LOW-COMPLEXITY MULTIPLE DESCRIPTION VECTOR QUANTIZATION WITH CONSTRAINED CENTRAL CODEBOOK

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ABSTRACT

Conventional multiple description vector quantizers (MDVQ) have high complexity, which limits their practical application. Two central-codebook-constrained MDVQ (CMDVQ) schemes are proposed to reduce the storage and search complexity. Simulation results show that for low channel loss rates, a tradeoff exists between choosing CMDVQ for its low complexity and the conventional MDVQ for its higher signal-to-noise ratio (SNR) performance. For medium to high channel loss rates, CMDVQ is preferred for its low complexity and comparable SNR performance to the conventional MDVQ.

1. INTRODUCTION

The increasing demand for communication over unreliable channels, e.g., voice over IP, imposes a continuous pressure on developing more robust source coders to combat channel losses. While retransmission of the lost information is a widely used solution, it incurs large transmission delay and may not be appropriate for interactive applications with strict delay constraints. Multiple description (MD) coding, an alternative approach requiring no retransmissions, is a method of communicating information from a source over two or more channels such that the information received from any subset of channels can be used for source reconstruction, and the reconstruction quality improves with the size of the subset.

Multiple description vector quantizers (MDVQs) have been studied in [1–5]. There exist two different approaches of designing a MDVQ scheme in the literature. In an "index assignment" approach, the encoder consists of an ordinary VQ followed by an index assignment which maps the index of each VQ encoder partition to multiple channel codewords. The design of MD scalar quantizers was first studied in [1], wherein a heuristic algorithm is presented to find a good index assignment. Index assignment optimization algorithms are studied in [2, 3]. In a "channel optimized" approach [4, 5], the encoder maps the source sample directly to multiple channel codewords using a channel optimized VQ scheme [6], wherein the index assignment is not necessary. The decoder for each approach is performed in a reverse manner of the encoder.

Direct use of the unconstrained MDVQ (UMDVQ) suffers from a serious complexity barrier even for medium bit rates and VQ dimensions. In this paper, we aim for developing low complexity MDVQ schemes. The proposed central-codebook-constrained MDVQ (CMDVQ) restricts the manner of generating the central codebook, i.e., each codevector in the central codebook of a two-description CMDVQ scheme is calculated as a weighted sum of two codevectors in the side codebooks. The use of the constraint reduces both storage requirements and computational complexity. However, the reduction is achieved in exchange of lower signal-to-noise-ratio (SNR) performance. An enhanced CMDVQ scheme, called CMDVQ with side distortion compensation (CMDVQ-SC), is further proposed to improve the SNR performance of CMDVQ. The complexity of CMDVQ-SC is higher than CMDVQ but still much lower than UMDVQ.

All three MDVQ schemes are compared over erasure channels. Simulation results show that, for low channel loss rates, a tradeoff exists between choosing the high SNR performance offered by UMDVQ and the low computational complexity offered by CMDVQ and CMDVQ-SC. For medium to high loss rates, CMDVQ-SC is preferred for its low complexity and comparable SNR performance to UMDVQ.

2. REVIEW OF UNCONSTRAINED MDVQ

The block diagram of a two-description UMDVQ scheme is shown in Fig. 1. The encoder maps a $k-$dimensional source

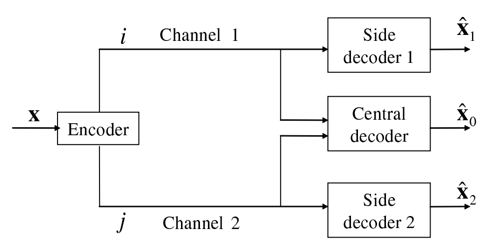

Fig. 1. System diagram of a two-description MDVQ scheme.

sample $\mathbf{x} \in \mathbb{R}^k$ to two channel codewords i and j at a bit rate R_1 and R_2, respectively, each to be sent over a separate independent erasure channel. If both channel codewords are received, a central decoder reconstructs the source sample as $\hat{\mathbf{x}}^0$ using a central codebook $\mathbf{C^0}$. If only one codeword is received, a side decoder reconstructs the source sample as $\hat{\mathbf{x}}^1$ or $\hat{\mathbf{x}}^2$ using a side codebook $\mathbf{C^1}$ or $\mathbf{C^2}$. If no codeword is received, the mean vector of the source is used for reconstruction.

