MAXIMUM-LIKELIHOOD NONCOHERENT LATTICE DECODING OF QAM

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ABSTRACT

We present a novel, maximum-likelihood (ML), lattice-decoding algorithm for noncoherent block detection of QAM signals. The computational complexity is polynomial in the block length; making it feasible for implementation compared with the exhaustive search ML detector. The algorithm works by enumerating the nearest neighbor regions for a *plane* defined by the received vector; in a conceptually similar manner to sphere decoding. Simulations show that the new algorithm significantly outperforms existing approaches.

1. INTRODUCTION

Noncoherent detection of digital signals over unknown fading channels has recently received significant attention especially for the case of the block-fading channel model. Applications include recovery from deep fades in pilot-symbol assisted modulation based schemes, eavesdropping, and blind channel estimation. Noncoherent detection is particularly applicable to systems exhibiting small coherence intervals where the use of training signals would result in a significant loss in throughput and capacity [1–3]. Under this noncoherent detection regime, it has been shown by numerical simulation that standard modulation techniques such as quadrature amplitude modulation (QAM) can achieve near-capacity in the single-antenna case [4]. Unfortunately, existing receiver designs are exponentially complex. For the constant envelope constellation of PSK, with known channel attenuation, efficient ML receiver algorithms have been developed [5], but the challenge remains for more general modulation classes and fading channels.

This paper focuses on the ML receiver design for QAM signals in fading channels. We first note that various suboptimal algorithms have been proposed. A blind phase recovery approach was proposed in [6] for noncoherent reception of QAM where the attenuation was assumed to be known exactly at the receiver. The noncoherent receiver considered in [7] involved using quantized channel estimates which would results in a significant loss of optimality for fading channels.

In this paper, we consider *optimal* noncoherent detection, which requires joint estimation of both channel and data. We propose an efficient noncoherent lattice decoding approach which provides the ML estimate with a complexity only *polynomial* in the block length. The lattice is defined by the QAM constellation and has dimension given by the block length. We show that the ML estimate of channel and data is identical to finding the closest possible transmitted lattice codeword (or lattice point) *in angle* to the plane described by the received lattice vector. We propose an algorithm that searches only in this plane, and yet it is guaranteed to find the ML estimate. We show that for a codeword of length T, the complexity of each detection is $O(T^3)$. This is a significant improvement over an exhaustive search which has exponential complexity in T.

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2. SYSTEM MODEL

2.1. Signal Model

The Gaussian integers are the set of all complex numbers $a + bi$ where both a and b are rational integers, and $i = \sqrt{-1}$. We define the odd Gaussian integers as those Gaussian integers having odd real and imaginary components.

Let $\mathbf{x} = [x_1, \ldots, x_T]' \in \mathcal{C}_M^T$ be a block of transmitted symbols (equivalently a lattice codeword) with each symbol x_i chosen from an M^2-ary square QAM constellation $\mathcal{C}_M$, where $\mathcal{C}_M$ is the set of odd Gaussian integers with the absolute value of both the real and imaginary components less than M. We also define $\mathcal{C}^T$ as the set of all $T \times 1$ lattice points containing only odd Gaussian integers.

We consider block fading channels and assume that the channel $h \in \mathbb{C}$ is constant for at least T symbols, as in [1–4]. Thus we can write the received codeword $\mathbf{y} = [\, y_1, \ldots, y_T \,]'$ as follows,

$$\mathbf{y} = h\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{n} = [\, n_1, \ldots, n_T \,]'$ is a vector of complex additive white Gaussian noise.

We will find it useful to separate each complex dimension into two real dimensions. In other words, to map points in complex space $\mathbb{C}^T$ to/from points in $\mathbb{R}^{2T}$. We will use the notation $\underline{\mathbf{v}}$ to denote the mapped version of some $\mathbf{v} \in \mathbb{C}^T$ as follows,

$$\underline{\mathbf{v}} = [\, \mathrm{Re}\{v_1\} \;\; \mathrm{Im}\{v_1\} \;\; \ldots \;\; \mathrm{Re}\{v_T\} \;\; \mathrm{Im}\{v_T\}]'. \tag{2}$$

We define the set $\underline{\mathcal{C}}_M^{2T} \subset \mathbb{R}^{2T}$ as the set produced by this (bijective) mapping from $\mathcal{C}_M^T \subset \mathbb{C}^{2T}$, and also the set $\underline{\mathcal{C}}^{2T}$ as the same mapping applied to $\mathcal{C}^T$.

