Transmit Correlation-aided Scheduling in Multiuser MIMO Networks

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Abstract – **The problem of joint scheduling and beamforming for a multiuser multiple-input multiple-output (MIMO) network with partial channel state information at the transmitter (CSIT) is addressed here. Unlike most previous work that rely on full instantaneous CSIT and require inacceptable overhead feedback to the transmitter, we point out here that useful information relevant to the scheduler lies untapped in the long term statistical information of the user's channels. We show how statistical CSIT can be efficiently combined with partial instantaneous CSIT to derive a scheduling rule for the downlink of multiuser MIMO systems.**

1. INTRODUCTION

The design of multiuser MIMO systems hinges on the problem of the joint design of a good antenna combining technique (e.g. beamforming, space time coding) with a properly matched channel access protocol that may include some degree of spatial division multiple access (SDMA). At the core of the problem lie the constraints of reasonably low feedback of CSIT and complexity.

It was shown that for the no-feedback case or error-prone feedback case, space-time coding combined with multiuser diversity time-division multiple access (TDMA)-like scheduling algorithms [1] seems a reasonable option [2]. However for a system encompassing even limited and reasonably accurate feedback of CSIT it is beneficial to exploit the spatial multiplexing capability of transmit antennas to several users at once rather than trying to maximize the reliability/diversity of a single user link. To realize this, optimal schemes based on the dirty paper coding approach have been proposed [3], as well as suboptimal greedy techniques for solving the precoding and multi-user power allocation problem [4]. Unfortunately the applicability of such schemes is limited due to 1-computational complexity and 2-the need for full CSIT *across all active users* which may lead to prohibitive feedback requirements in frequency-division duplex (FDD) systems or lack of robustness to CSIT errors in time-division duplex (TDD) setups with mobility.

In this paper we point out that the joint scheduling and beamforming design problem can be well approximated by a cascaded scheme where scheduling is first solved, then followed by beamforming design for the selected users. In this setting, we argue that it is the first stage (selecting N_t users out of K users, where N_t is the number of antennas at the base station, and where typically $N_t << K$) which is usually responsible for an inacceptable feedback overhead. The design of an optimal beamforming matrix for the N_t preselected users, however, can be done with little feedback and complexity overhead in comparison to the schemes known so far. Recently, we have exploited this idea by proposing a scheme where the scheduling stage is solved by launching N_t random orthogonal beams and selecting the users having the best SINRs under those beams, as per the opportunistic beamforming approach of [5]. The beamforming stage is then solved by requesting full CSIT and designing an optimal (e.g. MMSE) beamformer *for the selected users only* [6]. In comparison the scheme of [5] uses the random beams themselves as the beamforming solution to serve the users. The feedback required for such schemes for the scheduling stage is one scalar per user, signal-to-interference-plus-noise ratio (SINR).

Here we make the following key points:

- Schemes developed so far for MIMO scheduling do not exploit long-term statistical knowledge of CSIT. Because of its very long coherence period, statistical CSIT can be easily obtained by the mobile and fed back to the transmitter while causing almost negligible per-slot feedback overhead.

- Statistical CSIT can reveal information about the spatial separability of users, thus contains relevant information for the MIMO-SDMA scheduler. For instance, two users in very different areas of the cell are more likely to be separable than closely located users because their channels lie in two distinct cones of energy as seen by the base station (BTS), if reasonably limited angle spread at the BTS is assumed (which is typically the case [7]).

- The angle information is implicit in the transmit correlation matrix of the user's channel and need not be estimated.

Based on the above points we introduce a family of schemes

where the scheduling stage for multiuser MIMO is aided by the correlation matrix information and where this statistical CSIT is further augmented with a scalar instantaneous feedback (yielding a feedback overhead similar to previous schemes). Combining the second order channel statistics with the channel norm is addressed in [8], however it has not been exploited in an SDMA context. Here, we derive a correlation-based user separability metric. We show the gain of this type of approach over opportunistic schemes in various settings. In particular the performance of the proposed schemes is the same as the optimal joint scheduling and beamforming with full CSIT, when the path angle spread at the BTS decreases to zero.

