POWER-DISTORTION PERFORMANCE OF SUCCESSIVE CODING STRATEGY IN GAUSSIAN CEO PROBLEM

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ABSTRACT

In this paper, we investigate the power-distortion performance of the successive coding strategy in the so-called quadratic Gaussian CEO problem. In the CEO problem, L sensors will be deployed to observe independently corrupted versions of the source. They communicate information about their observations to the CEO through a Gaussian multiple access channel (MAC) without cooperating with each other. Two types of MAC are considered: orthogonal MAC and interfering (non-orthogonal) MAC. We address the problem from an information theoretic perspective and obtain the optimal tradeoff between the transmission cost, i.e., power, and the distortion D using Shannon's source-channel separation theorem. We also determine the optimal power allocation scheme based on the successive coding strategy to minimize the total power consumption in the sensor network.

1. INTRODUCTION

Smart environments require the information provided by the distributed wireless sensor networks (WSNs) [1]. New advances in hardware and wireless network technologies have reduced the cost, size, and power of micro-sensors which enables the application of distributed wireless sensing to a wide range of applications, including monitoring of remote locations, volcanic eruptions, the environment and traffic. However, the limited amount of energy available at wireless sensors has important effects on all aspects of WSNs, from the amount of information that the sensor node can process to the amount of information that can communicate to other nodes [2]. Since the bit-rate directly impacts transmission power consumption at a node, an efficient high ratio data compression can reduce energy consumption in a WSN. One of the enabling technologies is Distributed Source Coding, which refers to the compression of multiple correlated sensor outputs that do not communicate with each other. The outputs of these sensors will be sent to the Chief Executive Officer (CEO) for joint decoding. In the CEO problem defined in [3], L sensors observe independent noisy versions of the source signal X. Each sensor communicates information about its measurement to the CEO at rate $\{R_i\}_{i=1}^{L}$. The CEO desires to form an optimal estimate of X based on the information received from the sensors.

In practice joint decoding of all sensors' transmissions at the CEO is very difficult to implement. Hence, we consider the successively structured CEO problem, where the CEO problem is decomposed into a sequence of data fusion encoding and decoding blocks based on the noisy Wyner-Ziv results [4]. The distortion sum-rate tradeoff for the two sensor nodes with the same signal-to-noise ratio (SNR) is obtained in [4]. In [5], we extend the results of [4] from two equal-SNR sensors to L sensor nodes ($L > 2$) with different SNRs. We derive the optimal sum-rate distortion tradeoff based on the successive coding strategy for the Gaussian CEO problem and show that this is indeed the optimal sum-rate distortion function for the Gaussian CEO problem [5]. Hence, the successive coding strategy is an enabling technique with low complexity in order to achieve the optimal sum-rate distortion function of the Gaussian CEO problem.

Since the final goal in a sensor network is to reconstruct the measured phenomenon to whitin some prescribed distortion level at the smallest cost in the communication link [6], finding a suitable coding strategy to achieve this goal is critical. We obtain the power-distortion regions for Shannon's separation coding paradigm in the Gaussian sensor network with orthogonal and interfering MAC. We also determine the optimal power allocations to minimize the total power consumption in the network.

The rest of this paper is organized as follows: In Section 2, the problem statement is presented. In Section 3, we use the successive coding strategy and obtain the optimal power-distortion region and optimal power allocations for the Gaussian CEO problem under orthogonal and non-orthogonal MAC. Conclusions are given in Section 4.

2. PROBLEM STATEMENT

We consider the source-channel communication problem in sensor networks [6]. The distributed sensor network model studied in this paper is shown in Fig. 1. In this model, a firm's CEO is interested in the data sequence $\{X(t)\}_{t=1}^{\infty}$.

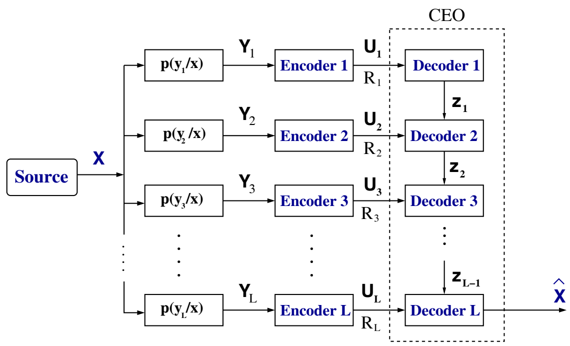

Fig. 1. Distributed sensor network model based on the successively structured CEO.

