PAPR REDUCTION FOR BEAMFORMING OFDM TRANSMITTERS

Timothy A. Thomas

Motorola Labs
Communications Systems Research Lab
Schaumburg, Illinois, USA

Douglas L. Jones

University of Illinois at Urbana-Champaign
Electrical and Computer Engineering
Urbana, Illinois, USA

ABSTRACT

A high peak-to-average-power ratio (PAPR) is a primary drawback of orthogonal frequency-division multiplexing (OFDM). Many methods have been proposed to reduce the OFDM PAPR for single-antenna and for MIMO systems. Some systems, however, use multiple antennas for frequency-selective transmit beamforming either to a single or multiple simultaneous receivers. This paper presents a new method for PAPR reduction that minimizes the peak power across all antennas while preserving the maximal ratio transmission (MRT) gain and zero-crosstalk criteria at the receivers. Because the gain and zero crosstalk is preserved at the receivers, no performance loss is seen by the receivers when using the PAPR-reduced transmit vector. With eight transmit antennas, the algorithm demonstrates an average of 5.5 dB PAPR reduction for single-user beamforming and about 4.5 dB PAPR reduction for four-user transmit spatial division multiple access (Tx-SDMA).

1. INTRODUCTION

Communications systems based on orthogonal frequency division multiplexing (OFDM) can achieve high performance in terms of data rate, spectral efficiency, and tolerance to channel impairments at moderate implementation complexity. These advantages have led to the adoption of OFDM in many modern wireless communications standards, including 802.11a,g (Wi-Fi), 802.16 (WiMAX), and the European terrestrial digital audio and video broadcast standards, and make OFDM a prime contender for many new standards.

Perhaps the greatest drawback of OFDM, however, is a relatively high peak-to-average-power (PAPR) ratio, which increases the back-off required in the transmitter power amplifier and thereby greatly reduces the transmitter efficiency. Many methods have been introduced for reducing the PAPR of OFDM, as summarized in [1]; approaches include setting aside some channels for peak cancellation signals, various coding methods, and selective phase manipulation. Practical peak reduction in the range of 3-4 dB has been reported using these methods. However, these methods entail a reduction in data rate due to the reserved channels, the coding redundancy, or the transmission of side information, respectively. A technique called active constellation extension (ACE) modifies data-bearing channels in a way that maintains data rate, margins, and bit error rate while reducing the PAPR by up to 4 dB [1], [2]. ACE can also be made compatible with most standard OFDM receivers while retaining virtually all of its PAPR reduction [3].

Multiple antennas can significantly improve the performance and capacity of wireless communication systems. The main approaches include exploiting diversity to reduce the probability of deep fades and dropouts, multi-input multi-output (MIMO) signaling to increase the overall data rate beyond that possible with a single antenna, and beamforming on either transmit or receive. Beamforming can steer the transmit energy more efficiently in the direction of the intended user, can optimally combine multiple received versions of the signal, or can reduce interference by steering nulls in the direction of other sources or receivers. Most emerging wireless standards allow or require the use of multiple antennas, and other systems can apply beamforming in ways that comply with single-channel standards. PAPR reduction methods for diversity and MIMO multi-antenna OFDM systems have recently been developed [4] [5], including generalizations of ACE for both diversity and MIMO OFDM signaling [5]. As in the single-channel case, MIMO ACE maintains data rate and performance and remains compatible with standard MIMO receivers; comparable peak power reductions of up to 3-4 dB are also observed.

We present here a method for PAPR reduction that is applicable to both maximal ratio transmission (MRT) and transmit spatial division multiple access (Tx-SDMA) [6]. Note that MRT is designed for transmission to a single receiver (mobile) and Tx-SDMA simultaneously transmits to multiple receivers (in the version of Tx-SDMA considered in this paper, a zero-crosstalk criterion is employed). The PAPR reduction method operates on the transmit vector on each subcarrier where the transmit vector is a product of the transmit symbol vector times the transmit weight matrix. The method is designed to reduce the PAPR while constraining the received signal at each mobile to be identical to the non-PAPR reduced transmit vector. This means that both the gain and zero-crosstalk is maintained by the PAPR-reduced transmit vector. The PAPR-reduced transmit vector is able to accomplish this by requiring a slightly increased transmit power (e.g., in simulation results, around 0.2 dB more power is needed on average over the non-PAPR reduced transmit vector). However, the substantial improvement in PAPR more than makes up for this small increase in transmit power. With eight base antennas, PAPR reduction of about 5.5 dB is observed with MRT transmission and about 4.5 dB with Tx-SDMA transmission with four mobiles.