2. SYSTEM MODEL

We consider the downlink of a single cell network with a BTS equipped with N_t antennas and K single-antenna users. The received signal $y_k(t)$ of user k at time slot t is mathematically described as

$$y_k(t) = \mathbf{h}_k \mathbf{x}(t) + n_k, \quad k = 1, \ldots, K \quad (1)$$

where $\mathbf{x}(t) \in \mathbb{C}^{N_t \times 1}$ is the transmitted symbol at time slot t, $\mathbf{h}_k \in \mathbb{C}^{1 \times N_t}$ is the channel gain vector, and n_k is the additive white Gaussian noise at receiver k. We assume that the channel vector $\mathbf{h}_k$ is perfectly known to the receiver, and that n_k have zero-mean complex Gaussian distribution with variance σ_n^2. The transmitter is subject to a peak power constraint P, $\mathrm{Tr}(\{\mathbf{x}\mathbf{x}^H\}) \leq P$. We let $\mathbf{H} \in \mathbb{C}^{K \times N_t}$ refer to the concatenation of all channels, where $\mathbf{H} = [\mathbf{h}_1^T, \ldots, \mathbf{h}_K^T]^T$.

3. OPTIMAL LINEAR BEAMFORMING SCHEMES

Although the downlink capacity (in the information theoretic sense) can be maximized by sending data to a number greater than N_t of simultaneous, spatially separated, users, we here limit to the case of linear beamforming schemes with exactly N_t users accessing the channel at the same time. In this case the joint scheduling and beamforming problem can be stated as follows.

Let $\mathbf{w}_k$ and s_k be the beamforming vector and transmitted symbol for user k respectively. Let $\mathcal{N} \in \mathbb{Z}_{N_t}^K$ be the set of all possible sets of size N_t of disjoint user indices among the total pool of users. Let $\mathcal{S} \in \mathcal{N}$, point at one such group of N_t users selected for transmission at time slot t, then $\mathbf{H}(\mathcal{S})$, $\mathbf{W}(\mathcal{S})$, $\mathbf{s}(\mathcal{S})$, $\mathbf{y}(\mathcal{S})$ are the concatenated channel vectors, beamforming vectors, uncorrelated transmitted symbols and received signals respectively for the scheduled set of users. We have:

$$\mathbf{y}(\mathcal{S}) = \mathbf{H}(\mathcal{S})\mathbf{W}(\mathcal{S})\mathbf{s}(\mathcal{S}) + \mathbf{n} \quad (2)$$

For the selected users in $\mathcal{S}$ we seek the beamforming vector matrix that will minimize the minimum mean-square error

(MMSE) at the receivers. This can be stated as

$$\mathbf{W}_{\mathrm{MMSE}}(\mathcal{S}) = \arg \min_{\substack{\mathbf{w} \\ \|\mathbf{W}\|^2 \leq P}} \mathbb{E}\left\{ \|\mathbf{s}(\mathcal{S}) - \mathbf{y}(\mathcal{S})\|^2 \right\} \quad (3)$$

The optimal filter matrix is given by [9]

$$\mathbf{W}_{\mathrm{MMSE}}(\mathcal{S}) = \left(\mathbf{H}(\mathcal{S})^H \mathbf{H}(\mathcal{S}) + \mu\mathbf{I}\right)^{-1} \mathbf{H}(\mathcal{S})^H \quad (4)$$

where μ is the non-negative Lagrange multiplier tuned to fulfill the transmit power constraint. Inserting this solution into to the minimization problem it can be shown as well that the MMSE level is given by

$$f_{\mathrm{MMSE}}(\mathcal{S}) = N_t - 2\Re\mathrm{Tr}\left(\mathbf{H}(\mathcal{S})\mathbf{W}_{\mathrm{MMSE}}(\mathcal{S})\right)$$
$$+ \mathrm{Tr}(\mathbf{H}(\mathcal{S})\mathbf{W}_{\mathrm{MMSE}}(\mathcal{S})\mathbf{W}_{\mathrm{MMSE}}(\mathcal{S})^H \mathbf{H}(\mathcal{S})^H) + N_t \sigma_n^2 \quad (5)$$

where $\Re$ is the real part. Defining $\mathbf{F}(\mathcal{S}) = \mathbf{H}(\mathcal{S})^H \mathbf{H}(\mathcal{S})$, the scheduler is optimized so as to select the set $\mathcal{S}$ that solves:

$$\mathcal{S}^* = \arg \max_{\forall \mathcal{S} \in \mathcal{N}} 2\Re\mathrm{Tr}\left\{ (\mathbf{F}(\mathcal{S}) + \mu\mathbf{I})^{-1} \mathbf{F}(\mathcal{S}) \right\}$$
$$- \mathrm{Tr}\left\{ \left((\mathbf{F}(\mathcal{S}) + \mu\mathbf{I})^{-1} \mathbf{F}(\mathcal{S})\right)^2 \right\} \quad (6)$$