Let the loss rates (erasure probabilities) for the two channels be p_1 and p_2 respectively. The channel optimized encoder of UMDVQ searches an optimal combination of i and j, where $0 \le i \le 2^{R_1} - 1, 0 \le j \le 2^{R_2} - 1$, to minimize

$$\begin{aligned} D_u = & (1-p_1)(1-p_2)||\mathbf{x} - \mathbf{c}^0_{ij}||^2 \\ & + (1-p_1)p_2||\mathbf{x} - \mathbf{c}^1_i||^2 + p_1(1-p_2)||\mathbf{x} - \mathbf{c}^2_j||^2, \end{aligned} \quad (1)$$

where the first term corresponds to the central distortion and the rest two terms correspond to the side distortion for each description. The distortion incurred when both channels fail does not depend on codebook search and is excluded in (1).

The MDVQ is unconstrained in the sense that both the central codebook and the two side codebooks have maximum degrees of freedom for codebook design. The two side codebooks have 2^{R_1} and 2^{R_2} entries and the central codebook has $2^{R_1+R_2}$ entries, with a k-dimensional vector for each entry. All entries of the codebooks can be freely chosen for optimization during the codebook design. The codebook design algorithm of the UMDVQ is similar to the conventional generalized Lloyd algorithm (GLA), where an iterative improvement algorithm is constructed based on two necessary optimality conditions to achieve a local minimum.

In UMDVQ, both the central codebook and the two side codebooks need to be stored for encoder searching and decoding. The storage requirement increases linearly with k and exponentially with R_1 or R_2. The computational complexity of UMDVQ is similarly high especially for large bit rates and VQ dimensions. The computational complexity of a MDVQ scheme is calculated by counting the numbers of arithmetic operations (e.g., multiplication and addition) per source sample required for encoding and decoding. The decoding complexity is generally low and thus ignored in our calculation. Our goal is to design MDVQ schemes with less storage requirement and computational complexity.

3. CONSTRAINED MDVQ

3.1. Central-codebook-constrained MDVQ

In the proposed CMDVQ scheme, we set a constraint on the manner of generating the central codebook. The central codevector $\mathbf{c}^0_{ij}$ is calculated as a weighted sum of the two side codevectors $\mathbf{c}^1_i$ and $\mathbf{c}^2_j$, i.e.,

$$\mathbf{c}^0_{ij} = w_1\mathbf{c}^1_i + w_2\mathbf{c}^2_j, \quad (2)$$

where w_1 and w_2 are the weight factors for the two descriptions. At the decoder, when i or j is lost, $\mathbf{c}^2_j$ or $\mathbf{c}^1_i$ is the decoder output. When both i and j are received, the weighted sum of $\mathbf{c}^1_i$ and $\mathbf{c}^2_j$ in (2) is the output. For balanced two descriptions ($R_1 = R_2$) and two symmetrical channels ($p_1 = p_2$), the value of w_1 and w_2 can be reasonably chosen as 0.5. In such case the central codevector is simply an average function of the two side codevectors. For unsymmetrical channels, w_1 and w_2 can be set adaptively according to channel characteristics. The optimal values of w_1 and w_2 can be found during codebook design, as described in section 3.4. At the encoder, the distortion measure of the CMDVQ for encoder search is

$$\begin{aligned} D_c = & (1-p_1)(1-p_2)||\mathbf{x} - (w_1\mathbf{c}^1_i + w_2\mathbf{c}^2_j)||^2 \\ & + (1-p_1)p_2||\mathbf{x} - \mathbf{c}^1_i||^2 + p_1(1-p_2)||\mathbf{x} - \mathbf{c}^2_j||^2. \end{aligned} \quad (3)$$

It is obvious that CMDVQ requires less memory storage than UMDVQ, since the central codebook can be calculated using a simple function of the two side codebooks and thus is unnecessary to be stored. As will be shown in section 3.3, the computational complexity of CMDVQ is also lower than UMDVQ. However, it can be inferred that the central codebook constraint causes the SNR performance of CMDVQ to be worse than that of UMDVQ, as the constraint in (2) reduces the degree of freedom for optimizing the central codebook. Hence, we present an enhanced CMDVQ scheme in the following section.