The target data cannot be observed directly. The CEO deploys a team of L sensors to observe the source data sequence. The sensors observe independent noisy versions of this sequence, represented by the set $\{Y_i(t)\}_{t=1}^{\infty}$ for L sensors. The sensors communicate information about their observed data to the CEO through a MAC under a transmission cost constraint. This constraint comes from the restrictions on the resources such as bandwidth or power that are available at sensor nodes. Sensors cannot cooperate to exploit their correlation. Here, the transmission cost constraint is in the form of

$$\frac{1}{n}\sum_{t=1}^{n} E\left[|U_i(t)|^2\right] \leq P_i \qquad i = 1, ..., L \tag{1}$$

which is in fact the constraint on the transmission power for each sensor. The CEO produces the source estimate $\widehat{\mathbf{X}}$ to an acceptable degree of fidelity D. The measure of the fidelity is the average distortion criterion, i.e., $\frac{1}{n}E\left[\sum_{i=1}^{n} d(x_i, \widehat{x}_i)\right]$ where $d(x, \widehat{x})$ is the mean-squared error distortion measure and n is the block length. The objective is to determine the power-distortion region achieved by separate source-channel coding for the Gaussian CEO problem in an information-theoretic sense. This includes determining optimal power allocations to minimize the total power consumption for any given distortion $D \geq 0$.

We consider the successive coding strategy introduced in [4] to characterize all achievable power-distortion pairs $(P_1, P_2, ..., P_L, D)$. This strategy is a joint design of source coding, communication and data fusion steps. When a sensor node encodes a message, it considers its observation and its statistical knowledge about the messages that the decoder has already received from any other nodes in the network, as the "decoder side information". Note that all nodes are assumed to know the full joint statistical description of the source and observations. At the CEO, instead of joint decoding, messages from sensors are decoded sequentially in order to increase the fidelity of estimation at each decoding step. Using this strategy, the CEO problem can be decomposed into a sequence of "noisy" Wyner-Ziv cases [4]. This strategy gives a less complex way to attain the prescribed distortion in the CEO problem.

3. OPTIMAL POWER-DISTORTION TRADEOFF

Assume that the memoryless Gaussian source X is observed at each sensor node in independent additive white Gaussian noise (AWGN). If we represent the observation of the i^{th} sensor node by Y_i, then $Y_i = X + V_i$ where $X \sim N\left(0, \sigma_X^2\right)$ and $V_i \sim N\left(0, N_i\right)$. Observations are conditionally independent given the source.

There are two steps to characterize all power-distortion pairs $(P_1, P_2, ..., P_L, D)$ achievable by the successive coding strategy: (1) the source coding part which is to characterize the rate-distortion region and (2) the capacity of a MAC and applying Shannon's separation theorem for the source-channel communication problem in a wireless sensor network. Let $P = (P_1, P_2, ..., P_L)$. For any target distortion $D \geq 0$, the power-distortion region is defined by

$$\mathcal{P}(D) = \left\{(P_1, P_2, ..., P_L) \mid (P, D) \ is \ admissible\right\}$$

We say (P, D) is admissible if there is a coding scheme that can achieve a distortion close to D while satisfying the transmission cost constraints. For the separate source and channel coding, a power-distortion pair (P, D) is admissible if the rate-distortion region $R(D)$ and the capacity region $C(P)$ intersects, i.e., $R(D) \cap C(P) \neq \emptyset$ [6].

3.1. Sum-Rate Distortion Function

From the view-point of source coding for the CEO problem, we are only interested in the tradeoff between the estimation distortion and the total rate at which the sensors may communicate information about their observations to the CEO. The sum-rate distortion tradeoff based on the successive coding strategy for the Gaussian CEO problem consisting of L sensors can be described as follows [5].

Theorem 1: Let M denote the number of active sensors where M is the largest integer value between 1 and L such that

$$\frac{M}{N_M} \geq \left(\frac{1}{\sigma_X^2} + \sum_{i=1}^{M}\frac{1}{N_i} - \frac{1}{D}\right). \tag{2}$$

Then, the minimum sum-rate distortion tradeoff based on the successive coding strategy for the Gaussian CEO problem is

$$\overline{R}(D) = \frac{1}{2}\log_2\left\{\frac{\sigma_X^2}{D}\frac{1}{\prod_{i=1}^{M} N_i}\left(\frac{M}{\frac{1}{D^*(M)} - \frac{1}{D}}\right)^M\right\} \tag{3}$$

for $D^*(M) \leq D \leq \sigma_X^2$ where

$$D^*(M) = \left(\frac{1}{\sigma_X^2} + \sum_{i=1}^{M}\frac{1}{N_i}\right)^{-1}. \tag{4}$$

$D^*(M)$ is the minimum mean square error (MMSE) of X given $\{Y_1, ..., Y_M\}$. In [5], we show that this is indeed the sum-rate distortion function for the CEO problem by comparing the result of Theorem *1* with the results of [7] and [8]. This strategy gives a less complex way to attain the prescribed distortion and thus is simple to implement for the WSN.

