LOW COMPLEXITY ADMISSION IN DOWNLINK BEAMFORMING

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ABSTRACT

We study the downlink of a system with multiple antennas at the base stations and and propose two different schemes for admission control and beamforming selection when a new user enters the system. To keep the complexity low, only the power control, not the spatial signatures of beamformers for the already admitted users, are adjusted. Numerical examples illustrate the performance compared to joint optimization of all users.

1. INTRODUCTION

When antenna arrays are used in a wireless system, users can be scheduled not only in time and frequency but also in space. To fully utilize the available radio resources, a cross-layer design involving both scheduling and beamformer should be used. Several such schemes have recently been proposed in the literature, see for example [1–3]. A common subproblem in all these schemes is to determine if yet another user may be added in a certain time/frequency slot and what the resulting system performance is. This admission control problem is an issue both when setting up an initial schedule and when adaptively updating the scheduling when a new user wants to connect. Optimal downlink beamforming in a given fixed schedule is described in [4–6] and an approach for admission control is described in [7]. When zero-forcing beamformers are used, it is easy to update the beamforming scheme with a beamformer for a new user, as shown in [8]. However, when other beamforming schemes such as optimal downlink beamforming are used, the computational complexity of updating the beamforming scheme when a new user is added, is of the same order as solving the problem from scratch.

Here, we propose two sub-optimal schemes with lower complexity, based on the idea to only modify the power allocation of the existing users, not the spatial signature of the beamformers. These algorithms are presented in Sect. 2 and evaluated against the jointly optimal solution in Sect. 3.

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2. ALGORITHMS

Consider the downlink of a system where $N-1$ single antenna mobiles are served by one or more base stations, each equipped with an antenna array. Assume that all the mobiles share the same narrowband time/frequency slot (the results can readily be extended to frequency selective channels and beamformers, see [4]). Let $\mathbf{w}_i$ denote the beamformer used to transmit the signal $s_i(t)$ intended for mobile i and $\mathbf{h}_{i,j}$ denote the channel to mobile i from the base station serving mobile j. Then, the total received base band signal at mobile i is

$$r_i(t) = \sum_{n=1}^{N-1} \mathbf{h}_{i,n}^H \mathbf{w}_n s_n(t) + \nu_i(t) \; . \tag{1}$$

If the transmitted signals are independent with unit power and the noise power is $\sigma_i^2 = \mathrm{E}|\nu_i(t)|^2$, the signal to interference to noise ratio (SINR) is

$$\text{SINR}_i = \frac{\mathbf{w}_i^H \mathbf{R}_{i,i} \mathbf{w}_i}{\sum_{n\neq i} \mathbf{w}_n^H \mathbf{R}_{i,n} \mathbf{w}_n + \sigma_i^2} \; , \tag{2}$$

where $\mathbf{R}_{i,j} = \mathrm{E}[\mathbf{h}_{i,j}\mathbf{h}_{i,j}^H]$ averaged over the small-scale fading (if the channels are perfectly known, set $\mathbf{R}_{i,j} = \mathbf{h}_{i,j}\mathbf{h}_{i,j}^H$). The quality of service (QoS) requirement for each user is expressed in the form of a constraint $\text{SINR}_i \geq \gamma_i$, for some given SINR targets, γ_i.

Assume that we already have a feasible beamforming solution $\mathbf{w}_i = \sqrt{p_i}\mathbf{u}_i$, with $\|\mathbf{u}_i\| = 1$, $i = 1, \ldots, N-1$, where the power levels p_i (possibly also the beamformers $\mathbf{u}_i$) have been optimized so that all QoS requirements are fulfilled with equality, $\text{SINR}_i = \gamma_i$. When a new user enters the system, we wish to determine

1. if the user can be admitted, i.e. if there is any feasible beamforming solution for all the N users.
2. a good beamformer to use for the new user.

The only way to decisively verify that the new user cannot possibly be admitted, is to try to solve the jointly optimal beamforming problem which is briefly reviewed in Sect. 2.1. Here, the focus is on algorithms with lower complexity, see Sect. 2.2 and 2.3.

