputs and propose a novel technique that allows us to carry out SLM without having to transmit any side information and without causing any distortions.

1. INTRODUCTION

OFDM transmission has gained much popularity in the past decade. The large peak-to-average power ratio (PAR), or the crest factor (which is the square root of the PAR), is a "purported Achilles' heel" of multicarrier transmissions [8]. The occurrence of the occasional large peaks places stringent demands on the dynamic range and linearity of analog components. Because the power amplifier is a peak power limited device, a high PAR signal will have to be transmitted at a low average power level (thus lowering the power efficiency) if nonlinear distortion is to be avoided.

Generally speaking, PAR reduction algorithms fall into two categories. Algorithms with distortion such as clipping are relatively simple to implement, but the drawbacks are inband distortion and out-of-band spectral regrowth; see [8], [5] and references therein. Distortionless PAR reduction algorithms include coding, selected mapping (SLM), partial transmit sequence, tone reservation, tone injection, active constellation extension, etc; see [8], [5] and references therein. Distortionless methods tend to be more computationally intensive and may have the added burden of side information transmission. We are interested in the SLM method [1], [7] because it has good PAR reducing capability and is distortionless. The drawback of the SLM method in [1], [7] is that side information needs to be transmitted as well, thus lowering the data rate.

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In this paper, we describe a means of structuring SLM so that the side information can be recovered from the received data itself; i.e., blindly. The technique is applicable to the case where the frequency domain OFDM signal has a constant modulus; i.e., it is drawn from a phase shift keying (PSK) constellation. The proposed algorithm is simple to implement and preserves the impressive PAR reducing capability of the original SLM method.

2. A BLIND SLM ALGORITHM FOR OFDM WITH PSK INPUTS

2.1. Review of OFDM and SLM

Denote by $\{S_k\}_{k=0}^{N-1}$ the frequency domain OFDM signal, where k is the subcarrier index and N is the number of subcarriers. The Nyquist-rate sampled time-domain OFDM signal s_n is obtained as the inverse discrete Fourier transform (IDFT) of S_k; i.e.,

$$s_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \, e^{j\frac{2\pi kn}{N}}, \qquad 0 \le n \le N-1. \tag{1}$$

The peak-to-average power ratio (PAR) of s_n is defined as

$$\mathrm{PAR}\{s_n\} = \frac{\max_{0\le n\le N-1} |s_n|^2}{E[|s_n|^2]}. \tag{2}$$

In SLM [1], [7], we first form

$$U_k^{(d)} = S_k \, e^{j\phi_k^{(d)}}, \quad 1 \le d \le D, \; 0 \le k \le N-1, \tag{3}$$

as D different representations of the same signal S_k, where $\{\phi_k^{(d)}\}$ are fixed pseudo-random phase sequences that are available to both the transmitter and the receiver. By default, we set $\phi_k^{(1)} = 0, \forall k$, thus $U_k^{(1)} = S_k$, representing the original OFDM signal. Next, we obtain the corresponding time-domain signal $u_n^{(d)}$ according to (1). In SLM, the transmitted signal, $u_n^{(\bar{d})}$, has the lowest PAR among the D candidate OFDM signals (including the original s_n), where

$$\bar{d} = \arg\min_{1\le d\le D} \mathrm{PAR}\{u_n^{(d)}\}. \tag{4}$$

We remark that $\bar{d}$ is sensitive to the data S_k and typically varies from block to block.

It is shown in [10] that if each phase sequence $\{\phi_k^{(d)}\}_{k=0}^{N-1}$ is i.i.d. satisfying $E[e^{j\phi_k^{(d)}}] = 0$, and if the phase sequences are mutually independent, then for N large, the CCDF of the PAR of the SLM OFDM signal $u_n^{(\bar{d})}$ is

$$\Pr(\mathrm{PAR}\{u_n^{(\bar{d})}\} > \gamma) = \Big(\Pr(\mathrm{PAR}\{u_n^{(d)}\} > \gamma)\Big)^D, \quad (5)$$

which lowers as D is increased.