2.2. Detection

The log-likelihood function of the maximum likelihood (ML) detector (of both channel and data) is given by

$$L(\mathbf{y}; \mathbf{x}, h) = -\left\| \mathbf{y} - h\mathbf{x} \right\|^2 \tag{3}$$

where constant factors have been discarded and $\|\cdot\|$ represents the Euclidean norm. For a given codeword hypothesis $\hat{\mathbf{x}}$, the likelihood function is maximized by choosing

$$\hat{h} = \frac{\hat{\mathbf{x}}^\dagger \mathbf{y}}{\|\hat{\mathbf{x}}\|^2} \tag{4}$$

where $(\cdot)^\dagger$ denotes Hermitian transpose. The joint ML estimate of $\mathbf{x}$ and h produces the following data estimate

$$\hat{\mathbf{x}}^{\text{ML}} = \arg\max_{\hat{\mathbf{x}} \in \mathcal{C}_M^T} L\left(\mathbf{y}; \hat{\mathbf{x}}, \frac{\hat{\mathbf{x}}^\dagger \mathbf{y}}{\|\hat{\mathbf{x}}\|^2}\right) = \arg\max_{\hat{\mathbf{x}} \in \mathcal{C}_M^T} \frac{\left|\hat{\mathbf{x}}^\dagger \mathbf{y}\right|^2}{\|\hat{\mathbf{x}}\|^2}.$$

We calculate the ML channel estimate $\hat{h}^{\text{ML}}$ by substituting $\hat{\mathbf{x}}^{\text{ML}}$ into (4). This is equivalent to the Generalized Likelihood Ratio Test (GLRT). By expressing the complex vectors in $\mathbb{R}^{2T}$ as in (2) we obtain a useful geometric interpretation of (5) as follows. We define $\underline{\mathbf{Y}} \in \mathbb{R}^{2T \times 2}$ as a basis for the subspace $\mathbf{y}\mathbb{C}$ mapped into the real space $\mathbb{R}^{2T}$ (as defined by (2)), that is

$$\begin{aligned} \underline{\mathbf{Y}} &\triangleq \begin{bmatrix} \operatorname{Re}\{y_1\} & \operatorname{Im}\{y_1\} & \ldots & \operatorname{Re}\{y_T\} & \operatorname{Im}\{y_T\} \\ -\operatorname{Im}\{y_1\} & \operatorname{Re}\{y_1\} & \ldots & -\operatorname{Im}\{y_T\} & \operatorname{Re}\{y_T\} \end{bmatrix}' \\ &\triangleq \begin{bmatrix} \underline{Y}_{1,1} & \underline{Y}_{2,1} & \cdots & \underline{Y}_{2T-1,1} & \underline{Y}_{2T,1} \\ \underline{Y}_{1,2} & \underline{Y}_{2,2} & \cdots & \underline{Y}_{2T-1,2} & \underline{Y}_{2T,2} \end{bmatrix}'. \end{aligned}$$

Note that the columns of $\underline{\mathbf{Y}}$ are orthogonal and have equal norm. The projection matrix $\mathbf{P}(\mathbf{y}) \in \mathbb{R}^{2T \times 2T}$ is defined as

$$\mathbf{P}(\mathbf{y}) \triangleq \frac{\underline{\mathbf{Y}}\,\underline{\mathbf{Y}}'}{\|\mathbf{y}\|^2}$$

That is, the vector $\mathbf{P}(\mathbf{y})\underline{\mathbf{x}}$ is the projection of $\underline{\mathbf{x}}$ onto the plane $\underline{\mathbf{Y}}\mathbb{R}^2$. It can now easily be shown that

$$\underline{\hat{\mathbf{x}}}^{\text{ML}} = \arg\max_{\underline{\hat{\mathbf{x}}} \in \mathcal{C}_M^T} \cos^2\theta(\underline{\hat{\mathbf{x}}}, \mathbf{P}(\mathbf{y})\underline{\hat{\mathbf{x}}})$$

Thus the ML estimate $\hat{\mathbf{x}}^{\text{ML}}$, corresponds to the $\underline{\hat{\mathbf{x}}} \in \underline{\mathcal{C}}_M^{2T}$ closest in angle to the plane $\underline{\mathbf{Y}}\mathbb{R}^2$.