2.1. Jointly optimal beamforming

Jointly optimal downlink beamforming is often defined as the solution to

$$\begin{aligned} &\min_{\{\mathbf{w}_i\}} \sum_{k=1}^{N-1} \|\mathbf{w}_k\|^2 \\ &\text{s.t. } \mathrm{SINR}_i \geq \gamma_i, \quad i = 1, \dots, N-1 \end{aligned} \tag{3}$$

i.e. the total transmitted power is minimized under constraints on the SINR at each receiver. Several algorithms have been proposed to solve this problem, see [4–6]. An alternative formulation, treated in [6] is

$$\begin{aligned} &\max_{\{\mathbf{w}_i\}} \min_k \frac{\mathrm{SINR}_k}{\gamma_k} \\ &\text{s.t. } \sum_{k=1}^{N-1} \|\mathbf{w}_k\|^2 \leq P_{\max} \end{aligned} \tag{4}$$

Note that if the optimal cost function of (4) is ≥ 1, then the system is feasible.

2.2. Power-only update

Here, the main idea is to keep the beamformer vectors $\mathbf{u}_i$ of all the old users. Clearly it is impossible to admit the new user if also the power levels are kept fixed. Therefore, the goal is to determine new power levels p_i, $i = 1 \dots, N-1$ for the old users and a beamformer $\mathbf{w}_N$ for the new user, if possible. Introduce the $(N-1) \times (N-1)$ matrix $\mathbf{G}$, where

$$\mathbf{G}_{kl} = \begin{cases} (\mathbf{u}_k^H \mathbf{R}_{k,k} \mathbf{u}_k)/\gamma_k & k = l \\ -\mathbf{u}_l^H \mathbf{R}_{k,l} \mathbf{u}_l & k \neq l \end{cases} \tag{5}$$

and the vector $\boldsymbol{\sigma} = [\sigma_1^2, \dots, \sigma_{N-1}^2]^T$. Then, before adding the new user, the power vector $\mathbf{p}^{\text{old}} = [p_1, \dots, p_{N-1}]^T$ fulfills $\mathbf{p}^{\text{old}} = \mathbf{G}^{-1}\boldsymbol{\sigma}$, since we assumed SINR balanced power control. When the new user is added with a beamformer $\mathbf{w}_N$, the interference power at user i (i.e. the denominator of (2)) is increased by $\Delta_i = \mathbf{w}_N \mathbf{R}_{i,N} \mathbf{w}_N$. Viewing this increased interference as an increased noise level, it follows directly that the SINR balanced power allocation after adding the new user is given by $\mathbf{p}^{\text{new}} = \mathbf{G}^{-1}(\boldsymbol{\sigma} + \boldsymbol{\Delta}) = \mathbf{p}^{\text{old}} + \mathbf{G}^{-1}\boldsymbol{\Delta}$, where $\boldsymbol{\Delta} = [\Delta_1, \dots, \Delta_{N-1}]^T$. The SINR constraint for the new user can be written in the form

$$\begin{aligned} \mathbf{w}_N^H \mathbf{R}_{N,N} \mathbf{w}_N =& \gamma_N \left(\sum_{n=1}^{N-1} p_n^{\text{new}} \mathbf{u}_n \mathbf{R}_{N,n} \mathbf{u}_n + \sigma_N^2 \right) \\ =& \gamma_N (\boldsymbol{\alpha}^T \mathbf{p}^{\text{new}} + \sigma_N^2) \\ =& \gamma_N (\boldsymbol{\alpha}^T (\mathbf{p}^{\text{old}} + \mathbf{G}^{-1}\boldsymbol{\Delta}) + \sigma_N^2) \end{aligned} \tag{6}$$

where $[\boldsymbol{\alpha}]_n = \mathbf{u}_n \mathbf{R}_{N,n} \mathbf{u}_n$. Introduce the vector $\boldsymbol{\beta} = \mathbf{G}^{-T}\boldsymbol{\alpha}$ and the matrix $\mathbf{A} = \sum_n [\beta]_n \mathbf{R}_{n,N}$. Then, (6) takes the form

$$\mathbf{w}_N^H (\mathbf{R}_{N,N} - \gamma_N \mathbf{A}) \mathbf{w}_N = \gamma_N (\boldsymbol{\alpha}^T \mathbf{p}^{\text{old}} + \sigma_N^2) \tag{7}$$

Since the right hand side is a known positive constant, it follows that if only the power levels of the old users may be changed, the new user can be admitted to the system iff the matrix $\mathbf{R}_{N,N} - \gamma_N \mathbf{A}$ has at least one positive eigenvalue.

