A BICM Approach to Type-II Hybrid ARQ

Jeremy Roberson and Zhi Ding
Department of Electrical and Computer Engineering
University of California Davis
Davis, CA, 95616
Email: jlrobe@ece.ucdavis.edu, zding@ece.ucdavis.edu

***Abstract*— We propose a low complexity H-ARQ utilizing Bit Interleaved Coded Modulation (BICM). Our aim is to improve the reliability and bandwidth efficiency of wireless communications using high rate H-ARQ enabled by the proposed BICM option. This type-II hybrid ARQ generates incremental redundancy and diversity by varying the bit-to-symbol mapping during retransmission without relying on FEC design. The joint detection at the receiver through maximum likelihood demapping before error correction is simple and effective. Using identical the interleaver and convolutional FEC encoder, this scheme is inexpensive and is largely independent of the outer code selection.**

I. Introduction

Bit Interleaved Coded Modulation (BICM) is a coding scheme that consists of an FEC code, typically a convolution code, followed by a bit-level interleaver and finally a bit-to-symbol mapping. It was first proposed in [1] and formally analyzed in [2]. It has been discovered [3] that BICM could be much more powerful if it is decoded iteratively much like a Serially Concatenated Convolutional Code or (SCCC). When iteratively decoded, BICM is referred to as BICM-ID [3]. Considerable research results on finding suitable mappings and evaluating the performance of BICM-ID systems can be found in the literature, e.g. [4][5].

On the other hand, wireless channels are notoriously unreliable because of fading and distortions. As an adaptive error protection measure, Automatic Repeat reQuest (ARQ) can be used to ensure that data is received reliably when a feedback channel exists for a simple ACK or a NACK message from the receiver to the transmitter. Equipped with error detection capability, the receiver checks for packet errors and sends a NACK requesting retransmission to the transmitter if a packet is found erroneous. Type II hybrid ARQ (H-ARQ) allows receivers to detect the packet by jointly detecting the received signals from multiple retransmissions.

Traditionally, type II H-ARQ would send incremental FEC parity during retransmissions. Recently, it was shown in [6][7] that by designing different bit-to-symbol mappings during retransmissions, joint maximum likelihood (ML) detection at the receiver can achieve large gains for H-ARQ even in uncoded systems. This paper extends the work in [6][7] for integration with BICM, by redesigning bit-to-symbol mapping for better performance in the BICM-ID system.

This manuscript is organized as follows: Section II, presents the system model for BICM-ID. Section III formulates the problem of finding optimal mappings for BICM-ID in the H-ARQ context. Section IV covers various considerations for solving the problem and also presents the proposed mapping schemes, The performance of the proposed mappings is demonstrated in V followed by a brief conclusion VI.

Work supported by National Science Foundation under Grant ECS-0121469

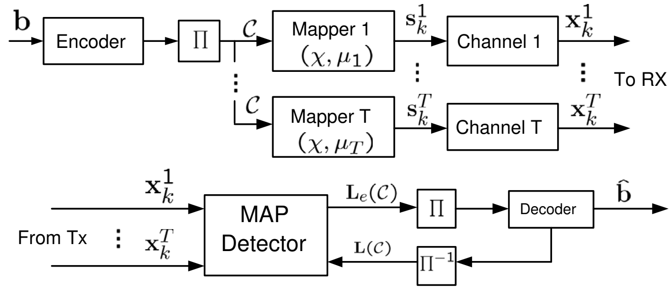

Fig. 1. H-ARQ system model driven by BICM-ID.

II. System Model

Figure 1 illustrates the use of BICM-ID system in H-ARQ. In the model, a particular packet has $T-1$ retransmission requests, and thus there are T total transmissions, leading to T channel outputs, x_k^i. This is in contrast to the traditional BICM-ID system which only assumes one channel output per packet. The diagram shows that we may only use one common convolutional encoder, and one interleaver. For the same constellation χ, there are T distinct bit-to-symbol mappings, μ_i and T received symbols $\mathbf{x}_k = [x_k^1, \cdots, x_k^T]^T$.