We explained in [10] that the simple choice of $\phi_k^{(d)} \in \{0, \pi\}$ with equal probability, which corresponds to $e^{j\phi_k^{(d)}} \in \{1, -1\}$ with equal probability, is as good as any other phase sequence in terms of the PAR reducing capability. In that case, the implementation of (3) becomes very simple: $U_k^{(d)} = S_k$ or $U_k^{(d)} = -S_k$ according to the sign table. Unless otherwise stated in this paper, we assume that $\phi_k^{(d)} \in \{0, \pi\}$.

Using the results of [9], we can show that if S_k is real-valued (e.g., BPSK), then the CCDF of the PAR of the SLM OFDM signal is given by

$$\begin{aligned} &\Pr(\mathrm{PAR}\{u_n^{(\bar{d})}\} > \gamma) \\ &= \Big(1 - (1 - \mathrm{erfc}(\sqrt{\gamma/2}))^2 (1 - e^{-\gamma})^{\frac{N-2}{2}}\Big)^D. \end{aligned} \quad (6)$$

On the other hand, if S_k is circularly symmetric (e.g., PSK with constellation size ≥ 3), then

$$\Pr(\mathrm{PAR}\{u_n^{(\bar{d})}\} > \gamma) = (1 - \big(1 - e^{-\gamma}\big)^N)^D. \quad (7)$$

The receiver must know the optimum sequence index $\bar{d}$ in order to decode. It is this side information transmission issue that has hindered the practical application of SLM; this is also the topic that we wish to address in the current paper. Blind SLM has been discussed in [6] and [2] by capitalizing on the finite alphabet nature of the input constellation. In [3], blind SLM was carried out in conjunction with pilot symbol assisted modulation in OFDM. We propose next a simple blind SLM algorithm that operates independently of any training sequences. It is simpler than the approaches in [6] and [2] and outperforms the blind SLM algorithm proposed in [4], by capitalizing on the constant modulus property of the PSK input.

2.2. Proposed blind SLM algorithm

In conventional SLM [1], [7], the phases of S_k are rotated but the amplitudes are left unchanged. We propose to scale the amplitudes and rotate the phases of S_k as follows:

$$X_k^{(d)} = S_k\, \rho_k^{(d)}\, e^{j\phi_k^{(d)}}, \;\; 1 \leq d \leq D,\; 0 \leq k \leq N-1, \quad (8)$$

where the $\phi_k^{(d)}$ table is constructed as usual. The amplitude sequences $\rho_k^{(d)} > 0$ and $\rho_k^{(d_1)}$ and $\rho_k^{(d_2)}$ take on different shapes when $d_1 \neq d_2$. By default, $\rho_k^{(1)} = 1$ and $\phi_k^{(1)} = 0, \forall k$, corresponding to the original OFDM signal.

We assume that S_k has a constant amplitude $|S_k| = A, \forall k$ (i.e., PSK). In order to ensure that the average signal energy remains unchanged; i.e., $N^{-1}\sum_{k=0}^{N-1} E[|X_k|^2] = E[|S_k|^2] = A^2$, we require

$$\sum_{k=0}^{N-1} [\rho_k^{(d)}]^2 = N. \quad (9)$$

Under the constant modulus assumption for S_k, we infer from (8) that

$$|X_k^{(d)}| = A\, \rho_k^{(d)}. \quad (10)$$

We take the IDFT of $X_k^{(d)}$ as in (1) to obtain $x_n^{(d)}$ and then select the one representation with the lowest PAR to transmit:

$$\bar{d} = \arg\min_{1\leq d\leq D} \mathrm{PAR}\{x_n^{(d)}\}. \quad (11)$$

At the receiver, after removing the cyclic prefix and taking the DFT, we obtain

$$Y_k = X_k^{(\bar{d})} H_k + V_k, \quad (12)$$

where V_k is the DFT of the zero-mean additive noise v_n, and H_k is the frequency response of the composite channel (the convolution of the transmit filter, the frequency selective channel, and the receive filter). Combining (10) and (12), we infer that $\rho_k^{(\bar{d})}$ can be estimated from

$$\hat{\rho}_k^{(\bar{d})} = \frac{|Y_k|}{A|H_k|}, \quad (13)$$

where we have assumed that the fading channel frequency response H_k is either known or has been accurately estimated.

