Construction of M-QAM STCC Based on QPSK STCC

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***Abstract*— Space-time convolutional code (STCC) is a technique that combines transmit diversity and coding to improve reliability in wireless fading channels. In this paper, we demonstrate a technique to design MQAM STCCs with high spectral efficiencies utilizing QPSK STCC as component code. The approach is based on a unique construction of M-QAM signal from several QPSK signals. The upper bound on the pairwise block error probability of the new scheme is derived. Simulation result demonstrates that the new technique outperforms existing M-QAM STCC design for $M = 16$.**

I. Introduction

Space-time code [1] has demonstrated remarkable performance by combining the capabilities of transmit spatial diversity with coding. In its original design, the space-time code required multiple transmit antennas in order to transmit encoded symbols simultaneously from different transmit antennas. In this paper, we present a technique that allows us to employ STCC to systems with only a single transmit antenna and multiple receiver antennas. The idea is to transmit more than one symbol from a single transmit antenna. We first present a technique that combines multiple symbols required to transmit in space-time code by superimposing them on each other. In order to combine these symbols at the transmitter and be able to separate them at the receiver, we multiply each symbol by different values that is known at the receiver, and then combine them. Therefore, many symbols can be sent out from a single transmit antenna in one symbol time period. This approach can be interpreted as creating additional transmit antennas, called virtual antennas. These virtual antennas create statistically dependent channels between virtual transmit antennas and each receive antenna that we call virtual paths. Once we show that one can transmit multiple symbols generated from the output of the STCC encoder using a single transmit antenna, we then demonstrate that a multiple-input multiple-output (MIMO) $n_T \times n_R$ system can be modeled equivalently with n_T distinct $1 \times n_R$ single-input multiple-output (SIMO) systems and apply STCC for each of $1 \times n_R$ SIMO systems. This aproach allows us to use QPSK STCCs as component codes and create STCCs with arbitrary high spectral efficiencies.

It has been shown in [2], [3], [4] that any M-QAM symbols with square constellation can be constructed with $\frac{\log_2 M}{2}$ QPSK symbols. This unique construction of M-QAM symbols is obtained by rotation and scaling of QPSK symbols using different scaling factors. In this paper, we construct M-QAM STCC utilizing QPSK STCC as component codes. This construction will allow us to have any M-QAM STCC constellation with a single QPSK STCC encoder.

The remainder of the paper is organized as follows. In section II, the basic system model and assumptions are presented. The construction of M-QAM symbol from QPSK symbols are described briefly in section III. In section IV, we present our approach and derive the upper bound on the pairwise block error probability. Simulation results and conclusion are presented in sections V and VI respectively.

II. System Model

We consider a wireless communication system utilizing n_T transmit and n_R receive antennas. A block error occurs when the decoded data sequence

$$\mathbf{e} = e_1^1 e_1^2 \dots e_1^{n_T} e_2^1 e_2^2 \dots e_2^{n_T} \dots e_l^1 e_l^2 \dots e_l^{n_T}$$

is different from the transmit data sequence

$$\mathbf{c} = c_1^1 c_1^2 \dots c_1^{n_T} c_2^1 c_2^2 \dots c_2^{n_T} \dots c_l^1 c_l^2 \dots c_l^{n_T}$$

where l is the number of symbols in one block. The channel path gain from antenna i to receive antenna j is denoted by $h_{i,j}$. These path gains are constant during a frame and change independently from one frame to another. The received data r_t^j at antenna j and time t, can be written as

$$r_t^j = \sum_{i=1}^{n_T} h_{i,j} c_t^i \sqrt{E_s} + n_t^j, \qquad 1 \leq j \leq n_R \tag{1}$$

where c_t^i is the complex transmit symbol with unit average power sent from antenna i at time t, n_t^j is the additive white Gaussian noise sample with zero mean and variance $\frac{N_0}{2}$ per dimension, and E_s is the signal constellation contraction factor.

