ITERATIVE MMSE DECODER FOR TRACE-ORTHOGONAL SPACE-TIME CODING

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ABSTRACT

Trace-orthogonality is an important property of linear space-time encoders that has emerged relatively recently. In this work we carry out the theoretical performance analysis of a low complexity decoder for Trace-Orthogonal space-time codes based on the linear MMSE estimator. We derive the diversity order of such a decoder in the case of MIMO systems affected by uncorrelated flat Rayleigh fading, when information symbols are carved from a QPSK constellation, and are encoded using what we term a Unitary Trace-Orthogonal Design. Then we propose an iterative decoder that dramatically improves the performance over the linear MMSE decoder. Simulation results give evidence of the effectiveness of the proposed scheme.

1. INTRODUCTION

Multi-antenna Multi-Input Multi-Output (MIMO) systems have attracted a lot of research in the recent years because of their potential great increment of spectral efficiency over scattering–rich wireless channels [1]. A huge literature is by now available on space-time coding, as the basic tool to exploit the potentials of multi-antenna systems. The basic problem in the design of space-time coding systems is how to strike the best balance between three fundamental issues: i) performance, typically expressed in terms of bit error rate (BER) or, more appropriately for fading channels, average BER or out of service probability; ii) capacity, and iii) receiver complexity. Orthogonal space-time block coding [2], [3] is an example of method capable to collect the full diversity gain, using a very simple scalar decoder, but it is not optimal from the point of view of capacity [4]. On the contrary, Vertical-BLAST methods [5] guarantee full rate, but at the expenses of diversity gain or complexity. Hassibi and Hochwald in [6] proposed a rather general method to design linear dispersion (LD) codes, where the transmitted symbols are dispersed over space and time through spreading matrices that are built in order to maximize the ergodic capacity of the MIMO system. In the effort of designing codes that are guaranteed to have good performance in terms of *both* rate and BER, Ma and Giannakis in [7] and El Gamal and Damen in [8] provided general methods for building codes capable of being information lossless while guaranteeing, at the same time, full-diversity gain. An important property of linear space-time encoders that emerged relatively recently, is *trace-orthogonality* [9], [10], [11], [12], [13] that is the property of a linear space-time code to have encoding matrices orthogonal with respect to the trace inner product between matrices. Trace-orthogonality is the key property for lossless information transfer, and it comes in useful to find suboptimal methods to achieve a good balance between rate, BER and complexity. Trace-orthogonality guaranteeing *both* full rate and full diversity was proposed in [13]. In this work we start from the results of [10] where the information invariance property of trace-orthogonality is proven and a low complexity scalar detector for Trace-Orthogonal space-time codes based on linear MMSE estimator is devised. In particular, in [10] the authors derived the condition under which the proposed detector achieves minimum BER, for any channel realization, and provided the expression of such a minimum BER as a function of SNR. Using these results, in this paper we take explicitly into account the statistics of the MIMO channels and carry out a theoretical performance analysis of the decoder, with the objective to determine its diversity order. Then, we propose an iterative receiver based on the linear MMSE estimator, that dramatically improves the performance. The paper is organized as follows. The system model is outlined in Section 2 with a summary of some results from [10] useful for subsequent evaluation. In Section 3, we carry out the theoretical performance analysis of linear MMSE decoder deriving its diversity order for uncorrelated flat Rayleigh fading channels. In Section 4 we propose an iterative detection scheme based on a property of the linear MMSE estimator. Section 5 follows with numerical simulations showing the effectiveness of the proposed scheme and some conclusive remarks.

2. SYSTEM MODEL

Consider a flat fading MIMO system with n_T transmit and n_R receive antennas. After matched filtering and symbol-rate sampling, the input-output relation can be written as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v}, \tag{1}$$

where $\boldsymbol{H} \in \mathbb{C}^{n_R \times n_T}$ is the channel matrix, $\boldsymbol{x} \in \mathbb{C}^{n_T}$ is the vector of transmitted symbols and $\boldsymbol{v} \in \mathbb{C}^{n_R}$ is the noise vector assumed to be zero mean circularly symmetric complex Gaussian with covariance matrix $\sigma_v^2\boldsymbol{I}$. Assuming the channel constant over Q consecutive channel uses (quasi-static fading), stacking the transmitted vectors and the received ones in matrices, we obtain the following relation