2. TRANSMIT BEAMFORMERS FOR OFDM SYSTEMS

OFDM is an ideal modulation method for beamforming because the channel vector seen by the mobile (assuming a single receive antenna) on a given subcarrier is flat faded. This means that any beamforming method designed for flat fading can be easily applied to OFDM on each subcarrier as long as channel state information is available at the base. Assuming that there are P transmit antennas at the base and B beams (one for each mobile) are to be transmitted, the $1 \times P$ signal sent from the base is given as (where $\mathbf{z}_k$ is the transmit vector on subcarrier k):

$$\mathbf{z}_k = \sum_{b=1}^{B} \mathbf{w}_{b,k}^T s_{b,k} \tag{1}$$

where $\mathbf{w}_{b,k}$ is the $P \times 1$ transmit weight for user b on subcarrier k and $s_{b,k}$ is the transmitted symbol for user b on subcarrier k. In matrix form, (1) can be written as:

$$\mathbf{z}_k = \mathbf{s}_k^T \mathbf{W}_k \tag{2}$$

where $\mathbf{s}_k$ is the $B \times 1$ vector of symbols for each mobile on subcarrier k and $\mathbf{W}_k$ is a $B \times P$ matrix whose b^{th} row is $\mathbf{w}_{b,k}^T$. The received signal on subcarrier k at mobile b is given as:

$$r_{k,b} = \mathbf{z}_k \mathbf{h}_{b,k} = \mathbf{s}_k^T \mathbf{W}_k \mathbf{h}_{b,k} \tag{3}$$

where $\mathbf{h}_{b,k}$ is the $P \times 1$ channel vector on subcarrier k for user b. Note that for this paper it is assumed that the transmitter has perfect channel knowledge.

For either MRT (i.e., $B = 1$) or Tx-SDMA the transmit weight matrix can be expressed as:

$$\mathbf{W}_k = \frac{\left(\mathbf{H}_k^H \mathbf{H}_k\right)^{-1} \mathbf{H}_k^H}{\sqrt{trace\left(\left(\mathbf{H}_k^H \mathbf{H}_k\right)^{-1}\right)}} \tag{4}$$

where $\mathbf{H}_k$ is a $P \times B$ matrix whose b^{th} column is $\mathbf{h}_{b,k}$. Note that the term in the denominator is present so that the average transmit power is one. Also note that for Tx-SDMA, the numerator enforces a zero-crosstalk constraint on the different beams.

3. PEAK POWER REDUCTION FOR MRT AND TX-SDMA BEAMFORMERS

MRT and Tx-SDMA beamformer weights [6] minimize the total radiated transmit energy required to achieve a certain signal level at the receiver while maintaining a zero (or low) crosstalk between each receiver. In OFDM, these beamforming methods operate on a per-subcarrier basis and thus the resulting transmit vector will still have a high PAPR similar to that of regular OFDM. Since the energy consumed by typical power amplifiers is mostly a function of the peak power, the MRT/Tx-SDMA weights will generally not minimize the energy consumed by a practical transmitter circuit. We instead must solve the following optimization problem: minimize the peak power $\max_{p,t} |a_p(t)|$, where $a_p(t)$ is the transmit waveform on antenna p, $p = 1, 2, \ldots, P$ over analog (continuous) time t. The continuous waveform is interpolated (perhaps with a shaping filter) from the frequency-domain transmit signal for the p^{th} transmit antenna, $z_{p,k}$ ($z_{p,k}$ is the p^{th} element of the transmit vector, $\mathbf{z}_k$):