Since the $\rho_k^{(d)}$ templates are known to both the transmitter and the receiver, we can then estimate $\bar{d}$ by finding the template that best matches the estimate in (13). For example,

$$\hat{\bar{d}} = \arg\min_{1\leq d\leq D} \sum_{k=0}^{N-1} [|Y_k| - A|H_k|\rho_k^{(d)}]^2. \quad (14)$$

Since $\rho_k^{(d)}$ and $\phi_k^{(d)}$ are pre-determined functions and H_k is assumed known, once $\bar{d}$ has been found, it is then straightforward to obtain the S_k estimate from (c.f. (8), (12)) $\hat{S}_k = Y_k/[H_k\, \rho_k^{(\bar{d})}]\, e^{-j\phi_k^{(\bar{d})}}$, followed by minimum distance decoding.

The amplitude functions $\rho_k^{(d)}$ should be sufficiently different and satisfy the condition in (9). For example, when $D = 4$, we can have

$$\rho_k^{(1)} = 1, \quad \forall k, \quad (15)$$

$$\rho_k^{(2)} = \begin{cases} \sqrt{\alpha}, & 0 \leq k \leq \frac{N}{2} - 1, \\ \sqrt{2-\alpha}, & \frac{N}{2} \leq k \leq N-1, \end{cases} \quad (16)$$

$$\rho_k^{(3)} = \begin{cases} \sqrt{2-\alpha}, & 0 \leq k \leq \frac{N}{2} - 1, \\ \sqrt{\alpha}, & \frac{N}{2} \leq k \leq N-1, \end{cases} \quad (17)$$

$$\rho_k^{(4)} = \begin{cases} \sqrt{2-\alpha}, & 0 \leq k \leq \frac{N}{4} - 1, \frac{3N}{4} \leq k \leq N-1, \\ \sqrt{\alpha}, & \frac{N}{4} \leq k \leq \frac{3N}{4} - 1. \end{cases} \quad (18)$$

2.3. PAR thresholding

Under the constraint in (9), we will have $|X_k^{(d)}|^2 > |S_k|^2$ at some of the subcarriers but $|X_k^{(d)}|^2 < |S_k|^2$ at the other subcarriers. When noise is present, bit error rate (BER) or symbol error rate (SER) may increase at the receiver. Take for example, the amplitude functions given in (15)-(18). A larger α $(1 < \alpha < 2)$ makes the $\rho_k^{(d)}$ functions more distinct from each other, which reduces the detection error rate in $\bar{d}$. The downside is that SER at the weaker signal subcarriers can be so high that it dominates the overall SER. Such trade-offs should be carefully considered in the design of the $\rho_k^{(d)}$ functions in order to minimize the overall SER.

For many practical systems, a PAR threshold γ is usually pre-determined according to the system power efficiency or linearity constraints. The objective is often to ensure that the transmitted signal has a PAR that exceeds γ only very rarely (e.g., no more than 10^{-4} in probability). In other words, minimizing or reducing the PAR to much below γ is not necessary. Under this PAR thresholding paradigm, we carry out SLM until the first $x_n^{(d)}$ that meets the PAR threshold has been found. The number of actual mappings conducted is anywhere between 1 and D. Sometimes, the given OFDM block already has a PAR that is smaller than γ; in that case, we do not perform any PAR reduction and transmit the block as is. On the other hand, if after all D mappings, the resulting PAR is still larger than γ, we will transmit $x_n^{(\bar{d})}$, despite of the fact that it will be clipped. This approach not only lowers the computational cost, but also ensures that we do not perform SLM unnecessarily.