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{V}, \tag{2}$$

where $\boldsymbol{V}$ is the $n_R \times Q$ received noise matrix, and $\boldsymbol{X}$ is the space-time $n_T \times Q$ code matrix. We assume that information symbols are independent with zero mean and variance σ_s^2, and are encoded into $\boldsymbol{X}$ using a full-rate *Unitary Trace-Orthogonal Design* (UTOD) [10]. That is a vector of $n_s = n_T Q$ (full-rate condition) complex symbols $\boldsymbol{s} = (s_1\ s_2\ \cdots\ s_{n_s})^T$ is mapped onto the code matrix $\boldsymbol{X}$ according to the rule

$$\boldsymbol{X} = \sum_{k=1}^{n_s} \boldsymbol{A}_k s_k \tag{3}$$

where encoding matrices $\boldsymbol{A}_k$ $(k = 1, \ldots, n_s)$ satisfy the following relations which define a Unitary Trace-Orthogonal Design:

$$\operatorname{tr}\left(\boldsymbol{A}_k^H \boldsymbol{A}_j\right) = \delta_{jk}, \qquad k, j \in \{1, \cdots, n_s\}, \tag{4}$$

where δ_{jk} denotes the Kronecker delta, and

$$\boldsymbol{A}_i\boldsymbol{A}_i^H = \frac{1}{n_T}\boldsymbol{I}_{n_T}, \qquad i \in \{1,\cdots,n_s\}. \tag{5}$$

In [10], and [11] it was proven that (4) is a necessary and sufficient condition for lossless information transmission, and the low complexity linear MMSE (LMMSE) estimator for UTOD was derived, given by

$$\hat{s}_k = \mathrm{tr}(\boldsymbol{A}_k^H\boldsymbol{W}\boldsymbol{Y})\,, \tag{6}$$

with

$$\boldsymbol{W} = (\boldsymbol{H}^H\boldsymbol{H} + \frac{1}{\gamma}\boldsymbol{I}_{n_T})^{-1}\boldsymbol{H}^H, \tag{7}$$

where $\gamma = \sigma_s^2/\sigma_v^2$ is the SNR.
Moreover, in the same references, it was proved that (5) is a necessary and sufficient condition, for the decoder composed by the cascade of (6) followed by hard decision, to achieve minimum BER when information symbols are carved from a QPSK constellation. In particular the minimum BER[1], for any realization $\boldsymbol{H}$ of the channel, is given by

$$\overline{P_e^{\min}}(\boldsymbol{H},\gamma) = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{1}{2}\left(\frac{n_T}{\mathrm{tr}\,(\boldsymbol{W}\boldsymbol{H})}-1\right)^{-1}}\right], \tag{8}$$

where $\boldsymbol{W}$ is defined in (7).

3. THEORETICAL PERFORMANCE ANALYSIS OF LMMSE DECODER

The results summarized in the previous section hold, independently of the statistics of the channel matrix $\boldsymbol{H}$. However, to carry out a performance analysis we need to specify the fading distribution. We assume flat Rayleigh fading channels with uncorrelated channel coefficients. Information symbols are carved from a QPSK constellation and are encoded using a full-rate[2] Unitary Trace-Orthogonal Design [10].

Under these hypotheses, using (8), the BER averaged over the channel statistics is given by

$$\mathcal{P}_e(\gamma) = \mathbb{E}\left\{\overline{P_e^{\min}}(\boldsymbol{H},\gamma)\right\}, \tag{9}$$

where γ is the SNR. We are interested in characterizing the asymptotic behavior of $\mathcal{P}_e(\gamma)$ as $\gamma \to \infty$. To this end let us consider the argument of expectation $\overline{P_e^{\min}}(\boldsymbol{H},\gamma)$ which can be recast as

$$\overline{P_e^{\min}}(\boldsymbol{H},\gamma) = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\omega}{2(1-\omega)}}\right], \tag{10}$$

with

$$\omega = \frac{1}{n_T}\,\mathrm{tr}(\boldsymbol{W}\boldsymbol{H}) = \frac{1}{n_T}\sum_{k=1}^{n_T}\frac{\lambda_k}{\lambda_k+\frac{1}{\gamma}}\,, \tag{11}$$

where $\lambda_1,\ldots,\lambda_{n_T}$ are the eigenvalues of $\boldsymbol{H}^H\boldsymbol{H}$. Firstly we need to ascertain the condition under which