$$a_p(t) = \text{interp}\left(\text{IFFT}\left(z_{p,k}\right)\right) \tag{5}$$

In the frequency domain and as given in (3), the signal recovered at receiver b is $\mathbf{s}_k^T \mathbf{W}_k \mathbf{h}_{b,k}$ after passing through the MISO channel, where $\mathbf{W}_k$ and $\mathbf{h}_{b,k}$ are the transmit beamformer and the MISO channel from the transmit array to the single receiver antenna at mobile b. Since the receiver requires a consistent gain and phase in each channel as measured from pilot symbols to equalize and recover $\hat{s}_{b,k}$, the frequency-domain beamformer coefficients for mobile b must be constrained to always produce the same overall complex gain (with $\mathbf{w}_{b,k}$ given in (4))

$$g_{b,k} = \mathbf{w}_{b,k}^T \mathbf{h}_{b,k} \tag{6}$$

Note that (6) enforces the zero-crosstalk criteria and that satisfying this requirement uses up only one degree of freedom per mobile leaving ample opportunity for adjusting the remaining degrees of freedom to reduce PAPR (as long as $B < P$). Also note that we make the standard beamforming assumptions that the transmitter has access to accurate channel state feedback and that the channel is essentially stationary between updates.

Since the constraints that the peak is less than a threshold, retention of the MRT reception, and nulls in the direction of other receivers are all convex, projection onto convex sets (POCS) theory tells us that an iterative algorithm based on successive projections (clipping and beamformer constraint restoration) converges toward a solution if one exists. Furthermore, since a small clip represents a gradient step in terms of peak reduction [2], an algorithm that successively reduces the peak to a minimum achievable value with beamformer constraint restoration is therefore a gradient-project algorithm. These observations place the following iterative algorithm on a solid theoretical foundation and confirm its convergence toward an optimal solution.

The iterative algorithm performs the following steps:

1. Assign frequency-domain constellation points for all B mobiles, $\mathbf{s}_k$, according to the input data and set the iteration number, $i = 0$.

2. Create the projection matrix on each subcarrier as $\mathbf{Q}_k = \mathbf{H}_k \left(\mathbf{H}_k^H \mathbf{H}_k\right)^{-1} \mathbf{H}_k^H$.

3. Create the transmit vector on each subcarrier, $\mathbf{z}_k$, from (2) using the transmit weights in (4) and set $\hat{\mathbf{z}}_k^{(i)} = \mathbf{z}_k$.

4. Reconstruct the time-domain signals $a_p(t)$ with a zero-padded inverse FFT according to:

   $a_p(t) = \text{interp}\left(\text{IFFT}\left(\hat{z}_{p,k}^{(i)}\right)\right)$

   where $\hat{z}_{p,k}^{(i)}$ is the p^{th} element of $\hat{\mathbf{z}}_k^{(i)}$; an oversampling factor of four for the IFFT has experimentally been found to be ample.

5. Clip $a_p(t)$ to a predetermined level (e.g., a level of 3 dB above the average power was used in the simulations). Note that either $a_p(t)$ can be clipped separately on each antenna or $a_p(t)$ can be clipped to a level relative to the average power across all antennas. The second option may be useful when the power transmitted from each antenna varies and an absolute maximum power input to the power amplifier is desired.

6. Return to the frequency domain for all antennas via FFTs to obtain $\tilde{\mathbf{z}}_k^{(i)}$; zero all frequencies not corresponding to data-bearing channels.

7. For each k compute the Tx-SDMA-restoring projection:

   $\hat{\mathbf{z}}_k^{(i+1)} = \tilde{\mathbf{z}}_k^{(i)} - \mathbf{Q}_k \tilde{\mathbf{z}}_k^{(i)} + \mathbf{z}_k$

8. $i = i + 1$, return to Step 4 and iterate until the desired peak level is achieved for all time samples, the algorithm converges, or the maximum allowed number of iterations is exceeded.