For joint detection and decoding, the receiver structure of BICM-ID exploits the standard Turbo Process and has been extensively studied in [3][4][5][9]. The only difference in the system for this paper is that the Soft Input Soft Output detector LLR should be adjusted to accommodate more than one transmission during demapping.

$$L_e(c_i) = \frac{\sum_{l_n \in \Omega_0^i} \exp(\frac{\|\mathbf{x}_k - \mu(l_n)\|^2}{2\sigma_n^2})}{\sum_{l_n \in \overline{\Omega}_0^i} \exp(\frac{\|\mathbf{x}_k - \mu(l_n)\|^2}{2\sigma_n^2})} \exp(L(c_i)) \qquad (1)$$

in which $\mu(l_k) = [\mu_1(l_n), \cdots, \mu_T(l_n)]^T$.

The problem that we study involves the design of remapping functions during retransmission for better performance.

III. H-ARQ Remapping Design Formulation

A. H-ARQ Mapping Objective

The problem posed in this paper is finding good $T-1$ retransmission mappings, $\mu_2, \cdots, \mu_T$, that deliver superior performance for the joint MAP receiver, given the first mapping μ_1 as defined in [3][9]. Based on the soft extrinsic information on the detected packet from the joint detection and decoding after K transmissions, an iterative process can be used to design the $(K+1)-$th mapping successively.

Without considering the FEC effect, an optimized mapping for H-ARQ retransmission for ML detection were found [7]. In this paper, however, we seek to design H-ARQ mappings for the MAP detection of BICM. The main difference between the ML detector and the MAP detector is the additional consideration of the extrinsic information passed to the MAP detector from the Soft Input Soft Output (SISO) FEC decoder made popular by turbo and LDPC codes.

This work proposes to find the $T-1$ mappings by focusing on the minimization of the BER expression for MAP detection. The design procedure for multiple H-ARQ transmissions is generalized from the single transmission formulation.

B. General Mapping Cost Function

The BER of a QAM system can be upper-bounded and in fact closely approximated by the bit level pairwise error probability (PEP), shown as follows

$$P(\mathbf{c} \to \hat{\mathbf{c}}) = \sum_{i=1}^{m} \sum_{l_k \in \Omega_0^i} \sum_{l_n \in \overline{\Omega}_0^i} P(\mu(l_k) \to \mu(l_n)). \qquad (2)$$

This bit level PEP is a function of symbol level PEP. For linear flat fading channels in this paper, the symbol PEP is a function of the squared distance between symbols

$$P(s_k \to s_n) = f(|s_k - s_n|^2). \qquad (3)$$

The function f(.) is dependent on the physical channel, and discussed in section III-C. By definition, $s_n = \mu(l_n)$. The subset Ω_0^i represents the set of all labels that have a 0 in the i^{th} position while $\overline{\Omega}_0^i$ is simply the complementary subset.

While formulating the cost function for H-ARQ BICM mapping design, equation (2) can be simplified by restricting our focus to the two extreme cases of no prior and ideal prior. The former represents the case when the retransmission receives zero prior information about other bits from the FEC while the later represents the case when the FEC sends perfect prior information about bits not currently being considered for detection. Without prior information, the average Hamming distance is approximately

$$D^0(\mu) = \sum_{k=0}^{M-1} \sum_{n=k+1}^{M-1} H(\ell_k, \ell_n) f(|\mu(\ell_k) - \mu(\ell_n)|^2) \qquad (4)$$

We can model ideal-prior information as the ability to totally distinguish between the correct symbol and an erroneous one, if the Hamming distance between the two symbols is greater than 1. Hence we minimize part of the previous cost:

$$D^1(\mu) = \sum_{k=0}^{M-1} \sum_{n=k+1}^{M-1} \delta(H(\ell_k, \ell_n) - 1) f(|\mu(\ell_k) - \mu(\ell_n)|^2). \qquad (5)$$

The number of symbols in the constellation is M, the Hamming distance function is $H(.,.)$, and $\delta(.)$ is the kronecker delta function. By definition, $s_n = \mu(\ell_n)$, where the complex value, s_n, depends on the mapping μ. The term, ℓ_n, represents the label n (e.g. $\ell_{12} = 1100$). Finally, the function $f(.)$ represents a channel dependent cost that can approximate the symbol-wise PEP.