The conditional pairwise block error probablity has the upper bound [1]

$$P(\mathbf{c} \rightarrow \mathbf{e} \mid h_{i,j}, 1 \leq i \leq n_T, 1 \leq j \leq n_R) \leq \exp\left(-d^2(\mathbf{c}, \mathbf{e}) \frac{Es}{4N_0}\right), \tag{2}$$

where $d^2(\mathbf{c}, \mathbf{e})$ is given by

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^{n_R} \sum_{t=1}^{l} \left| \sum_{i=1}^{n_T} h_{i,j}(c_t^i - e_t^i) \right|^2 \tag{3}$$

This upper bound on the pairwise block error probability can be written in matrix format as

$$P(\mathbf{c} \rightarrow \mathbf{e} \mid h_{i,j}, 1 \leq i \leq n_T, 1 \leq j \leq n_R) \leq \exp\left(\sum_{j=1}^{n_R} \Omega_j \mathbf{A} \Omega_j^* \frac{Es}{4N_0}\right), \tag{4}$$

where $\Omega_j = [h_{1,j}, \dots, h_{n_T,j}]$ and $\mathbf{A}(\mathbf{c}, \mathbf{e}) = \mathbf{B}(\mathbf{c}, \mathbf{e})\mathbf{B}^*(\mathbf{c}, \mathbf{e})$. The matrix $\mathbf{B}(\mathbf{c}, \mathbf{e})$ is defined as

$$\mathbf{B}(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} e_1^1 - c_1^1 & \dots & e_l^1 - c_l^1 \\ \vdots & \ddots & \vdots \\ e_1^{n_T} - c_1^{n_T} & \dots & e_l^{n_T} - c_l^{n_T} \end{pmatrix}.$$

It is easy to show that the matrix $\mathbf{A}(\mathbf{c}, \mathbf{e})$ is a Hermitian matrix. Therefore, there is a unitary matrix V and a real diagonal matrix D such that $\mathbf{A}(\mathbf{c}, \mathbf{e}) = V^* D V$. The diagonal elements of D are the eigenvalues of $\mathbf{A}(\mathbf{c}, \mathbf{e})$ denoted as λ_i, $1 \leq i \leq n_T$.

Define a vector $\beta_{1,j}, \dots, \beta_{n_T,j} = \Omega_j V^*$, then

$$\Omega_j \mathbf{A} \Omega_j^* = \sum_{i=1}^{n_T} \lambda_i |\beta_{i,j}|^2.$$

$|\beta_{i,j}|$ has a Rayleigh or Rician distribution depending on whether the channel paths have zero or non-zero mean distribution.

In the case of Rayleigh fading, the upper bound on the pairwise error probability of each block is further expressed as [1]

$$P(\mathbf{c} \rightarrow \mathbf{e}) \leq \left(\prod_{i=1}^{n_T} \frac{1}{1 + \frac{E_S}{4N_0} \lambda_i} \right)^{n_R} \tag{5}$$

In this paper, we assume that all the physical channel path gains are statistically independent with Rayleigh fading distribution. The extention of this work to Rician fading distribution is straightforward.

III. Construction of M-QAM Signal Constellation from QPSK Signals

The construction of M-QAM signal ($M = 2^{n_1}$) from multiple QPSK (x_i) signals can be obtained using the following equation [2], [3].

$$M - QAM = \sum_{i=1}^{\frac{n_1}{2}} 2^{i-1} \frac{\sqrt{2}}{2} x_i \exp\left(j \frac{\pi}{4}\right) \quad 1 \leq i \leq \frac{n_1}{2} \tag{6}$$

The QPSK modulation is realized by choosing x_i from the following set, $\{+1, +j, -1, -j\}$. In this equation, we use $\frac{n_1}{2}$ QPSK signals to construct an M-QAM symbol. Each M-QAM signal that is constructed based on (6) utilizes the QPSK symbols that are from the output of the STCC encoder. For example, if we want to create a 16-QAM signal constellation from QPSK symbols, we choose $n_1 = 4$ in (6). In the first operation to construct a 16-QAM signal, we apply to the two QPSK signals x_0 and x_1 a rotation and a scaling factor by multipliying them by $\exp(j\frac{\pi}{4})$ and $\frac{\sqrt{2}}{2}$. For the second operation, a scaling factor of 2^0 is applied to x_0 and 2^1 to x_1. The third and final operation is a summation of the new scaled x_0 and x_1 constellations as shown in figure 1.