The signal transmitted from the base is given by $\hat{\mathbf{z}}_k^{(i)}$ at the final iteration. Note that the computational complexity of applying the projection matrix can be made lower by taking advantage of the structure of the projection matrix. For example, in the single-user case, the projection matrix is rank one and can be expressed as a product of a vector times the Hermitian of the same vector. Thus a matrix-vector multiply is saved in Step 7.

This iterative algorithm is similar in spirit to that first introduced in [2] for the ACE algorithm, but in this case the transmit vector itself, rather than the constellation points, are modified within constraints guaranteeing receiver invariance. When applicable, this approach offers substantially more degrees of freedom than ACE (for $B < P$), so the peak reduction may be expected to be greater. This approach could also be combined with active constellation extension of the OFDM symbols to potentially reduce the peaks even further. As B approaches P there could be a significant benefit of combining the ACE algorithm with the proposed method.

Note that the step of enforcing the gain and zero-forcing constraints result in a small increase in the transmit power over the non-PAPR reduced MRT weights (e.g., simulations showed around a 0.2 dB increase in power on average). However, the fact that the PAPR is greatly reduced using the PAPR-reduced weights enables the power amplifier to operate more efficiently and thus the trivial power increase is a reasonable tradeoff.

4. SIMULATION RESULTS

Simulation results are now presented for an 802.16-like OFDM system with a system bandwidth of 5.0 MHz, a subcarrier spacing of 11.2 kHz, an FFT size of 512, and 432 subcarriers used for data. The base station has $P = 8$ transmit antennas (with an antenna spacing of 5.0 λ) and the mobiles have only a single receive antenna. The downlink channel was simulated with a 2.0μ sec RMS delay spread with a 15° angular spread at the base and 64-QAM modulation was used. The PAPR was measured on a per-antenna basis by taking an $8\times$ IFFT to simulate a continuous time signal. The PAPR was calculated on a per OFDM symbol basis (i.e., only the maximum peak per OFDM symbol was used in calculating the CDF) and the PAPR was averaged over all transmit antennas. The PAPR reduction algorithm used a clipping value of 3 dB above the average power and the clipping was done separately on each transmit antenna. For calculating the signal to be clipped, the PAPR reduction algorithm used a $4\times$ IFFT.

Figure 1 shows the CDF of the PAPR for the PAPR reduction algorithm for a varying number of mobiles (users) for a fixed number of iterations (nine). As can be seen, as the number of users increases, less PAPR reduction is obtained for a fixed number of iterations. In fact the eight user CDF is identical to the non-PAPR reduced beamformer CDF for any number of users. These results are expected since the degrees of freedom (which are used for PAPR reduction) in the transmit weights decrease as the number of users increases. Note that about 5.5 dB of reduction (measured at the 95% probability point) is seen for the PAPR reduced MRT weights (one user) and the PAPR reduced Tx-SDMA weights for four users sees about 4.5 dB of reduction.

Figure 2 shows the PAPR improvement as a function of the number of iterations for Tx-SDMA with four mobiles. Note that most of the PAPR improvement can be obtained with just a few iterations, and diminishing returns are seen after about five iterations.

Recall that the PAPR-reduced weights presented in this paper are designed to provide the same gain and zero crosstalk as the original weights by allowing the PAPR reduced weights to transmit with more power. The frame error rate (FER) performance of the PAPR reduced weights is thus identical to the non-PAPR reduced weights, but this is for a power-unfair comparison. To assess the level of power increase needed, a power-fair FER comparison is made where the PAPR-reduced transmission is normalized to unit transmit power like the non-PAPR reduced weights. The simulation results looked at the SNR loss at a 0.01 FER (for rate 3/4 turbo-coded 64-QAM) for the transmit weights with PAPR reduction versus no PAPR reduction. The simulation results showed that the PAPR-reduced MRT and Tx-SDMA weights needed about 0.2 dB more power on average regardless of the number of users (one to seven users was simulated) to match the same gain as the non-PAPR reduced weights when nine iterations of the algorithm were used. This 0.2 dB increased transmit power is trivial when compared against the substantial PAPR reduction possible with the algorithm.