$$\lim_{\gamma\to\infty}\mathcal{P}_e(\gamma) = 0, \tag{12}$$

as this guarantees the absence of BER floor. It easy to verify that (12) holds if and only if

$$\lim_{\gamma\to\infty}\overline{P_e^{\min}}(\boldsymbol{H},\gamma) = 0 \quad (a.s.), \tag{13}$$

where (a.s.) stands for *almost surely*, i.e. with probability 1. From (10) and (11) condition (13) is equivalent to

$$\lim_{\gamma\to\infty}\frac{1}{n_T}\sum_{k=1}^{n_T}\frac{\lambda_k}{\lambda_k+\frac{1}{\gamma}} = 1 \quad (a.s.), \tag{14}$$

which holds true if and only if $\lambda_1,\ldots,\lambda_{n_T}$ are almost surely non-null. These last are the eigenvalues of $\boldsymbol{H}^H\boldsymbol{H}$ and, since we are considering uncorrelated Rayleigh fading, this is possible if and only if $\boldsymbol{H}$ is almost surely full column rank, that is *if and only if* $n_R \geq n_T$.

In the sequel we assume that this condition is satisfied and thus $\mathcal{P}_e(\gamma)$ is infinitesimal as $\gamma \to \infty$. The next step is to quantify the infinitesimal order of $\mathcal{P}_e(\gamma)$, which gives the diversity order of the LMMSE decoder.

Towards this end let us consider $\overline{P_e^{\min}}(\boldsymbol{H},\gamma)$ in (10). It is a convex function of its argument $\omega \in [0,1)$, and since ω is the arithmetic mean of the values $\lambda_k\left(\lambda_k+\frac{1}{\gamma}\right)^{-1} \in [0,1)$, for $k = 1,\ldots,n_T$, the application of Jensen's inequality to (10) leads to the following upper bound

$$\overline{P_e^{\min}}(\boldsymbol{H},\gamma) \leq \frac{1}{2n_T}\sum_{k=1}^{n_T}\mathrm{erfc}\left(\sqrt{\frac{\lambda_k\gamma}{2}}\right), \tag{15}$$

which can be further upper bounded, resorting to the usual inequality $\mathrm{erfc}(x) \leq e^{-x^2}$, as

$$\overline{P_e^{\min}}(\boldsymbol{H},\gamma) \leq \frac{1}{2n_T}\sum_{k=1}^{n_T}e^{-\frac{\gamma}{2}\lambda_k}\,. \tag{16}$$

Inserting this bound in (9) we get, eventually

$$\mathcal{P}_e(\gamma) \leq \frac{1}{2n_T}\sum_{k=1}^{n_T}\mathbb{E}\left\{e^{-\frac{\gamma}{2}\lambda_k}\right\} = \frac{1}{2}\,\mathbb{E}\left\{e^{-\frac{\gamma}{2}\lambda}\right\}, \tag{17}$$

where λ denotes one of the unordered eigenvalues of $\boldsymbol{H}^H\boldsymbol{H}$. The probability density function of λ, under the above hypotheses of uncorrelated Rayleigh fading and $n_R \geq n_T$, assumes the following expression [1]

$$p_\lambda(\lambda) = \lambda^{n_R-n_T}e^{-\lambda}\left\{\frac{1}{n_T}\sum_{k=0}^{n_T-1}\frac{k!\left[L_k^{n_R-n_T}(\lambda)\right]^2}{(n_R-n_T+k)!}\right\}, \tag{18}$$

where $L_k^\nu(\lambda) = \sum_{i=0}^{k}\binom{k+\nu}{k-i}\frac{(-\lambda)^i}{i!}$ is the associated Laguerre polynomial[3] of order ν and degree k. The term between braces in (18) is a polynomial of degree $2(n_T-1)$. Denoting it by $\mathcal{Q}(\lambda) = \sum_{k=0}^{2(n_T-1)}c_k\lambda^k$, the expected value in (17) is computed as follows

$$\begin{aligned}\frac{1}{2}\,\mathbb{E}\left\{e^{-\frac{\gamma}{2}\lambda}\right\} &= \frac{1}{2}\sum_{k=0}^{2(n_T-1)}c_k\int_0^\infty\lambda^{n_R-n_T+k}e^{-\left(1+\frac{\gamma}{2}\right)\lambda}d\lambda\\ &= \sum_{k=0}^{2(n_T-1)}c_k\frac{(n_R-n_T+k)!\,2^{n_R-n_T+k}}{(\gamma+2)^{n_R-n_T+k+1}}\\ &= c_0\frac{(n_R-n_T)!\,2^{n_R-n_T}}{(\gamma+2)^{n_R-n_T+1}} + o\left(\frac{1}{\gamma^{n_R-n_T+1}}\right)\\ &= \frac{\binom{n_R}{n_T}\,2^{n_R-n_T}}{n_R-n_T+1}\left[\frac{1}{\gamma+2}\right]^{n_R-n_T+1} + o\left(\frac{1}{\gamma^{n_R-n_T+1}}\right),\end{aligned} \tag{19}$$

[1] Average BER for each block of n_s transmitted symbols, see [10].