3. SIMULATIONS

3.1. PAR reducing capability

We assess the PAR reducing capability by examining the CCDF of the resulting PAR values. In the following example, we assume that the input S_k is drawn from a quadrature PSK (QPSK) constellation with $E[|S_k|] = A = 1$. Each OFDM block has $N = 128$ subcarriers. The number of different mappings $D = 4$ (including the original OFDM sequence). Except for $d = 1$, $e^{j\phi_k^{(d)}}$ is i.i.d. ± 1 with equal probability. The amplitude sequences $\rho_k^{(1)}$ through $\rho_k^{(4)}$ are given in (15)-(18) with $\alpha = 1.4$. We simulated 10^5 OFDM blocks to construct the empirical CCDF curves. Fig. 1(a) shows the theoretical CCDF curves described by eq. (7) with $D = 1$ (solid line) and $D = 4$ (dashed line), respectively, for the original OFDM signal s_n and the SLM OFDM signal $x_n^{(\bar{d})}$. The crosses and the diamonds correspond to the empirical CCDF values obtained from the Monte Carlo simulations. The empirical CCDFs agreed with the closed-form expressions fairly well. At the 10^{-4} CCDF level, 2.7 dB of PAR reduction was achieved. The dash-dotted line corresponds to the case where

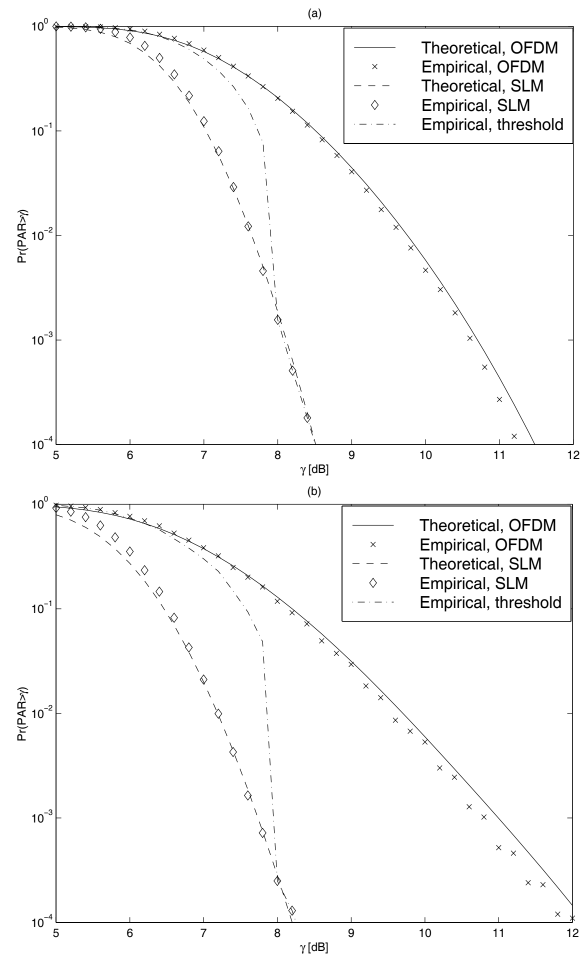

Fig. 1. Theoretical vs. empirical CCDF curves for the original OFDM signal, the proposed SLM method, and the PAR thresholding SLM method. (a) S_k is QPSK. (b) S_k is BPSK.

PAR thresholding (threshold = 8 dB) was used. We find that with 20.47% of the chance, the PAR of the original OFDM signal exceeds 8 dB. With SLM or PAR thresholding SLM, the chance that the resulting PAR exceeds 8 dB is reduced to 0.16%.