3.2. Capacity Region of a MAC

The capacity-region of the MAC, i.e, the set of all achievable rate pairs $(R_1, R_2, ..., R_L)$ when the channel inputs satisfy the power constraints, is determined when the messages of different users are independent [9]. But in the case of sensor networks, the messages are not independent. In this paper, we consider the problem of source-channel communication in Gaussian sensor networks under two kinds of MACs: orthogonal MAC and interfering MAC.

3.2.1. Orthogonal MAC*:*
By considering multiple-access schemes such as time/frequency/code division multiple access (TDMA/FDMA/CDMA), the channels between L sensors and the CEO are orthogonal [10]. In other words, the Gaussian MAC is reduced to an array of L independent single-user Gaussian channels. These channels can be modeled as AWGN channels with individual channel gains $\left\{\sqrt{g_i} : i = 1, 2, ..., L\right\}$ [11]. The sensor network model for this case is shown in Fig. 2. Since sensor i has a transmission power constraint of P_i, the capacity region can be represented as

$$\mathcal{C}(\mathcal{P}) = \left\{(R_1, R_2, ..., R_L) \mid 0 \le R_i \le \frac{1}{2}\log\left(1 + g_i P_i\right)\right\} \tag{5}$$

3.2.2. Interfering MAC*:*
Consider the Gaussian sensor network model of Fig. 3 where Z represents the AWGN of the interfering channel with the variance of σ_Z^2. The sum-rate capacity region of a Gaussian MAC with correlated data is not known. Instead, we use the upper bound on the sum-rate in which we assume arbitrary correlation between the inputs of the MAC. To obtain the upper bound, we use

$$\sum_{i=1}^{L} R_i \le I(U_1, U_2, ..., U_L; W) \tag{6}$$

where U_i's are inputs of the interfering MAC and W is its output. We assume the noise of the MAC is independent of the input signals of the channel. The mutual information in (6) can be obtained by

$$\begin{aligned} I(U_1, U_2, ..., U_L; W) &= h(W) - h(W \mid U_1, U_2, ..., U_L) \\ &= h(W) - h(Z) = \tfrac{1}{2}\log(\sigma_W^2/\sigma_Z^2) \end{aligned} \tag{7}$$

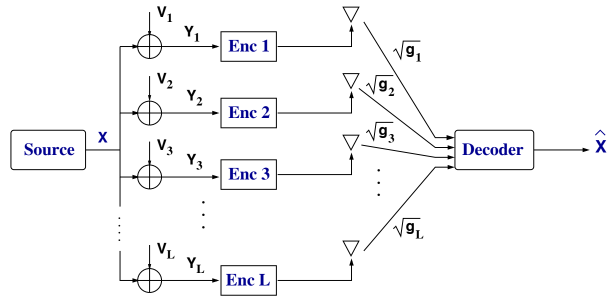

Fig. 2. Gaussian sensor network under orthogonal MAC. Channel gains are represented by $\sqrt{g_i}$'s.

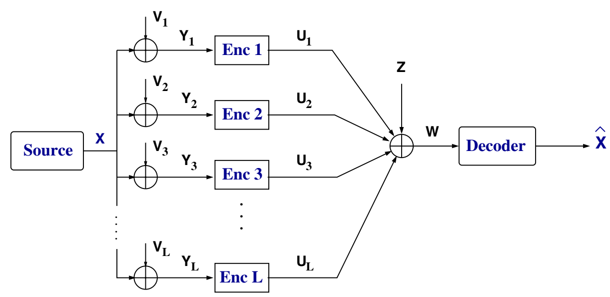

Fig. 3. Gaussian sensor network under interfering MAC. Z represents the AWGN of the channel.

where $h(X)$ denotes the differential entropy of X. By computing the variance of the channel output W, the following lemma will be obtained.

Lemma 1: An upper bound on the sum-rate of the Gaussian interfering MAC with dependent inputs; each input with a power constraint P_i, is as follows:

$$\sum_{i=1}^{L} R_i \le \frac{1}{2}\log\left(1 + \frac{\left(\sum_{i=1}^{L}\sqrt{P_i}\right)^2}{\sigma_Z^2}\right) \tag{8}$$

Note that to be consistent with [6], the channel gains are assumed to be 1. If the channel gain of the i^{th} channel in the interfering MAC is $\sqrt{g_i}$, the upper bound would be the same as (8) except P_i is replaced by $g_i P_i$.

3.3. Shannon's Separation Theorem

Shannon proved that separate source and channel code design is an optimal strategy for the ergodic point-to-point communication scenario [12]. Combining the sum-rate distortion achieved by the successive coding strategy in (3) and the capacity region of the MAC in (5) and (8), we obtain the optimal power-distortion region of the Gaussian CEO problem under orthogonal and non-orthogonal MAC.