3.2. CMDVQ with side distortion compensation

An enhanced CMDVQ scheme is proposed to improve the SNR performance of CMDVQ for erasure channels. Two additional codebooks $\mathbf{U}^1$ and $\mathbf{U}^2$ are used when channel loss occurs. When the channel codeword j is lost, $\mathbf{u}^1_i$ in $\mathbf{U}^1$ is used as the decoder output. The codebook $\mathbf{U}^2$ functions similarly as $\mathbf{U}^1$. The use of $\mathbf{U}^1$ and $\mathbf{U}^2$ adds more degrees of freedom for codebook design. The enhanced CMDVQ is called CMDVQ with side distortion compensation (CMDVQ-SC). The distortion measure of CMDVQ-SC for the encoder search is

$$\begin{aligned} D_{sc} = & (1-p_1)(1-p_2)||\mathbf{x} - (w_1\mathbf{c}^1_i + w_2\mathbf{c}^2_j)||^2 \\ & + (1-p_1)p_2||\mathbf{x} - \mathbf{u}^1_i||^2 + p_1(1-p_2)||\mathbf{x} - \mathbf{u}^2_j||^2. \end{aligned} \quad (4)$$

When $p_1 = p_2 = 0$, the distortion measure of CMDVQ-SC in (4) is identical to (3). Thus, the improvement of CMDVQ-SC over CMDVQ lies in the lossy case where $p_1 > 0$ and $p_2 > 0$. $\mathbf{U}^1$ and $\mathbf{U}^2$ can be viewed as codebooks invoked by erasure events.

3.3. Storage and complexity

To see the advantage of the proposed CMDVQ schemes on storage and computational complexity, we expand the channel optimized distortion measure of UMDVQ in (1), and discard the term including $||\mathbf{x}||^2$ which is independent of codebook search. For simplicity, balanced descriptions and symmetrical channels are considered below, i.e., $R_1 = R_2 = R$ and $p_1 = p_2 = p$. Minimizing (1) is equivalent to minimizing

$$\begin{aligned} D_u^{eq} = & -2(1-p)\mathbf{x}^T\mathbf{c}_{ij}^0 - 2p\mathbf{x}^T\mathbf{c}_i^1 - 2p\mathbf{x}^T\mathbf{c}_j^2 \\ & + (1-p)||\mathbf{c}_{ij}^0||^2 + p||\mathbf{c}_i^2||^2 + p||\mathbf{c}_j^2||^2. \end{aligned} \quad (5)$$

The last three terms in (5) can be calculated and stored before encoding. Let $N = 2^R$. In the encoding process, the first term and the next two terms of (5) require N^2k, Nk and Nk inner product operations, respectively. The computational complexity of UMDVQ, defined as the number of multiplication and addition operations, is shown in Table 1.

For CMDVQ and CMDVQ-SC, the distortion measure for the encoder search in (3) and (4) can be written equivalent to

$$\begin{aligned} D_c^{eq} &= [2w_1(1-p)+2p]\mathbf{x}^T\mathbf{c}_i^1 + [2w_2(1-p)+2p]\mathbf{x}^T\mathbf{c}_j^2 \\ &\quad - (1-p)||w_1\mathbf{c}_i^1 + w_2\mathbf{c}_j^2||^2 - p||\mathbf{c}_i^1||^2 - p||\mathbf{c}_j^2||^2, \\ D_{sc}^{eq} &= [2w_1(1-p)+2p]\mathbf{x}^T\mathbf{c}_i^1 + [2w_2(1-p)+2p]\mathbf{x}^T\mathbf{c}_j^2 \\ &\quad + 2p\mathbf{x}^T\mathbf{u}_i^1 + 2p\mathbf{x}^T\mathbf{u}_j^2 - (1-p)||w_1\mathbf{c}_i^1 + w_2\mathbf{c}_j^2||^2 \\ &\quad - p||\mathbf{u}_i^1||^2 - p||\mathbf{u}_j^2||^2. \end{aligned}$$

Conducting a similar analysis as for UMDVQ, the storage requirement and search complexity for CMDVQ and CMDVQ-SC can be calculated and the results are summarized in Table 1. It can be seen that the storage and search complexity increase in the order of CMDVQ, CMDVQ-SC and UMDVQ. Fig. 2 shows that the complexity increases exponentially as