The sum rate is then obtained by summing the single user capacity $log_2(1+\Gamma)$ where Γ is the user SINR, over the users of $\mathcal{S}$. This optimum scheduler however requires full CSIT for *all* K users which is not practical in many cases.

4. BEAMFORMING WITH STATISTICAL KNOWLEDGE

We consider the problem of designing an approximation of the scheduling metric above using limited CSIT. We investigate using second-order statistics combined with a single scalar instantaneous feedback per user given by the energy $\|\mathbf{h}_k\|$. Let us assume $\mathbf{R}_k$, the transmit correlation matrix of user k

$$\mathbf{R}_k = \mathbb{E}\left[\mathbf{h}_k^H \mathbf{h}_k\right] \quad (7)$$

to be known at the base station. We now propose two approaches on how to use this information in the scheduler. In the first, a modified MMSE criterion is derived that exploits the average channel behavior of the users. In the second, the combined channel statistics and instantaneous norm are exploited to build a coarse channel estimate for each user, which is used for the sole purpose of selecting users. In both cases the intuition behind the method is to pick users whose channels span spatially separated cones of multipath and have good channel gains.

4.1. Greedy Approach

Here we add users one by one to the set. We define a threshold γ_{th} and consider only users with channel norm above it. γ_{th} is defined by selecting the top γ percent channel gains, where γ is a parameter. For a given set $\mathcal{S}$, we define

the concatenated correlation matrix $\mathbf{R}(\mathcal{S}) = \sum_{\forall k \in \mathcal{S}} \mathbf{R}_k = \mathbb{E}\left\{\mathbf{H}(\mathcal{S})^H \mathbf{H}(\mathcal{S})\right\}$. The key observation is that Equation (6) can be approximated by replacing $\mathbf{F}(\mathcal{S})$ by its statistical estimate $\mathbf{R}(\mathcal{S})$. This gives rise to the following algorithm.

We find a suboptimal set $\mathcal{S}_{gr}$ according to the following algorithm:

At each time slot t

1. Initialize $\mathcal{S}_{gr} = \emptyset$, $\mathcal{G} = \emptyset$ and let $\mathcal{K} \subseteq \mathbb{Z}^K$ be the set of all user indices in the system.

2. Select the users that fall above the threshold γ_{th}

$$\mathcal{G} = \{\forall k \in \mathcal{K}| \quad \|\mathbf{h}_k\| \geq \gamma_{th}\} \tag{8}$$

3. Store the user with the highest channel norm in $\mathcal{S}_{gr}$

$$k_{max} = \arg \max_{\forall k \in \mathcal{G}} \|\mathbf{h}_k\| \tag{9}$$

$$\mathcal{S}_{gr} \leftarrow \mathcal{S}_{gr} \cup \{k_{max}\}, \quad \mathcal{G} \leftarrow \mathcal{G} \setminus \mathcal{S}_{gr} \tag{10}$$

4. As long as $|\mathcal{S}_{gr}| < N_t$, let $\hat{\mathbf{R}}(\{k\}) = \mathbf{R}(\mathcal{S}_{gr} \cup \{k\})$ and repeat adding a user k^* according to the following

$$k^* = \arg \max_{\forall k \in \mathcal{G}} 2\Re\mathrm{Tr}\left\{\left(\hat{\mathbf{R}}(\{k\}) + \mu\mathbf{I}\right)^{-1} \hat{\mathbf{R}}(\{k\})\right\}$$

$$- \mathrm{Tr}\left\{\left(\left(\hat{\mathbf{R}}(\{k\}) + \mu\mathbf{I}\right)^{-1} \hat{\mathbf{R}}(\{k\})\right)^2\right\} \tag{11}$$

$$\mathcal{S}_{gr} \leftarrow \mathcal{S}_{gr} \cup \{k^*\}, \quad \mathcal{G} \leftarrow \mathcal{G} \setminus \mathcal{S}_{gr} \tag{12}$$