[2] That is $n_s = Q\cdot n_T$.

[3] The reported expression of $L_k^\nu(\lambda)$ is easily derived from its Rodrigues representation $L_k^\nu(\lambda) = \frac{e^\lambda\lambda^{-\nu}}{k!}\frac{d^k}{d\lambda^k}\left(e^{-\lambda}\lambda^{k+\nu}\right)$.

where in deriving the last equality c_0 was computed from (18) as

$$c_0 = \frac{1}{n_T} \sum_{k=0}^{n_T-1} \frac{k! \left[L_k^{n_R-n_T}(0)\right]^2}{(n_R - n_T + k)!} = \frac{\binom{n_R}{n_T}}{(n_R - n_T + 1)!} .$$

Substituting (19) in (17) we conclude that as the SNR γ increases the following asymptotic upper bound holds for the average error probability

$$\mathcal{P}_e(\gamma) \leq \binom{n_R}{n_T} \frac{2^{n_R-n_T}}{n_R - n_T + 1} \gamma^{-(n_R-n_T+1)} \quad (\gamma \to \infty) , \quad (20)$$

and thus the *diversity order* for LMMSE detector is $n_R - n_T + 1$. In [14] and [15], authors argued that one can expect LMMSE receiver to have the same diversity order of Zero-Forcing (ZF) one, namely $n_R - n_T + 1$, since the former converges to ZF as SNR goes to infinity, but no formal proof was given to support this result.

4. ITERATIVE LMMSE DECODER

The LMMSE decoder has the advantage of very low complexity but it suffers from a quite low diversity order. In this section we consider the possibility to improve the performance of LMMSE decoder resorting to an iterative procedure based on some properties of MMSE estimators. To be more precise, let us assume that $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are independent zero mean real random vectors with i.i.d. components having variance σ_s^2, collecting information symbols, and $\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{0}, \sigma_w^2 \boldsymbol{I})$, with $\boldsymbol{w}$ independent of $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$, is the noise vector. Assume that we observe $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ through the vector $\boldsymbol{y}$ given by

$$\boldsymbol{y} = \boldsymbol{H}_1 \boldsymbol{x}_1 + \boldsymbol{H}_2 \boldsymbol{x}_2 + \boldsymbol{w} , \quad (21)$$

where $\boldsymbol{H}_1$ and $\boldsymbol{H}_2$ are known real matrices. The linear MMSE estimate of $\boldsymbol{x}_i$ $(i = 1, 2)$ is given by

$$\hat{\boldsymbol{x}}_i = \boldsymbol{H}_i^T \boldsymbol{G}^{-1} \boldsymbol{y} = \boldsymbol{x}_i + \boldsymbol{e}_i \quad (22)$$

where $\boldsymbol{G} = (\boldsymbol{H}_1 \boldsymbol{H}_1^T + \boldsymbol{H}_2 \boldsymbol{H}_2^T + \frac{1}{\gamma} \boldsymbol{I})$, with $\gamma = \sigma_s^2 / \sigma_w^2$, and the last equality expresses each estimate as the actual value plus estimation error. Unlike $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ that are statistically independent, the estimation error vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are correlated according to the following error covariance matrix $\boldsymbol{K}_e$

$$\boldsymbol{K}_e = \begin{bmatrix} \boldsymbol{K}_{11} & \boldsymbol{K}_{12} \\ \boldsymbol{K}_{21} & \boldsymbol{K}_{22} \end{bmatrix} , \quad (23)$$

where $\boldsymbol{K}_{ij} = \mathbb{E}\{\boldsymbol{e}_i \boldsymbol{e}_j^T\} = \sigma_s^2 (\delta_{ij} \boldsymbol{I} - \boldsymbol{H}_i^T \boldsymbol{G}^{-1} \boldsymbol{H}_j)$, with δ_{ij} the Kronecker delta.