How should $\mathbf{w}_N$ be chosen? A natural choice is to minimize the increase in total power, $\|\mathbf{w}_N\|^2 + \mathbf{1}^T(\mathbf{p}^{\text{new}} - \mathbf{p}^{\text{old}})$. Introducing $\boldsymbol{\lambda} = \mathbf{G}^{-T}\mathbf{1}$ and $\mathbf{B} = \sum_n [\boldsymbol{\lambda}]_n \mathbf{R}_{n,N} + \mathbf{I}$, we get $\mathbf{1}^T(\mathbf{p}^{\text{new}} - \mathbf{p}^{\text{old}}) + \|\mathbf{w}_N\|^2 = \mathbf{w}_N^H \mathbf{B} \mathbf{w}_N$, so the optimal beamformer is found as the solution of

$$\begin{aligned} &\min_{\mathbf{w}_N} \mathbf{w}_N^H \mathbf{B} \mathbf{w}_N \\ &\text{s.t. } \mathbf{w}_N^H (\mathbf{R}_{N,N} - \gamma_N \mathbf{A}) \mathbf{w}_N = \gamma_N (\boldsymbol{\alpha}^T \mathbf{p}^{\text{old}} + \sigma_N^2) \end{aligned} \tag{8}$$

i.e. the eigenvector with largest eigenvalue μ of the following generalized eigenvalue problem [9], properly scaled to fulfill the constraint of (8),

$$(\mathbf{R}_{N,N} - \gamma_N \mathbf{A}) \mathbf{w}_N = \mu \mathbf{B} \mathbf{w}_N \ . \tag{9}$$

If the algorithm is repeated several times with the same initial scenario and different prospective new users, note that only $\mathbf{A}$ and the eigenvalue test of $\mathbf{R}_{N,N} - \gamma_N \mathbf{A}$ has to recalculated to check the feasibility of each new user. In addition, $\mathbf{B}$ and (8) has to recalculated for each user to determine the optimal beamformer and the corresponding increase in power.

2.3. Exploiting the virtual uplink

The algorithms in [5, 6] for joint optimal downlink beamforming are based on a reformulation into an equivalent "virtual uplink problem", which is solved by iteratively determining virtual uplink power levels and beamformers. Assuming that the beamforming solution for the original $N-1$ users was obtained using one of these algorithms, it may be tempting to try an alternative approach and keep not only the normalized beamformers $\mathbf{u}_i$, but also the virtual uplink powers q_i of the old users. Then, the normalized beamformer $\mathbf{u}_N$ of the new user can be calculated using the following single step from the above mentioned algorithms,

$$\mathbf{u}_N = \arg\max \frac{\mathbf{u}_N^H \mathbf{R}_{N,N} \mathbf{u}_N}{\mathbf{u}_N^H (\sum_{n=1}^{N-1} \frac{q_n}{\sigma_n^2} \mathbf{R}_{n,N} + \mathbf{I}) \mathbf{u}_N} \tag{10}$$

Once this $\mathbf{u}_N$ has been determined, the feasibility of the resulting system and the downlink power levels for all the beamformers, can be calculated from the SINR balancing power control equation.

There is a close connection to the approach of Sect. 2.2. It can easily be shown that the virtual uplink powers are related to the vector $\boldsymbol{\lambda}$ by $q_i = \sigma_i^2 [\boldsymbol{\lambda}]_i$, so (10) is equivalent to solving the generalized eigenvalue problem,

$$\mathbf{R}_{N,N} \mathbf{w}_N = \mu \mathbf{B} \mathbf{w}_N \ , \tag{11}$$

which should be compared to (9).