4.2. Channel estimation with partial CSIT

Here we suggest to generate a maximum likelihood channel estimate based on the limited CSIT available, *for the sole purpose of scheduling* (i.e. the optimal MMSE beamformer is computed once the set of users is decided, using full feedback). We assume that the channel follows $\mathbf{h}_k \sim \mathbb{CN}(0, \mathbf{R}_k)$, denoting a multivariate circularly-symmetric zero-mean complex Gaussian random vector with probability density function (PDF)

$$p_{\mathbf{h}}(\mathbf{h}_k) = \frac{1}{\pi^{N_t} \det(\mathbf{R}_k^*)} \exp\left\{-\mathbf{h}_k^* \mathbf{R}_k^{*-1} \mathbf{h}_k^T\right\} \tag{13}$$

Assuming that the norm of the channel of user k is known and given by $\|\mathbf{h}_k\| = \rho$, we attempt to find the channel vector $\mathbf{h}_k$ that maximizes the PDF and satisfies the channel norm constraint. Maximizing (13) is equivalent to minimizing $\mathbf{h}_k^* \mathbf{R}_k^{*-1} \mathbf{h}_k^T$ which results in the following constrained optimization problem:

$$\max_{\mathbf{h}_k} \quad \mathbf{h}_k^* \mathbf{R}_k^* \mathbf{h}_k^T$$
$$s.t. \quad \|\mathbf{h}_k\| = \rho \tag{14}$$

The solution of (14) is given by $\hat{\mathbf{h}}_k = \alpha\mathbf{u}_k^H$, where $\mathbf{u}_k$ is the eigenvector associated with the largest eigenvalue of $\mathbf{R}_k^*$

and α is a constant defined such that the constraint on the channel norm is satisfied. Thus, the channel estimate for user k is given by

$$\hat{\mathbf{h}}_k = \rho\mathbf{u}_k^H \tag{15}$$

The optimum set $\mathcal{S}_{ce}$ under this channel estimation method is then given by

$$\mathcal{S}_{ce} = \arg \max_{\forall \mathcal{S} \in \mathcal{N}} 2\Re\mathrm{Tr}\left\{\left(\hat{\mathbf{F}}(\mathcal{S}) + \mu\mathbf{I}\right)^{-1} \hat{\mathbf{F}}(\mathcal{S})\right\}$$

$$- \mathrm{Tr}\left\{\left(\left(\hat{\mathbf{F}}(\mathcal{S}) + \mu\mathbf{I}\right)^{-1} \hat{\mathbf{F}}(\mathcal{S})\right)^2\right\} \tag{16}$$

where $\hat{\mathbf{F}}(\mathcal{S}) = \hat{\mathbf{H}}(\mathcal{S})^H \hat{\mathbf{H}}(\mathcal{S})$ is obtained from the concatenation of the channel estimates for the users in $\mathcal{S}$. A more efficient channel estimator with partial CSIT can be derived in the context of random beamforming [10].

5. MODELS AND NUMERICAL RESULTS

5.1. Specular model

We adopt a specular model where the signal propagation from user k to the BTS is assumed to follow a finite number of paths U. Each of these paths have an angle of incidence respect to the BTS broadside of $\theta_{k,u}$. The angles are asssumed to be distributed according to a Gaussian distribution with mean $\bar{\theta}_k$. The spread of angles around its mean is given by the root-mean square deviance, $\sigma_{\theta_k} = \sqrt{\mathrm{E}\left[|\theta_{k,u} - \bar{\theta}_k|^2\right]}$. The channel between BTS and MS k can be described as

$$\mathbf{h}_k^T = \frac{1}{\sqrt{U}} \sum_{u=1}^{U} \gamma_{k,u} \mathbf{a}(\theta_{k,u}) \tag{17}$$

Here $\gamma_{k,u}$ is the gain of the uth path seen at the MS. It is assumed to be a zero-mean complex Gaussian distributed random variable. All the paths are assumed to have unit variance. The steering vectors $\mathbf{a}(\theta_{k,u})$ are defined as

$$\mathbf{a}(\theta_{k,u}) = \left[1, e^{-j2\pi\frac{d\cos(\theta_{k,u})}{\lambda}}, \ldots, e^{-j2\pi\frac{(N_t-1)d\cos(\theta_{k,u})}{\lambda}}\right]^T \tag{18}$$

where d is the antenna spacing at the BTS and λ is the wavelength (here for a 2GHz system).