Now, suppose that we have knowledge of the current outcome of vector $\boldsymbol{x}_2 = \bar{\boldsymbol{x}}_2$, which, in turn, implies that we are able to compute the corresponding outcome of the error vector $\boldsymbol{e}_2$ from (22) as $\bar{\boldsymbol{e}}_2 = \hat{\boldsymbol{x}}_2 - \bar{\boldsymbol{x}}_2$. Exploiting the correlation between $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ and the knowledge of $\bar{\boldsymbol{e}}_2$ we may estimate $\boldsymbol{e}_1$ as

$$\hat{\boldsymbol{e}}_1 = \boldsymbol{K}_{11} \boldsymbol{K}_{22}^{-1} \bar{\boldsymbol{e}}_2, \quad (24)$$

and try to improve the estimate of $\boldsymbol{x}_1$ using

$$\hat{\hat{\boldsymbol{x}}}_1 = \hat{\boldsymbol{x}}_1 - \hat{\boldsymbol{e}}_1. \quad (25)$$

It is worthwhile noting that (25) actually coincides with a linear MMSE estimate of $\boldsymbol{x}_1$. Indeed it is possible to prove that the following identity holds true

$$\hat{\hat{\boldsymbol{x}}}_1 = \left(\boldsymbol{H}_1^T \boldsymbol{H}_1 + \frac{1}{\gamma} \boldsymbol{I} \right)^{-1} \boldsymbol{H}_1^T (\boldsymbol{y} - \boldsymbol{H}_2 \bar{\boldsymbol{x}}_2), \quad (26)$$

where the right-hand side of (26) is the linear MMSE estimate of $\boldsymbol{x}_1$ when $\boldsymbol{x}_2$ is *known* and is equal to $\bar{\boldsymbol{x}}_2$. Equivalently (26) is the linear MMSE estimate of $\boldsymbol{x}_1$ after removing from $\boldsymbol{y}$ the ISI due to $\boldsymbol{x}_2$, namely $\boldsymbol{H}_2 \bar{\boldsymbol{x}}_2$.

The procedure described so far suggests a way to improve iteratively the soft estimate of the symbols not yet decoded ($\boldsymbol{x}_1$), provided that we have a *reliable* estimate of the previously decided symbols ($\boldsymbol{x}_2$). A critical point is the reliability of the decisions already taken, which can be achieved only if we are able to choose "cleverly" the symbols to decode at each iteration. In this regard, note that the LMMSE decoder in (6) has the property that the estimates are affected by errors with the *same* variance [10]. This, in turn, implies that the symbol estimates have the same SINR, thus implying that the SINR value cannot be used as a selection metric. However, the estimation errors have zero mean, and since they can be assumed to be (approximately) Gaussian, large values for them are less likely. This led us to decode at each iteration the symbol whose MMSE estimate is closest to a constellation point. The effectiveness of this choice will be later assessed by simulations.

For each block of n_s complex ($2\, n_s$ real) symbols the receiver performs the iterative procedure listed below. We use the convention that when $\mathcal{A}$ is a set, then $\boldsymbol{s}_{\mathcal{A}}$ (or $\boldsymbol{e}_{\mathcal{A}}$) denotes a vector collecting symbols (estimation errors) whose indices belong to $\mathcal{A}$.

Iterative LMMSE decoder

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1. Set $\mathcal{A} = \{1, 2, \ldots, 2\, n_s\}$, and $\mathcal{B} = \emptyset$
2. Compute in $\hat{\boldsymbol{s}}_{\mathcal{A}}$ the linear MMSE estimates of the symbols with indices in $\mathcal{A}$
3. Find the component of $\hat{\boldsymbol{s}}_{\mathcal{A}}$ nearest to a constellation point; denote it by $\hat{s}_k$, where $k \in \mathcal{A}$
4. Decode it as $\tilde{s}_k = Dec[\hat{s}_k]$
5. Compute the error affecting the k-th estimate as $\bar{e}_k = \hat{s}_k - \tilde{s}_k$
6. Set $\mathcal{A} = \mathcal{A} - \{k\}$, and $\mathcal{B} = \mathcal{B} \cup \{k\}$
7. Estimate the error vector $\hat{\boldsymbol{e}}_{\mathcal{A}}$ using $\bar{\boldsymbol{e}}_{\mathcal{B}}$
8. Update undecoded estimates $\hat{\boldsymbol{s}}_{\mathcal{A}} = \hat{\boldsymbol{s}}_{\mathcal{A}} - \hat{\boldsymbol{e}}_{\mathcal{A}}$
9. If $\mathcal{A} \neq \emptyset$ goto 3 else stop