Using MAP detection, the bit level PEP for multiple transmissions in the two cases are

$$D^0(\mu_1, \cdots, \mu_T) = \sum_{k=0}^{M-1} \sum_{n=k+1}^{M-1} H(\ell_k, \ell_n) f(\sum_{i=1}^{T} |\mu_i(\ell_k) - \mu_i(\ell_n)|^2) \qquad (6)$$

$$D^1(\mu_1, \cdots, \mu_T) = \sum_{k=0}^{M-1} \sum_{n=k+1}^{M-1} \delta(H(\ell_k, \ell_n) - 1) f(\sum_{i=1}^{T} |\mu_i(\ell_k) - \mu_i(\ell_n)|^2). \qquad (7)$$

The term μ_i represents the mapping during transmission i and T represents the number of transmissions.

Now, we can formulate the optimization problem that will be utilized to design the optimal retransmission mappings:

$$\arg \min_{(\mu_1 \cdots \mu_T)} D(\mu_1 \cdots \mu_T). \qquad (8)$$

The purpose of this paper is to find the $T-1$ retransmission mappings $\{\mu_i\}$ that minimize the PEP cost in equation (7) in order to improve the ARQ BICM-ID performance. Such an optimization problem, (8), is an NP-hard quadratic assignment problem (QAP), [8], that can be approximately solved using various fast techniques that will be briefly discussed later.

C. Channel Models

Consider several important classes of wireless channels. We can determine the PEP function $f(.)$ for these cases. Specifically, we limit our discussions to noncoherent detection under AWGN channel, coherent detection in AWGN channels, coherent detection in AWGN channels, and coherent detection in Rayleigh fading channels. For these three cases, the symbol PEP function is given, respectively, by

$$f_1(x) = \exp(-\frac{E_s}{2\sigma_n^2} x), \qquad (9)$$

$$f_2(x) = Q\left(\sqrt{\frac{E_s}{2\sigma_n^2} x}\right), \qquad (10)$$

and

$$f_2(x) = \frac{1}{x}. \qquad (11)$$

For one transmission, x is $|\mu(\ell_k) - \mu(\ell_n)|^2$, while for multiple transmissions, x becomes $\sum_{i=1}^{T} |\mu_i(\ell_k) - \mu_i(\ell_n)|^2$. These channel models have been discussed in works of [2].

IV. H-ARQ Mappings

A. Retransmission Mappings

Given the formulated cost function, $D(\mu_1 \cdots \mu_T)$, we do not need to find all T mappings at once. Because of H-ARQ, a new ($T-$th) retransmission is only needed when previous $T-1$ transmissions fail to deliver the packet. Hence, a reasonable design approach is to solve the problem iteratively by designing the current mapping μ_T given previous mappings $\{\mu_1 \cdots \mu_{T-1}\}$.

In the first transmission, the mapping design follows those of systems without H-ARQ. Hence, the first initial mapping μ_1 can be found using already known methods, such as the Modified Set Partitioning (MSP) [5] and the Maximum Squared Euclidean Weight (MSEW) [9]. In addition to the MSEW and MSP mapping, another mapping could also be used during the first transmission by solving (7) [4].

To begin, we can use the MSP mapping, then the mapping used for the first retransmission can be found by minimizing

$$\mu^{ret1} = \arg\min_{\mu_2} D(\mu^{(MSP)}, \mu_2).$$

Subsequently, to find the third mapping, one can continue by

$$\arg\min_{\mu_3} D(\mu^{(MSP)}, \mu^{ret1}, \mu_3).$$

This process continues until all T transmissions are exhausted for a packet in H-ARQ.

Strictly speaking, because of H-ARQ, we do not tackle the big problem (8) but rather a collection of single mapping cost functions generated by (8). Both $D^1(\mu_1 \cdots \mu_T)$ and $D^0(\mu_1 \cdots \mu_T)$ can be similarly dealt with iteratively to yield different retransmission mappings.

As examples, we show some mappings in Figure 2 that result from the optimization approach presented. The mappings were found under the AWGN channel model of (10) assuming 0dB SNR. In general, the performance was not noticeably different when solving the problem with an SNR between -2 and 2 dB. The first set of mappings MSP and MSEW were obtained for conventional single transmission only. The mappings proposed for the 2nd transmission of MSP and MSEW based BICM are MSP $\mu^{(ip)}$ and MSEW $\mu^{(ip)}$, respectively, for minimization of $D^1(\mu_1, \mu_2)$ with ideal prior. The the MSP and MSEW mappings $\mu^{(zp)}$ resulting from iteratively solving $D^0(\mu_1, \mu_2)$, are also shown as useful mapping at very low SNR.