It is important to note that two forms of ambiguity exist for this noncoherent detection problem. The first is the well-known phase ambiguity. For square QAM constellations there will be four indistinguishable $(\hat{h}_{\text{ML}}, \hat{\mathbf{x}}^{\text{ML}})$ pairs with the same $|\hat{h}_{\text{ML}}|$ corresponding to the four $\pi/2$ rotations of the constellation. We will assume that this type of ambiguity can be resolved, for example, by using the phase of the last symbol from the previous codeword [4], or by using differential encoding [8]. The second type of ambiguity we call a divisor ambiguity and arises when there are multiple points in $\mathcal{C}_M^T$ that lie within the same subspace through the origin (e.g. $[1+i, 1+i, 1+i]$ and $[3+3i, 3+3i, 3+3i]$ for 16-ary QAM with $T = 3$ and here $i = \sqrt{-1}$). This produces a lower bound on the noncoherent block detection error rate as discussed and analyzed in [9, 10]. Whenever this ambiguity arises we choose the estimate for which the corresponding $|\hat{h}_{\text{ML}}|$ is largest.

3. MAXIMUM LIKELIHOOD AND NEAREST NEIGHBOR REGIONS

Here, we consider the 'nearest neighbor set' of a *plane* segment which is defined by the received codeword. We then present a theorem stating that the nearest neighbor set of this plane segment contains $\underline{\hat{\mathbf{x}}}^{\text{ML}}$, which provides the basis for a low-complexity ML algorithm.

Definition 1 *We define $NN(\underline{\mathbf{v}})$ as the set of points in $\underline{\mathcal{C}}^{2T}$ which are the nearest neighbors to $\underline{\mathbf{v}} \in \mathbb{R}^{2T}$. That is, if $\underline{\mathbf{d}} \in NN(\underline{\mathbf{v}})$ then $\|\underline{\mathbf{v}} - \underline{\mathbf{d}}\| \leqslant \|\underline{\mathbf{v}} - \underline{\mathbf{u}}\|$ for all $\underline{\mathbf{u}} \in \underline{\mathcal{C}}^T$.*

Of course, $NN(\underline{\mathbf{v}})$ will usually have only one element.

Definition 2 *For QAM, we define $\mathcal{N}(\mathcal{S}, \underline{\mathbf{Y}})$ as the subset of a point set $\mathcal{S}$, such that $\underline{\mathbf{u}} \in \mathcal{N}(\mathcal{S}, \underline{\mathbf{Y}})$ if and only if: $\underline{\mathbf{u}} \in \mathcal{S}$; and there exists some $\boldsymbol{\lambda} \in \mathbb{R}^{2\times 1}$ such that $\|\underline{\mathbf{u}} - \underline{\mathbf{Y}}\boldsymbol{\lambda}\| \leqslant \|\underline{\mathbf{z}} - \underline{\mathbf{Y}}\boldsymbol{\lambda}\|$ for all $\underline{\mathbf{z}} \in \mathcal{S}$. We denote $\mathcal{N}(\mathcal{S}, \underline{\mathbf{Y}})$ as the nearest neighbor set of $\mathcal{S}$ with respect to the plane $\underline{\mathbf{Y}}\mathbb{R}^2$.*

Note that for any given $\boldsymbol{\lambda} \in \mathbb{R}^{2\times 1}$, the points $\boldsymbol{\lambda}\underline{\mathbf{y}}$ and $\underline{\mathbf{u}} \triangleq NN(\underline{\mathbf{Y}}\boldsymbol{\lambda})$ have the following property

$$\left|\underline{u}_i - [\,\underline{Y}_{i,1}\ \underline{Y}_{i,2}\,]\boldsymbol{\lambda}\right| \leqslant 1 \tag{5}$$

for all $i = 1, \ldots, 2T$. The converse also holds.

Definition 3 *We define the* clipping function *$f_M(u)$ as*

$$f_M(u) = \begin{cases} u & \text{if } -M+1 \leqslant u \leqslant M-1, \\ -M+1 & \text{if } u < -M+1, \\ M-1 & \text{if } u > M-1. \end{cases} \tag{6}$$

We also define that when u is complex, $f_M(u)$ implies that the clipping function is applied to the real and imaginary components. Moreover, $f_M(\mathbf{u})$ implies that the scalar clipping function is applied to each element of the vector $\mathbf{u}$, and likewise $f_M(\mathcal{S})$ implies that the clipping is applied to each element of the set $\mathcal{S}$.