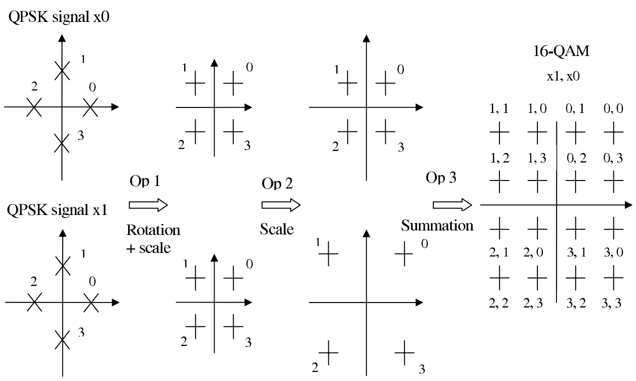

Fig. 1. Construction of a 16-QAM signal constellation from two QPSK symbols

IV. Problem Formulation

We will derive the upper bound pairwise block error probability to evaluate the performance of this system and compare that with the performance of the original STCC design. The search space for STCCs increases exponentially with the increase in the constellation size. However, utilizing QPSK STCC as component code, we can design STCC with large constellation size without any need for such search. We will first compute the diversity and coding gain with only a single transmit antenna by creating virtual path gains as long as we have multiple receive antennas. The construction of M-QAM STTCs with high spectral efficiencies will be done using QPSK STCC based on (6).

Assuming the number n_T of transmit antennas equals to 1, (1) can be written as

$$r_t^j = h_{1,j} C_t \sqrt{E_s} + n_t^j. \qquad 1 \leq j \leq n_R \tag{7}$$

The coefficients $h_{1,j}$ are modeled as independent samples of complex Gaussian random variables with zero complex mean and variance 0.5 per dimension. We propose to modify the transmit signal C_t using (6).

$$C_t = \sum_{i=1}^{n_V} 2^{i-1} \frac{\sqrt{2}}{2} c_t^i \exp\left(j\frac{\pi}{4}\right) \tag{8}$$

n_V is the number of statistically dependent virtual path gains created between the transmit antenna and each receive antenna. Combining (7) and (8) leads to

$$r_t^j = \sum_{i=1}^{n_V} h'_{i,j} c_t^i \sqrt{E_s} + n_t^j, \qquad 1 \leq j \leq n_R \tag{9}$$

where

$$h'_{i,j} = h_{1,j} 2^{i-1} \frac{\sqrt{2}}{2} \exp\left(j\frac{\pi}{4}\right) \tag{10}$$

is called a virtual path gain.

We have created a MIMO ($n_V \times n_R$) system from a SIMO ($1 \times n_R$) system. The actual number of physical channel path gains are n_R, therefore, the rank of the new channel matrix is unchanged. $d^2(\mathbf{c}, \mathbf{e})$ is now given by

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^{n_R} \sum_{t=1}^{l} |h_{1,j}|^2 \left| \sum_{i=1}^{n_V} 2^{i-1} \frac{\sqrt{2}}{2} (c_t^i - e_t^i) \right|^2 \tag{11}$$

Equivalently, it can be written as

$$d^2(\mathbf{c}, \mathbf{e}) = \sum_{j=1}^{n_R} |h_{1,j}|^2 \mathbf{A}'(\mathbf{c}, \mathbf{e}), \tag{12}$$

where

$$\mathbf{A}'(\mathbf{c}, \mathbf{e}) = \sum_{t=1}^{l} \left| \sum_{i=1}^{n_V} 2^{i-1} \frac{\sqrt{2}}{2} (c_t^i - e_t^i) \right|^2 . \tag{13}$$

The conditional pairwise block error probablity can be upper bounded as

$$\begin{gathered} P(\mathbf{c} \to \mathbf{e} \mid h_{1,j}, 1 \leq j \leq n_R) \leq \\ \prod_{j=1}^{n_R} \exp\left(- |h_{1,j}|^2 \mathbf{A}'(\mathbf{c}, \mathbf{e}) \frac{Es}{4N_0} \right) \end{gathered} \tag{14}$$