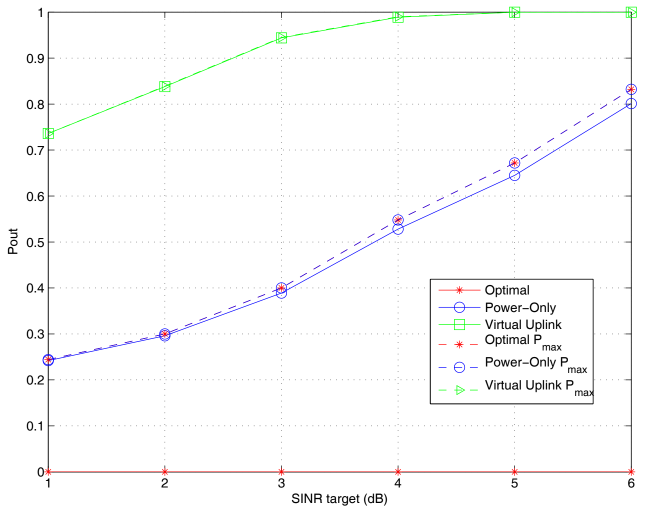

Fig. 1. Outage probability when adding a new user to a system with 4 existing users.

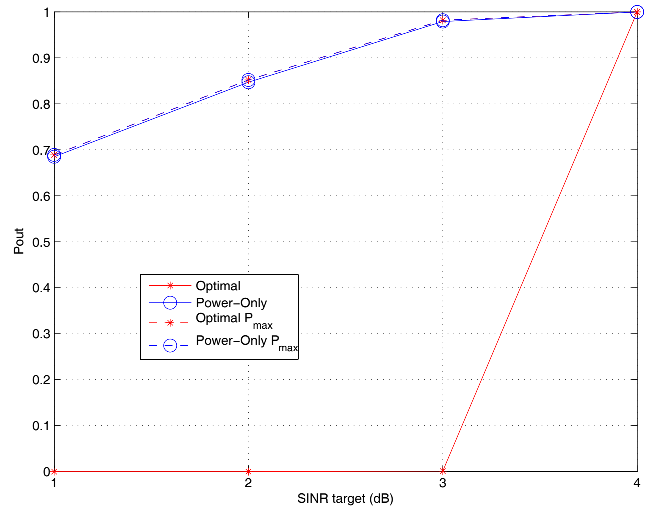

Fig. 2. Outage probability when adding a new user to a system with 5 existing users.

Unfortunately, this algorithm performs significantly worse, as is shown below. However, as can be seen by comparing (9) and (11), the difference is small if the SINR target γ_N is small.

3. NUMERICAL EXAMPLES

The algorithms have been evaluated in a simulated scenario where a single base station equipped with a 4 element uniform linear array serves a number of randomly positioned mobile terminals. The spatial signature corresponds to line of sight propagation and the path loss and shadow fading was calculated using the COST 231 TU channel model, using a carrier frequency of 2.4MHz, a noise power at the receiver of -101dBm and 500m cell radius.

Figures 1 and 2 show the outage probability as a function of the SINR target when the number of users is increased from 4 to 5 and from 5 to 6, respectively. Here, the outage is defined as the probability that the new user cannot be admitted. Two curves are shown for each algorithm, one for the case that there is no limit on the transmit power and one (marked $P_{\max}$), where an outage is declared also if the total transmit power exceeds 20W. In Fig. 1, the joint optimal algorithm without power limits always found a solution for the 5th user, so the outage is 0, in contrast to the "power-only" algorithm of Sect. 2.2. However, when combined with a practical limit on the transmit power, both algorithms provide almost identical outage levels. On the other hand, the algorithm based on existing virtual uplink powers performs significantly worse and the outage level when going from 5 to 6 users was actually 100% (not included in Fig. 2). Note that no zero-forcing beamforming solution exists in these scenarios, since the number of users is larger then the number of antennas.