The following theorem establishes a condition on the ML estimate $\underline{\hat{\mathbf{x}}}^{\text{ML}}$, which will allow us to bound the search region.

Theorem 1 *For noncoherent detection of square M-ary QAM codewords of length T, the ML estimate has the following property:*

$$\underline{\hat{\mathbf{x}}}^{ML} \in f_M(\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}})). \tag{7}$$

Proof 1 *First, consider the case where $\underline{\hat{\mathbf{x}}}^{ML} \in \mathcal{N}(\underline{\mathcal{C}}_M^{2T}, \underline{\mathbf{Y}})$, and note that since $\mathcal{N}(\underline{\mathcal{C}}_M^{2T}, \underline{\mathbf{Y}}) \subset f_M(\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}}))$, then clearly $\underline{\hat{\mathbf{x}}}^{ML} \in f_M(\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}}))$.*

Now consider the case when $\underline{\hat{\mathbf{x}}}^{ML} \notin \mathcal{N}(\mathcal{C}_M^{2T}, \underline{\mathbf{Y}})$. It can be shown that in this case a point exists in the region $\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}})$ (i.e., outside $\mathcal{C}_M^{2T}$ but inside $\underline{\mathcal{C}}_{M+2T}^{2T}$) which maps to $\underline{\hat{\mathbf{x}}}^{ML}$ when clipped according to (6). To show this, consider the point $\underline{\mathbf{v}}$ on the plane $\underline{\mathbf{Y}}\mathbb{R}^2$ given by

$$\underline{\mathbf{v}} = \frac{\left\|\underline{\hat{\mathbf{x}}}^{ML}\right\|^2}{\left\|\mathbf{P}(\mathbf{y})\underline{\hat{\mathbf{x}}}^{ML}\right\|^2}\mathbf{P}(\mathbf{y})\underline{\hat{\mathbf{x}}}^{ML}.$$

We first show that $\underline{\hat{\mathbf{x}}}^{ML}$ is actually the clipped version of the nearest neighbor of $\underline{\mathbf{v}}$, i.e. $\underline{\hat{\mathbf{x}}}^{ML} = f_M(NN(\underline{\mathbf{v}}))$. Since we are considering $\underline{\hat{\mathbf{x}}}^{ML} \notin \mathcal{N}(\mathcal{C}_M^{2T}, \underline{\mathbf{Y}})$, we know that the plane $\underline{\mathbf{Y}}\mathbb{R}^2$ does not pass through the nearest neighbor region of $\underline{\hat{\mathbf{x}}}^{ML}$, and hence $\underline{\mathbf{v}}$ (which is on the plane) is not in the nearest neighbor region of $\underline{\hat{\mathbf{x}}}^{ML}$. Therefore, the nearest neighbor of $\underline{\mathbf{v}}$ in $\mathcal{C}^{2T}$ is not $\hat{\mathbf{x}}^{ML}$; and in fact other points in $\mathcal{C}^{2T}$ can be closer to $\underline{\mathbf{v}}$ as well. These closer points must therefore lie within a hypersphere of radius $\left\|\underline{\mathbf{v}} - \underline{\hat{\mathbf{x}}}^{ML}\right\|$ centered at $\underline{\mathbf{v}}$. With $\underline{\mathbf{v}}$ defined as above, the vector $\underline{\hat{\mathbf{x}}}^{ML}$ is tangential to the hypersphere, and therefore it is also the case that these points are closer in angle *to the plane than $\hat{\underline{\mathbf{x}}}^{ML}$. So, since $\hat{\underline{\mathbf{x}}}^{ML}$ is the closest point in $\mathcal{C}^{2T}$ to the plane $\underline{\mathbf{Y}}\mathbb{R}^2$ in angle (by definition), therefore the points within the hypersphere must lie outside the constellation $\mathcal{C}_M^{2T}$. It can then be shown that $\hat{\underline{\mathbf{x}}}^{ML}$ is actually the codeword obtained by clipping the nearest neighbor of $\underline{\mathbf{v}}$ (which is inside the hypersphere and outside $\mathcal{C}^{2T}$), i.e. $\hat{\underline{\mathbf{x}}}^{ML} = f_M(NN(\underline{\mathbf{v}}))$. To complete the proof it remains to determine the domain of $f_M(NN(\underline{\mathbf{v}}))$. This can be found by determining the largest possible value of $\underline{v}_i$ for any i. It can be shown that $\underline{v}_i < M+2T-1$ and therefore $f_M(NN(\underline{\mathbf{v}})) < M+2T$.*

It follows that in both cases $\hat{\mathbf{x}}^{ML} \in f_M(\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}}))$. For more details see [11].