3.3.1. Orthogonal MAC*:*

Theorem 2: The optimal power-distortion region for the

Gaussian CEO problem under orthogonal MAC is:

$$\mathcal{P}^o(\mathcal{D}) = \Big\{(P_1, P_2, ..., P_M) \mid \prod_{j=1}^{M}(1+g_jP_j) \geq \left(\frac{\sigma_X^2}{D}\frac{1}{\prod_{i=1}^{M}N_i}\left(\frac{M}{\frac{1}{D^*(M)}-\frac{1}{D}}\right)^M\right)\Big\} \quad (9)$$

Proof: Combining (3) with (5) and doing some manipulations will result in (9). ■

Now we want to minimize the total power consumption, i.e., $P_{sum} = \sum_{i=1}^{M} P_i$, in the sensor network. In other words, we want to find $(P_1, P_2, ..., P_M)$ for the following problem:

$$\begin{cases} min & \sum_{i=1}^{M} P_i \\ s.t. & (P_1, P_2, ..., P_M) \in \mathcal{P}^o(\mathcal{D}) \end{cases} \quad (10)$$

Theorem 3: Optimal power allocations based on the successive coding strategy for any given distortion $D \geq 0$ can be expressed as

$$P_i^{opt} = \left\{\left(\frac{M}{\frac{1}{D^*(M)}-\frac{1}{D}}\right)\left(\frac{\sigma_X^2}{D}\frac{1}{\prod_{j=1}^{M}g_jN_j}\right)^{\frac{1}{M}} - \frac{1}{g_i}\right\} \quad (11)$$

for $i = 1, 2, ..., M$.

Proof: Let $Q = \left(\frac{\sigma_X^2}{D}\frac{1}{\prod_{i=1}^{M}N_i}\left(\frac{M}{\frac{1}{D^*(M)}-\frac{1}{D}}\right)^M\right)$. To minimize the total power consumption under the constraint of $\prod_{j=1}^{M}(1+g_jP_j) = Q$ we use the Lagrange multiplier method, as the minimization of

$$J = P_1 + P_2 + \cdots + P_M + \lambda\left(\prod_{j=1}^{M}(1+g_jP_j) - Q\right) \quad (12)$$

Differentiating with respect to P_k and P_l and setting the results to 0 leads to

$$1 + g_lP_l = \frac{g_l}{g_k}(1+g_kP_k) \quad (13)$$

Using the constraint $\prod_{j=1}^{M}(1+g_jP_j) = Q$ and doing some manipulations gives the optimal power allocation of (11). ■

3.3.2. Interfering MAC*:*

Theorem 4: For any $D \geq 0$, the successive coding strategy achieves the following power-distortion region:

$$\mathcal{P}^I(\mathcal{D}) = \Big\{(P_1, P_2, ..., P_M) \mid \left(\sum_{i=1}^{M}\sqrt{P_i}\right)^2 \geq \left(\frac{\sigma_X^2}{D}\frac{\sigma_Z^2}{\prod_{i=1}^{M}N_i}\left(\frac{M}{\frac{1}{D^*(M)}-\frac{1}{D}}\right)^M - \sigma_Z^2\right)\Big\} \quad (14)$$

Proof: By combining (3) with (8) and doing some manipulations we obtain the result of (14). ■

Again, we use the Lagrange multiplier method to minimize the total power consumption while achieving a given average distortion D. The proof involves simple calculations and is omitted due to space constraints.

Theorem 5: For the case of interfering MAC, the optimal power allocation method allocates equal rates to different sensors which is equal to

$$P_i^{opt} = \frac{\sigma_Z^2}{M^2}\left(\frac{\sigma_X^2}{D}\frac{1}{\prod_{j=1}^{M}N_j}\left(\frac{M}{\frac{1}{D^*(M)}-\frac{1}{D}}\right)^M - 1\right) \quad (15)$$

We see that all P_i's are the same. Because in the upper bound for the sum-rate capacity of the interfering MAC, we assumed that the channel gains are 1. If the channel gain of the i^{th} channel in the interfering MAC is $\sqrt{g_i}$, then the optimal power allocation is the same as (15) except σ_Z^2/M^2 is replaced by $\left(g_i\sigma_Z^2\right) / \left(\sum_{j=1}^{M} g_j\right)^2$.

4. CONCLUSION

Power-distortion performance of the successive coding strategy in the Gaussian CEO problem is evaluated. Two types of MAC are considered: orthogonal MAC and interfering MAC. We obtained the power-distortion tradeoff for both cases using Shannon's separation theorem. We also determined the optimal power allocation scheme which minimizes the total power consumption for any given distortion D in the sensor network.

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