Due to identity (26), the proposed procedure is equivalent to iteratively performing detection and ISI cancelation. In this regard, it could suffer severely from error propagation due to erroneous decisions. However simulations confirm that its use dramatically increases the performance over the non-iterative MMSE decoder. The relation (25) can be useful to explain this behavior. In fact, the performance of linear MMSE detector severely degrades compared to ML detection due to its (in general of linear detectors) inability to properly adapt its decision regions to the noise statistics [16]. It should be noted, in fact, that multiplication by Wiener filter matrix introduces correlations of the noise components which make componentwise symbol detection suboptimal. Now, let us consider (25), that can be recast, using (22), as

$$\hat{\hat{\boldsymbol{x}}}_1 = \hat{\boldsymbol{x}}_1 - \hat{\boldsymbol{e}}_1 = \boldsymbol{x}_1 + \boldsymbol{e}_1 - \hat{\boldsymbol{e}}_1. \quad (27)$$

Note that the new estimate $\hat{\hat{\boldsymbol{x}}}_1$ in (27) is affected by error $(\boldsymbol{e}_1 - \hat{\boldsymbol{e}}_1)$ which is uncorrelated with $\boldsymbol{e}_2$, due to the Orthogonality Principle [17]. In words, the effect of the iterations is to whiten, even if only approximately, the estimation errors. This, in turn, improves the performance of subsequent componentwise symbols detection.

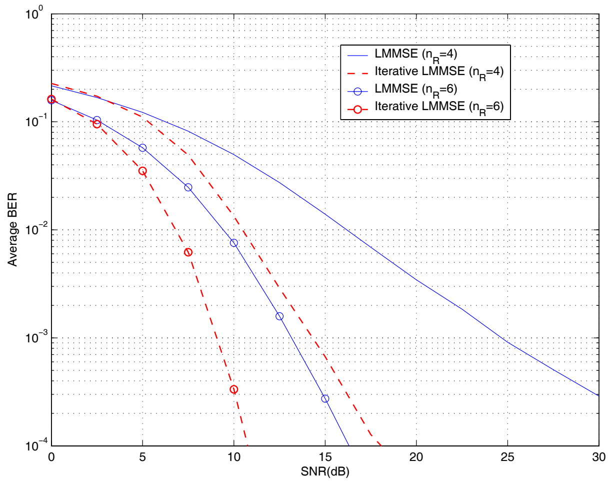

Fig. 1. Average BER for QPSK in a MIMO system with $n_T = 4$, and $n_R = 4, 6$. UTOD with LMMSE (continuous line) decoder compared to iterative LMMSE (dashed line) decoder.

5. SIMULATION RESULTS AND CONCLUSION

In Figure 1 we compare the average BER performance of LMMSE decoder versus iterative LMMSE one. We assume Rayleigh fading channels with uncorrelated channel coefficients. The curves are obtained via Monte Carlo simulations averaging over 10^5 channel realizations. The SNR is defined as $P_T/(n_T\sigma_v^2)$, where P_T is the total transmission power, n_T is the number of transmitting antennas, and σ_v^2 is the noise variance. Information symbols are carved from the QPSK constellation with Gray mapping and are encoded using the *shift and multiply basis* introduced in [9], which constitutes a Unitary Trace-Orthogonal Design. We consider MIMO systems with $n_T = 4$ and $n_R = 4, 6$. Since $n_R \geq n_T$ we are guaranteed that BER floor is avoided. Continuous curves denote LMMSE decoder and give evidence of the diversity order which is respectively 1 (for 4×4 system) and 3 (for 6×4 system) in agreement with (20). The performance of iterative LMMSE decoder (dashed lines) improves dramatically over LMMSE as the marked increase of slope, i.e. diversity order, emphasizes. We should attribute this remarkable gain mainly to two aspects: the use of a Unitary Trace-Orthogonal Design as the space-time coding strategy, since it guarantees minimum BER for LMMSE decoder[4]; the adoption of an effective selection metric[5] that settles the decoding order. This last operation is indeed the most critical point since the effectiveness of the procedure strongly depends on the reliability of taking decisions.

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