4. EFFICIENT MAXIMUM LIKELIHOOD ALGORITHM

We now propose the following approach for reducing the search space needed to find the ML estimate. Theorem 1 implies that $\hat{\underline{\mathbf{x}}}^{\mathrm{ML}}$ can be found by first enumerating all $\hat{\underline{\mathbf{x}}} \in f_M(\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}}))$ and then examining each such $\underline{\mathbf{x}}$ using the metric in (5) to find $\hat{\mathbf{x}}^{\mathrm{ML}}$. The algorithm can be visualized as performing a search across a plane, enumerating and testing all the nearest neighbor regions, which are hypercubes, through which the plane $\mathbf{Y}\mathbb{R}^2$ passes.

It is therefore necessary to find and enumerate these nearest neighbor regions. This can be done by solving for all the points $\underline{\mathbf{Y}}\boldsymbol{\lambda}$ on the plane $\mathbf{Y}\mathbb{R}^2$, where $\boldsymbol{\lambda} \triangleq [\lambda_1 \lambda_2]' \in \mathbb{R}^2$, which correspond to the intersections of the plane $\underline{\mathbf{Y}}\mathbb{R}^2$ with each of the $2T-1$ dimensional hyperplanes which represent the boundaries between the nearest neighbor regions.

These hyperplanes are defined by the QAM constellation as $\{\underline{\mathbf{y}} \mid \underline{y}_i = k\}$ for all $i \in \{1, \ldots, 2T\}$ and for all $k \in \mathcal{K} = \{0, \pm 2, \ldots, \pm(M-2), \pm(M+2T)\}$ (note that values of k between M and $M+2T-2$ are not included, as discussed below). Note that due to the presence of noise, the plane $\underline{\mathbf{Y}}\mathbb{R}^2$ is almost surely not a subset of any of the hyperplanes and therefore the intersections are lines. Recall that $\mathrm{Re}\{y_j\} = \underline{y}_{2j-1}$ and $\mathrm{Im}\{y_j\} = \underline{y}_{2j}$ and therefore the lines of intersection between the hyperplanes and the plane $\mathbf{Y}\mathbb{R}^2$ are given as follows. For each odd i and $k \in \mathcal{K}$ we have a pair of orthogonal lines given by

$$\lambda_1 \underline{y}_i - \lambda_2 \underline{y}_{i+1} = k, \tag{8}$$

$$\lambda_1 \underline{y}_{i+1} + \lambda_2 \underline{y}_i = k. \tag{9}$$

An example of these lines is shown in Figure 1 which is (of course) in the plane $\underline{\mathbf{Y}}\mathbb{R}^2$. The figure considers 16-QAM and codewords of length $T = 3$. It shows lines generated for the randomly chosen received codeword $\mathbf{y} = [\,-0.0195 - 0.3179i, -0.0482 + 1.0950i, 0.0000 - 1.8740i\,]' \in \mathbb{C}^3$. Each enclosed region in the figure has an associated point $\hat{\underline{\mathbf{x}}}$ which is in the reduced search space defined by Theorem 1. The outer bold square in the figure shows the outer boundary of the reduced search region. This arises because Theorem 1 states that $\hat{\underline{\mathbf{x}}}^{\mathrm{ML}} \in f_M(\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}}))$.

To summarize, the proposed efficient ML search algorithm involves finding all the intersection lines (e.g. as shown in Figure 1), enumerating all the regions enclosed by the lines, and for each region calculating the likelihood metric for the corresponding codeword. Clearly, this is a significantly reduced search space compared with the space of all possible codewords.

One important technical point to note is that the regions between the bold squares correspond to points in $\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}})$ but not in $\underline{\mathcal{C}}_M^{2T}$. The corresponding lattice point in each region will be clipped back to a lattice point in $\underline{\mathcal{C}}_M^{2T}$, as described in Theorem 1 and is why $\mathcal{K}$ was defined without the values between M and $M+2T-2$.