5. CONCLUSIONS

This paper presents a new method of PAPR reduction for transmit-beamformed OFDM systems. The method operates on the transmit vector on each OFDM subcarrier (as opposed to the transmit weights) to reduce the PAPR of per-subcarrier MRT/Tx-SDMA transmission while preserving the MRT gain and zero crosstalk at each receiver. Because the gain and zero crosstalk is preserved at the receivers, no performance loss is seen by the receivers when using the PAPR-reduced transmit vector. In simulation results for an 802.16-like system with eight transmit antennas, significant PAPR reduction was seen (e.g., about a 5.5 dB reduction for the PAPR-reduced MRT weights and about a 4.5 dB reduction for the PAPR-reduced Tx-SDMA weights for four receivers).

6. REFERENCES

[1] B.S. Krongold and D.L. Jones, “Peak Power Reduction via Active Constellation Extension,” *IEEE Transactions on Broadcasting*, Vol. 49, No. 3, September 2003, pp. 258–268.

[2] D.L. Jones, “Peak Power Reduction in OFDM and DMT via Active Channel Modification,” *Proc. 33st Asilomar Conference on Signals, Systems, and Computers*, 1999, pp. 1076–1079.

[3] T.E. Detwiler and D.L. Jones, “OFDM Receiver Design for Active Constellation Extension,” *Proc. 39th Asilomar Conference on Signals, Systems, and Computers*, 2005.

[4] H.-C. Lee, D.N. Liu, W.-J. Zhu, and M.P. Fitz, “Peak power reduction using a unitary rotation in multiple transmit antennas,” *IEEE International Conference on Communications*, 2005, pp. 2407–2411.

[5] G. Woo and D.L. Jones, “Peak Power Reduction in MIMO OFDM via Active Channel Extension,” *IEEE International Conference on Communications*, 2005.

[6] J. K. Cavers, “Single-User and Multiuser Adaptive Maximal Ratio Transmission for Rayleigh Channels,” *in IEEE Transactions on Vehicular Technology*, Vol. 49, No. 6, November 2000.

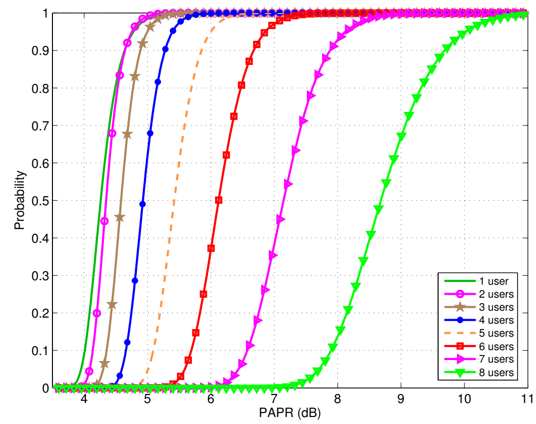

Fig. 1. CDF of PAPR for varying number of beams/users for 9 iterations of the PAPR reduction algorithm.

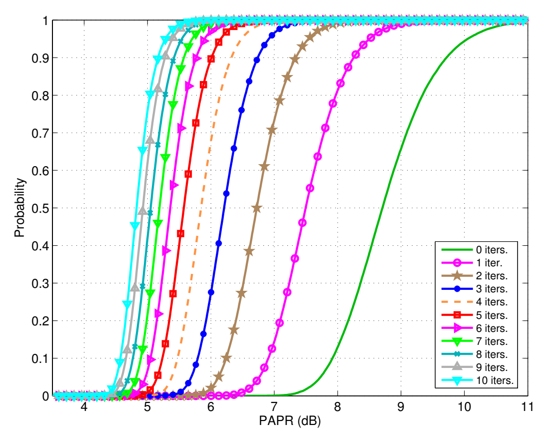

Fig. 2. CDF of PAPR for varying number of iterations of the PAPR reduction algorithm for Tx-SDMA weights with four beams/users (i.e., $B = 4$).