	Storage	Complexity
CMDVQ	$2Nk + N^2$	$2Nk + 2N^2$
CMDVQ-SC	$4Nk + N^2$	$4Nk + 4N^2$
UMDVQ	$N^2k + 2Nk + N^2$	$N^2k + 2Nk + 3N^2$

Table 1. Storage requirement and complexity of CMDVQ, CMDVQ-SC and UMDVQ. $R_1 = R_2 = R$ and $N = 2^R$.

the bit rate increases. For the bit rates $R_1 = R_2 = 4$ bits/sample and VQ dimension $k = 4$, the computational complexity of CMDVQ and CMDVQ-SC is decreased by 66% and 33%, respectively, comparing with UMDVQ. For higher bit rates and VQ dimensions, the reduction is more significant.

3.4. Codebook design and optimal weight factors

As in the conventional VQ codebook design, an iterative GLA-style training algorithm is applied in the proposed CMDVQ schemes. The two side codebooks $\mathbf{C}^1$ and $\mathbf{C}^2$ are first initialized. In CMDVQ-SC, the two codebooks ($\mathbf{U}^1$, $\mathbf{U}^2$) are also initialized. The GLA-style training algorithm is then performed iteratively based on two necessary optimal conditions. The *nearest neighbor* optimality condition is straightforward to derive. The *centroid condition* for the partition region $\mathcal{R}_{ij}$

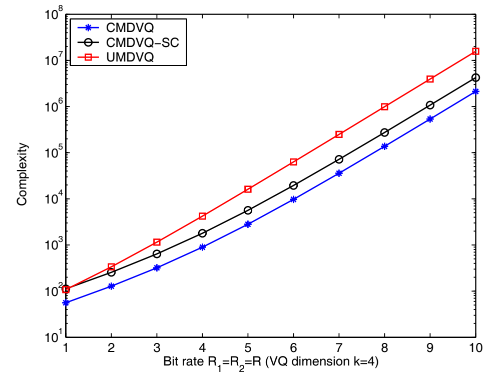

Fig. 2. The complexity of CMDVQ, CMDVQ-SC and UMDVQ for various bit rates. The VQ dimension $k = 4$.

is given by

$$(\mathbf{c}_i^{1*}, \mathbf{c}_j^{2*}) = \arg\min_{\mathbf{c}_i^1, \mathbf{c}_j^2} \sum_{i=0}^{2^{R_1}-1} \sum_{j=0}^{2^{R_2}-1} \sum_{\mathbf{x}\in\mathcal{R}_{i,j}} D_c$$

$$(\mathbf{c}_i^{1*}, \mathbf{c}_j^{2*}, \mathbf{u}_i^{1*}, \mathbf{u}_j^{2*}) = \arg\min_{\mathbf{c}_i^1, \mathbf{c}_j^2, \mathbf{u}_i^1, \mathbf{u}_j^2} \sum_{i=0}^{2^{R_1}-1} \sum_{j=0}^{2^{R_2}-1} \sum_{\mathbf{x}\in\mathcal{R}_{i,j}} D_{sc},$$

where D_c and D_{sc} are obtained from (3) and (4). We have replaced the expectation by summation over the entire set of source training samples. The optimal codevectors ($\mathbf{c}_i^{1*}$, $\mathbf{c}_j^{2*}$) and ($\mathbf{u}_i^{1*}$, $\mathbf{u}_j^{2*}$) can be calculated by first inserting (3)-(4) into one of the above equations and setting partial derivatives to zero, and then solving the associated equation respectively. The optimal weight factors w_1 and w_2 in (3)-(4) for specific channel loss rates can be calculated in a similar manner as obtaining the optimal codebooks.

4. PERFORMANCE

We compare the three MDVQ schemes over erasure channels for a memoryless i.i.d. Gaussian source with zero mean and unit variance. During the codebook design process, the side codebooks of CMDVQ are randomly initialized. The trained side codebooks of CMDVQ are used as the initialization of CMDVQ-SC. The codebooks for UMDVQ are initialized the same as for CMDVQ, i.e., the side codebooks of UMDVQ are initialized to be the same as those in CMDVQ and the central codebook is initialized using equation (2). 100 different trials were run for all schemes. The design with the lowest distortion amongst the trials is selected.