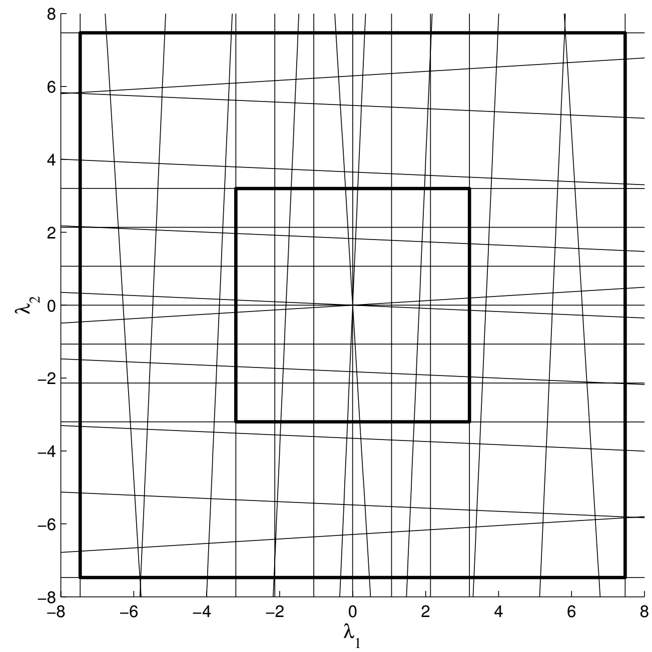

Fig. 1. Plot of lines $\mathcal{L}$, vertices $\mathcal{V}$ and regions $\mathcal{R}$ given by (10) for the vector $\mathbf{y} = [\,-0.0195 - 0.3179i, -0.0482 + 1.0950i, 0.0000 - 1.8740i\,]' \in \mathbb{C}^3$. The regions within the inner bold square correspond to codewords in $\mathcal{N}(\underline{\mathcal{C}}_M^{2T}, \underline{\mathbf{Y}})$. The regions between the bold squares correspond to lattice points in $\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}})$ but not in $\underline{\mathcal{C}}_M^{2T}$, and will be 'clipped' to a valid codeword.

We turn our attention to enumerating all the enclosed regions in the plane. Let us define $\mathcal{Q}$ to be the region in the (λ_1, λ_2)-plane, formed by the four lines corresponding to the intersection of the plane $\underline{\mathbf{Y}}\mathbb{R}^2$ with the 4 subspaces, $\mathrm{Re}\{y_i\} = \pm(M+2T)$ and $\mathrm{Im}\{y_i\} = \pm(M+2T)$ (i.e. this is the region inside the outer bold square in Figure 1). The choice of i here is such so as to minimize the area of $\mathcal{Q}$, that is $i = \max_j |y_j|^2$.

We denote $\mathcal{L}$ as the set of lines described in (8) and (9), also denote $\mathcal{V} \subset \mathcal{Q}$ as the set of intersection points of these lines, and $\mathcal{R} \in \mathcal{Q}$ as the set of enclosed regions. Note that each $r \in \mathcal{R}$ is a convex region.

Each enclosed region r on the plane is a polygon, uniquely corresponding to a codeword in $\mathcal{N}(\underline{\mathcal{C}}_{M+2T}^{2T}, \underline{\mathbf{Y}})$. To enumerate these regions, we first enumerate the vertices $\mathcal{V}$, and then use them to find at least one point within each region. For each region we use the point(s) to find a codeword estimate $\hat{\underline{\mathbf{x}}}^E$ corresponding to the region and then calculate the corresponding metric (5).

The vertices $\mathcal{V}$ are found by simple calculation of the intersection points of the lines given in (8) and (9) as follows,

$$\boldsymbol{\lambda}(i, j, k_1, k_2) \triangleq \begin{bmatrix} \underline{Y}_{i,1} & \underline{Y}_{i,2} \\ \underline{Y}_{j,1} & \underline{Y}_{j,2} \end{bmatrix}^{-1} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \tag{10}$$

for all $k_1, k_2 \in \mathcal{K}$, $i = 0, 1, \ldots, 2T$ and $j = i+1, i+2, \ldots, 2T$.