By averaging (14) with respect to independent Rayleigh distribution of $|h_{1,j}|^2$, we get

$$P(\mathbf{c} \to \mathbf{e}) \leq \left(1 + \mathbf{A}'(\mathbf{c}, \mathbf{e})\right)^{-n_R} \tag{15}$$

This result clearly demonstrates that in order to reduce the upper bound on the pairwise block error probability, we need to maximize the minimum Euclidean distance of the STCC. There are several techniques in literature to design STCC that maximizes the minimum Euclidean distance [5], [6], [7].

V. Simulation Results

For this particular example, each STCC encoder contains 130 symbols which is equivalent of 520 bits per frame. The channel is Rayleigh fading channel and it is constant during each frame of 130 symbols and changes independently from one frame to another. A coherent detection is assumed at the receiver that requires perfect knowledge of the channel.

Figure 2 shows that in the case of SIMO 1×2 and 1×5 systems, the coding gain is much less than MIMO 2×2 and 2×5 systems respectively, which is expected because the virtual channels are statistically dependent.

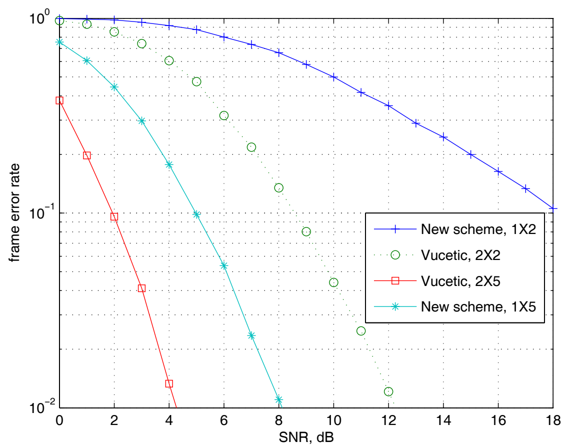

Fig. 2. Frame error rate performance of QPSK STCC 4-State 2×2 and 2×5 systems from [5] is compared with the 1×2 and 1×5 systems using the new scheme.

Figure 3 demonstrates the frame error rate performance of 2×3 and 2×5 MIMO systems utilizing 16-QAM STCC of [1] and the new scheme combining 2 QPSK STCCs of [5]. The simulation result demonstrates that the new scheme outperforms the existing 16-QAM STCC of [1] by as much as 2 dB.

VI. Conclusion

We have shown that a 16-QAM STCC constellation can be built from 2 QPSK STCCs. Higher QAM constellation can also be generated by using (6). This approach recommends to model a MIMO $n_T \times n_R$ system as equivalent of n_T distinct $1 \times n_R$ SIMO systems. Then, STCC is applied to each $1 \times n_R$ SIMO system. Our derivation for the upper bound on the pairwise block error probability demonstrates that this upper bound will be minimized if STCC is designed considering maximum Euclidean distance of the code similar to the design of [5]. Therefore, the QPSK STCC component code were selected from [5].

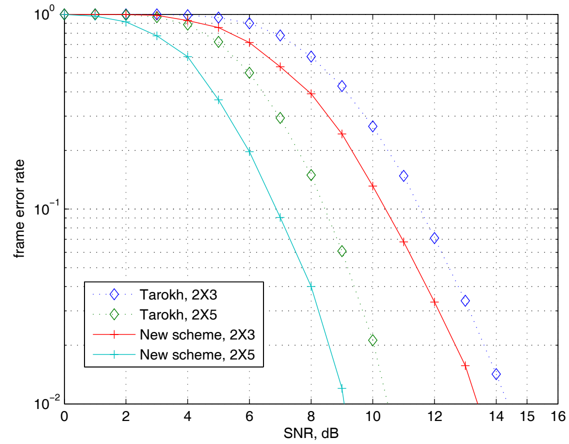

Fig. 3. Frame error rate performance of a 16-QAM STCC 2×3 and 2×5 systems from [1] are compared with the new scheme using 16-QAM constellation constructed from two STCC QPSK of [5].

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