B. Solving the Optimization Problems

The full solution to even the iterative mapping problem μ_i is NP-hard. In our approach, reality requires that the problem be solved suboptimally using a heuristic solver called the Binary Switching Algorithm (BSA) proposed in [4].

The idea of the Binary Switching Algorithm is to choose a seed mapping. With the seed mapping, compute every possible swap of two indices, and swap the pair of indices that results in the smallest overall cost. Take this new mapping, and recompute every possible swap of indices and perform the best swap again. Keep iterating this process until no improved minimum is found, after which the algorithm stops and reports the minimum for that run. Several runs with different seeds are then performed until the same minimum is repeated several times. It is assumed that this minimum is close enough to global.

In addition to the BSA, near exact solutions may be used to search for better minima using a more powerful QAM solver [11]. In our experiment, it was found that the BSA algorithm

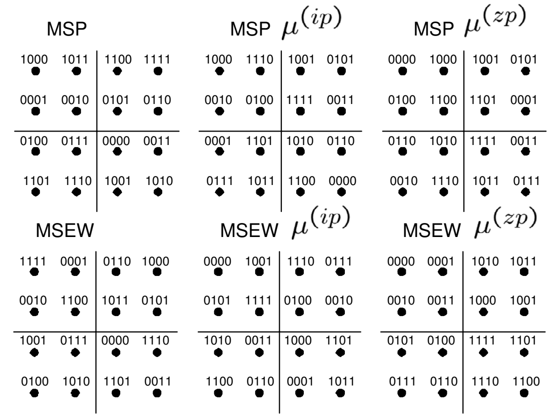

Fig. 2. Mappings for AWGN at 0dB SNR.

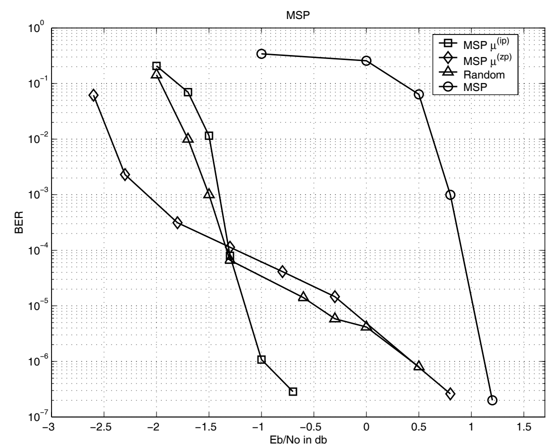

Fig. 3. BER Performance of MSP BICM-ID H-ARQ Using the Various Retransmission Schemes

and the more powerful QAP solver attained very similar mappings. Since the heuristic BSA algorithm is computationally faster, we utilize it in our simulation examples throughout.

V. Simulation Results

A. Summary of Examples

Two different experiments were tested. Each experiment is identical aside from the fact that the mapping used for the first transmission in the first and second experiment is the MSP and MSEW mapping, respectively. There are a total of 2 transmissions in H-ARQ used. The simulation results for the MSP and MSEW scenarios are shown in Figure 3 and Figure 4, respectively.

As shown in each figure, there are 4 curves, representing 4 different remapping schemes used in the 2nd H-ARQ transmission. We compare the performance of both $\mu^{(zp)}$, $\mu^{(ip)}$ against random remappings and the trivial repetition mapping, and measure the retransmission gain with respect to the repetition mapping after two H-ARQ transmissions and detection. In the simulation, we average over 20 randomly generated remappings for the "random" remapping case.