Now to calculate a point within each region it is sufficient to

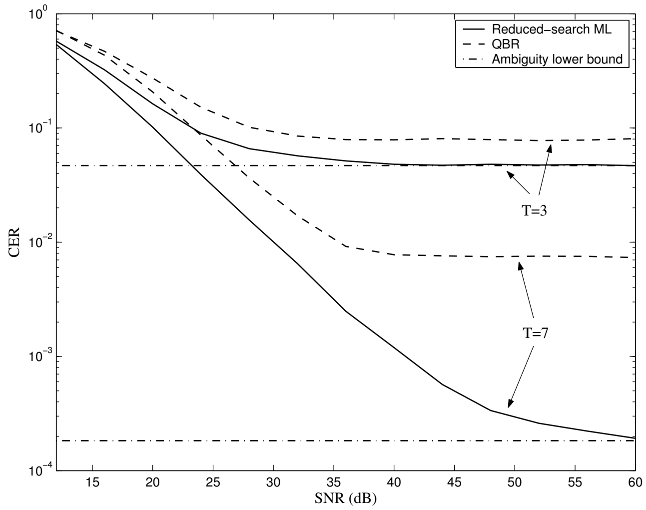

Fig. 2. Plot of Codeword Error Rate (CER) as a function of SNR for a 16-ary square QAM system.

calculate a point on the open line segments in the directions $[\ 1\ 0\ ]'$ and $[\ -1\ 0\ ]'$ between each vertex $\boldsymbol{\nu}$ and the first intersection with a line in $\mathcal{L}$. An $O(T)$ method is possible for this, however we can simply calculate $\boldsymbol{\nu}^+ = \boldsymbol{\nu} + [\ \epsilon\ 0\ ]'$, and $\boldsymbol{\nu}^- = \boldsymbol{\nu} - [\ \epsilon\ 0\ ]'$ for some small $\epsilon > 0$, which if sufficiently small should enumerate all $r \in \mathcal{R}$. To calculate the corresponding codeword $\hat{\underline{\mathbf{x}}}^+$, for the point $\boldsymbol{\nu}^+$ we simply calculate

$$\hat{\underline{\mathbf{x}}}^+ = f_M\big(NN\big(\underline{\mathbf{Y}}\boldsymbol{\mu}^+\big)\big). \tag{11}$$

The codeword $\hat{\underline{\mathbf{x}}}^-$ is calculated similarly. To complete the algorithm, the metric (5) is applied to each calculated codeword to obtain $\hat{\mathbf{x}}^{\mathrm{ML}}$.

The algorithmic complexity is governed by the computational cost of enumerating $\mathcal{V}$. There are $O(T^2)$ intersections to enumerate, each of constant computational expense. For each $\boldsymbol{\nu} \in \mathcal{V}$ generated by (10), we are required to solve for $\boldsymbol{\nu}^+$ and $\boldsymbol{\nu}^-$, which is $O(T)$ (or by using the suboptimal procedure described here is $O(1)$). Each calculation of the corresponding $\hat{\underline{\mathbf{x}}}^+$ (and $\hat{\underline{\mathbf{x}}}^-$) using (11) is $O(T)$, which is also the complexity of the metric calculation (5). Therefore, the overall complexity is $O(T^3)$.

A reduction in computational expense, without any loss in optimality, can be achieved by noticing that in Figure 1, the plot is invariant to rotations of $\pi/2$; a manifestation of the phase ambiguity for square QAM. Thus only a quarter of $\mathcal{V}$ needs to be enumerated.

5. SIMULATION RESULTS

Figure 2 presents the codeword error rate (CER) for 16-QAM as a function of SNR, for four codeword lengths $T = 3$ and 7. In the simulations, we have assumed that the phase ambiguities have been removed within each codeword, (for example, by the use of differential encoding [8]).

For comparison we show the performance of the quantization-based receiver (QBR) considered in [7]. For the sake of comparison, we assume a block independent Rayleigh fading channel and have chosen the attenuation estimates for QBR in [7] uniformly from the CDF of a Rayleigh fading distribution. The phase estimates for QBR are uniformly spaced. For fairness, the number of attenuation estimates for QBR was varied while keeping the total number of codeword estimates equal to the maximum number that potentially could be produced by our new algorithm. As noted in [7], there is an inherent suboptimality introduced by quantizing the unbounded channel attenuation in QBR, and the performance of our reduced search ML algorithm is clearly superior.

As discussed in Section 2.2, there exists divisor ambiguities resulting in an unavoidable lower bound on the probability of codeword detection error. Expressions for this lower bound were provided using a number-theoretic analysis in [9] and are shown in the figure. Clearly, for high SNR our reduced-search ML algorithm achieves these bounds, whereas the QBR approach does not.

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