5.2. Results

In this section we evaluate the methods using the sum rate for a $N_t = 2$ antenna system, as a function of K, antenna spacing d, and the angle spread σ_θ respectively. We compare the two proposed schemes with the MMSE beamforming with full CSIT at the transmitter and with a random beamforming-based scheduling approach. In all the methods, once the scheduling set is obtained, the transmitter
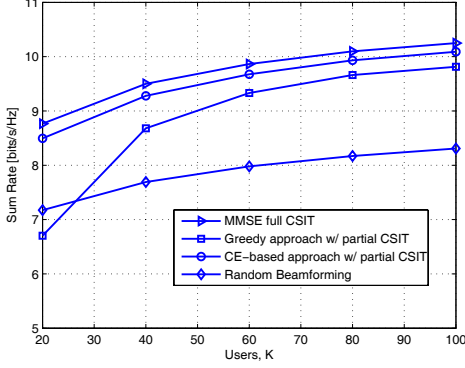
Figure 1: Sum rate as a function of the number of users for various user selection schemes with $N_t = 2$, antenna spacing $d = 0.5\lambda$ and $\sigma_\theta = 0.1\pi$.

obtains full CSIT for the selected users and designs a full MMSE beamformer. We use 10000 channel realizations and 100 slot averaging for the statistics. The parameter μ is chosen according to [9] as $\mu = N_t/P$. We use SNR of 10 dB. Typical measurements report angle spreads in the region less than 5-20 degrees at the BTS for outdoor networks [7]. Our methods show a clear gain over random beamforming for angle spread less than 35 degrees making them practical approach for cellular outdoor systems. Note that the actual beamwidths span $2\sigma_\theta$ according to our definitions. Insterestingly, the antenna spacing can be optimized and it is found that about 0.4λ gives optimal results, as it gives the best tradeoff between resolution and suppression of spatial aliasing. Note that a small spacing reduces antenna diversity, however multiuser diversity compensates for that through the scheduler.
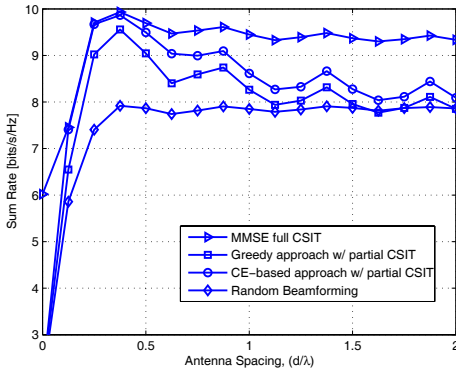

Figure 2: Sum rate as a function of antenna spacing for various user selection schemes with $N_t = 2$, $\sigma_\theta = 0.1\pi$ and $K = 50$ users.

6. CONCLUSION

We derive new scheduling metrics for the downlink of multiuser MIMO networks which have the advantage of accomodating statistical channel information and limited instantaneous channel feedback. The proposed joint scheduler
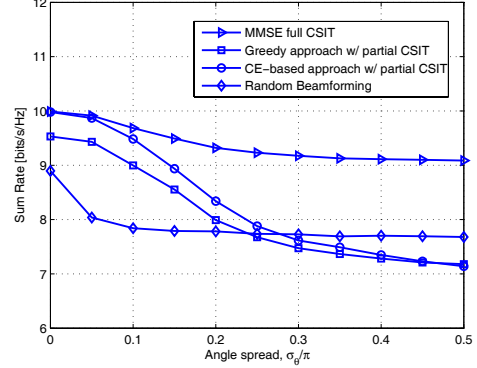

Figure 3: Sum rate as a function of angle spread for various user selection schemes with $N_t = 2$, antenna spacing $d = 0.5\lambda$ and $K = 50$ users.

and beamformer offers performance close to the full CSIT scheme when the multipath angular spread per user at the BTS is small enough, making this approach suitable to wireless systems with elevated BTS such as outdoor cellular networks.

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