The SNR performance for the three schemes over erasure channels for $R_1 = R_2 = 4$ bits/sample and $k = 4$ is shown in Fig. 3. CMDVQ-SC performs the same as CMDVQ for $p = 0$. CMDVQ-SC outperforms CMDVQ for all $p > 0$, since the improvement of CMDVQ-SC over CMDVQ is on reducing the side distortion for lossy channels, as described in section 3.2. For $p = 0$ and small p, UMDVQ performs better

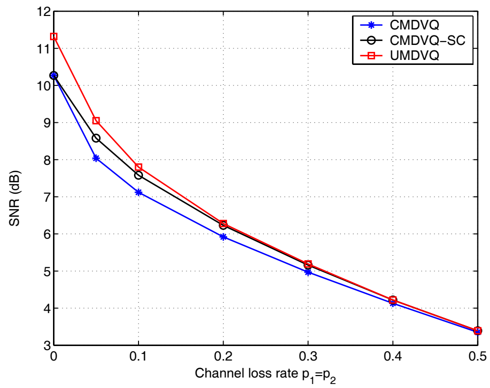

Fig. 3. Performance of CMDVQ, CMDVQ-SC and UMDVQ for erasure channels. $R_1 = R_2 = 4$ bits/sample and $k = 4$.

than the other two schemes. While CMDVQ and CMDVQ-SC offer advantages of moderate computational complexity and storage, their performance for small p is sacrificed. The central codebook of UMDVQ has $2^{R_1+R_2}$ degrees of freedom for optimization, while CMDVQ and CMDVQ-SC have only $2^{R_1} + 2^{R_2}$ degrees of freedom. A suboptimal central codebook is obtained after codebook design in CMDVQ and CMDVQ-SC. Hence, for small p where the central codebook plays more important role than the side codebooks, the SNR performance of CMDVQ and CMDVQ-SC is worse than that of UMDVQ. This reveals a tradeoff existing between choosing the high SNR performance offered by UMDVQ and the low complexity offered by CMDVQ. In a special case when $R_1 = R_2 = 1$, UMDVQ has the same degrees of freedom as the two CMDVQ schemes. In such case, the SNR performance of CMDVQ-SC is identical to UMDVQ for all p and all three schemes have identical SNR performance for $p = 0$.

For medium to large p ($p > 0.1$), the performance of CMDVQ-SC is comparable to that of UMDVQ. In such case, CMDVQ-SC is a better scheme than UMDVQ for its low complexity. Fig. 4 shows an example where with $p = 0.2$, CMDVQ-SC outperforms UMDVQ in SNR for the same complexity.

5. CONCLUSIONS

Two constrained MDVQ schemes, CMDVQ and CMDVQ-SC, have been proposed to reduce the storage and complexity of the conventional MDVQ. The reduction factor increases as the bit rate or VQ dimension increases. For low channel loss rates, the complexity reduction offered by the two CMDVQ schemes is accompanied with lower SNR performance. For medium to high channel loss rates, CMDVQ-SC is superior for its low complexity and high SNR performance. While two-description MDVQ schemes are the focus in this paper, the proposed methods are readily extended to MDVQ with more descriptions.

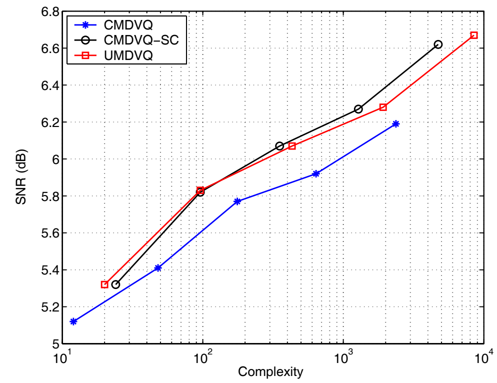

Fig. 4. Performance of CMDVQ, CMDVQ-SC and UMDVQ and their complexities for $p_1 = p_2 = 0.2$. The points on the curves, from left to right, correspond to VQ dimension $k = 1, 2, 3, 4, 5$, all for a fixed bit rate of 2 bits/sample.

The three MDVQ schemes considered in this paper requires the knowledge of the channel loss rate p. If p is varying sufficiently slowly, the receiver can estimate p and request the transmitter to choose one of the schemes that has been optimized for the specific value of p. It is also worthy to note that all three MDVQ schemes perform encoding via an exhaustive search algorithm. The complexity can be further reduced by developing a faster but suboptimal search algorithm.

6. REFERENCES

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