Another way to evaluate the performance of the "power-only" algorithm, is illustrated in Figures 3 and 4, which show the difference in total transmit power, compared to the jointly optimal solution, for the scenarios where the "power-only" algorithm finds any feasible solution. Fig. 3 shows the cumulative density function of the power difference results when the resulting number of users is 5. For example, for an SINR target of $\gamma = 3$dB, the power difference is less than 4dB in 80% of the scenarios. When the SINR target is 6dB, it is so hard to find a solution at all, so in over 90% of the cases, the solutions of the two algorithms are identical up to the numerical precision of the optimization algorithm. Fig. 4 shows the corresponding CDF when the total number of users is 6.

4. CONCLUSIONS

We have studied the problem of admission control and beamforming for a new user entering a running system where a set of downlink beamformers is already in use for the existing users. In many scheduling algorithms [1–3], this is a common subproblem, which has to be solved repeatedly. A new algorithm has been proposed, based on the idea to keep the beamforming signatures and only adjust the power levels for the existing users. Numerical simulations show that a user that can be admitted to the system using jointly optimal downlink beamforming can almost always also be admitted using this low complexity algorithm, at least if there is a limit on the transmit power. Also, for the admitted users, the performance of the resulting beamformers is not large, in most cases.

An alternative solution, based on a single step from the iterative algorithm in [6] for jointly optimal beamforming has been shown to perform significantly worse, even though it may look intuitively appealing.

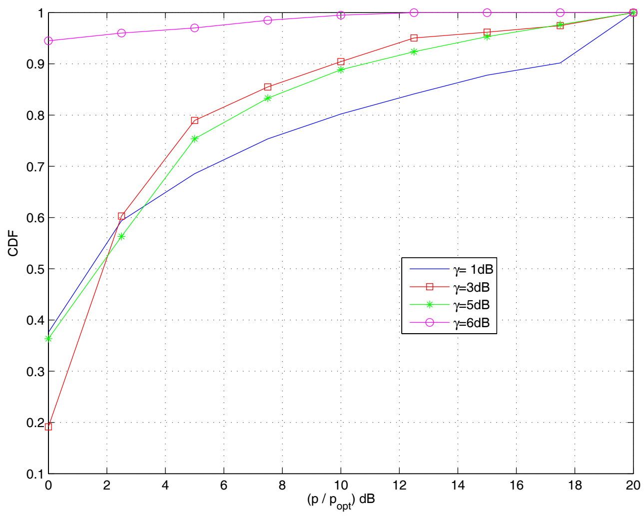

Fig. 3. CDF of the difference in total power between the proposed "power-only" algorithm and the joint optimum when a new user has been added to 4 existing users.

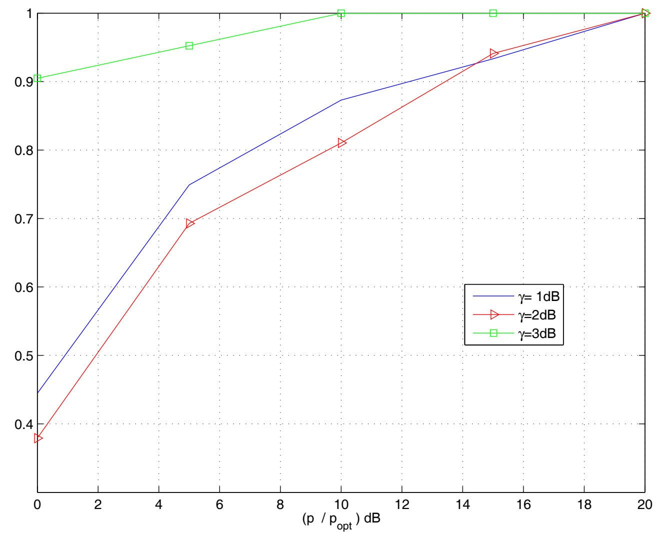

Fig. 4. CDF of the difference in total power between the proposed "power-only" algorithm and the joint optimum, when a new user has been added to 5 existing users.

5. REFERENCES

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