Not surprisingly, for moderate bit error rates around 10^{-3}, the $\mu^{(zp)}$ mapping is the best, and has a gain of 2.9 and 3.8 dB

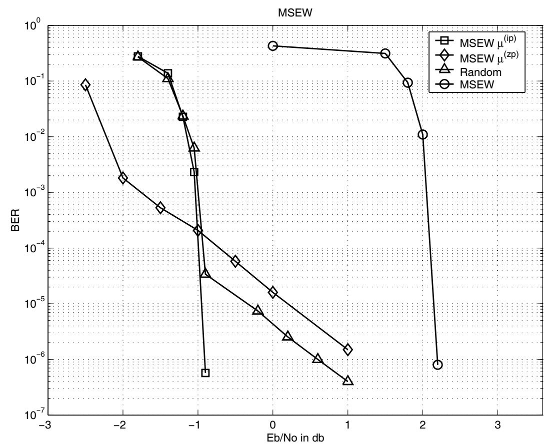

Fig. 4. BER Performance of MSEW BICM-ID H-ARQ Using the Various Retransmission Schemes

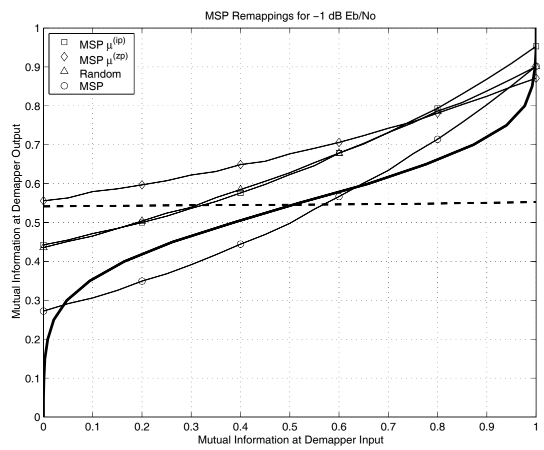

Fig. 5. Information Reliability of MAP Detector Using Various Remappings of MSP.

for MSP and MSEW respectively. At error rates around 10^{-6} for MSP, $\mu^{(ip)}$ achieves a gain of 2.2 dB over the repetition mapping and a gain of 1.5dB over random remapping. For MSEW $\mu^{(ip)}$ achieves a 3.2 dB and 1.6dB gain over the repetition and random mapping schemes.

B. Discussions

As in many turbo receivers, EXIT chart provides a great design tool to understand the bit error rate performance [10]. Figures 5 and 6 are the EXIT charts for the MSP and MSEW schemes at -1 dB and -.8dB respectively. The SNR points were chosen to provide the most meaningful explanation of the simulation results.

As can be seen, at these SNR levels for MSEW and MSP, the repetition code is below the inverse EXIT curve of the convolutional FEC and thus the turbo receiver does not converge well. This explains the poorer BER for repetition mapping at this SNR. The EXIT chart also shows that there is a large enough "tunnel" generated by all other retransmission mappings for the turbo receiver to iterate between demapping and FEC soft decoding to achieve a lower BER. It is clear from

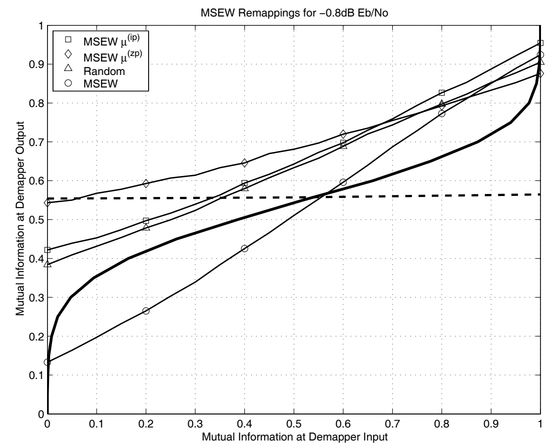

Fig. 6. Information Reliability of MAP Detector Using Various Remappings of MSEW.

the EXIT chart that $\mu^{(ip)}$ should have the best performance, consistent with the BER simulation result.

VI. Conclusion

This paper proposes a simple H-ARQ retransmission strategy that utilizes the advantages of mapping redesign for BICM. The transmitters and receivers are simple and are flexible to allow integration with other FEC codes and turbo equalizers. Without exhaustively searching for optimal mapping designs, significant performance gains have been established for this simple H-ARQ against simple repeat (type I H-ARQ) and type II H-ARQ using randomly selected remapping. Our approach can be generalized for different channel types and is compatible with OFDM and MIMO wireless systems.

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