Similar parameters were used in generating the curves in Fig. 1(b), except that S_k was drawn from the BPSK constellation. The solid line in Fig. 1(b) was calculated from eq. (6) with $D = 1$; the dashed line was calculated from eq. (6) with $D = 4$. At the 10^{-4} CCDF level, 3.7 dB of PAR reduction was achieved, 1 dB better than for the QPSK case! We find that with 11.81% of the chance, the PAR of the original OFDM signal exceeds 8 dB. With SLM or PAR thresholding SLM, the chance that the resulting PAR exceeds 8 dB is drastically reduced to 0.025%.

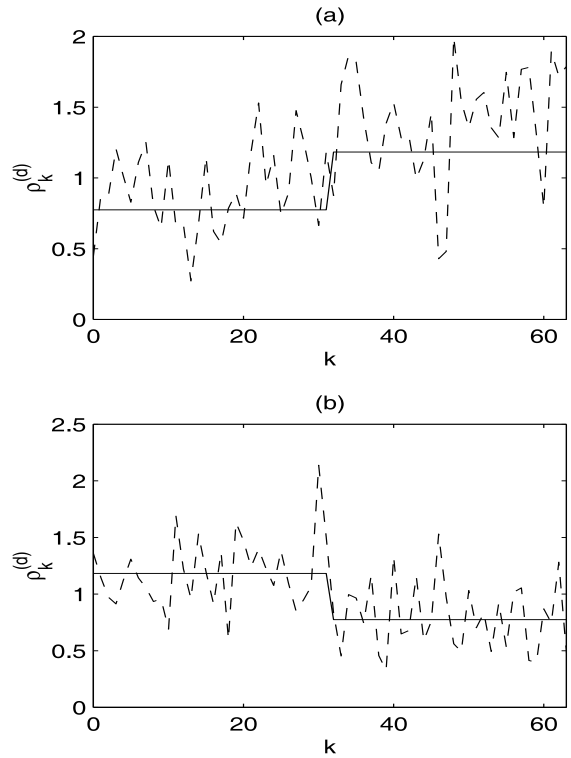

Fig. 2. Estimated (dashed line) and the best matched $\rho_k^{(d)}$ function (solid line) from the set of D known amplitude functions. (a) True and estimated $\bar{d} = 3$; (b) true and estimated $\bar{d} = 2$.

3.2. Blind detection of $\bar{d}$

In the following example, we assume an additive white Gaussian noise (AWGN) channel with SNR=5 dB. The input S_k is drawn from the QPSK constellation; the OFDM block length $N = 64$ and the number of mappings $D = 4$. Fig. 2 shows the $\hat{\rho}_k^{(\bar{d})}$ estimates (dashed lines) for two different realizations (OFDM blocks). The solid line depicts the $\rho_k^{(d)}$ function that best matched the $\hat{\rho}_k^{(\bar{d})}$ estimate from among the set of D candidate amplitude functions. Correct $\bar{d}$ estimates, $\bar{d} = 3$ for Fig. 2(a) and $\bar{d} = 2$ for Fig. 2(b), were obtained.

4. DISCUSSIONS AND CONCLUSIONS

We are able to overcome the side information transmission issue in SLM by linking the phase (or the sign) sequence index d to a set of distinct amplitude functions. A similar concept was explored in [4]. In [4], $X_k^{(d)} = -\mu S_k$ for up to 10% of the subcarriers (whose positions are pre-determined), and $X_k^{(d)} = S_k$ for the rest. $\mu > 1$ is a carefully chosen constant to ensure that "the minimum distance between modified signal point and adjacent signal point is larger than the minimum distance between the original signal points." The authors of [4] reported a performance degradation in terms of the PAR reducing capability as well as an increase in the average transmit power. Our explanation for the loss in the PAR reducing capability is that their $E[e^{j\phi_k^{(d)}}] \neq 0$.

Our proposed method enjoys the same PAR reduction performance as the original SLM method. It does not lead to any average power increase, and is simple to implement. We are able to accomplish this because we have explicitly taken advantage of the constant modulus nature of the PSK input.

5. REFERENCES

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