EQUALIZATION FOR MIMO ISI SYSTEMS USING CHANNEL INVERSION UNDER CAUSALITY, STABILITY AND ROBUSTNESS CONSTRAINTS

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ABSTRACT

The problem of linear equalization for frequency selective multiple-input multiple-output wireless channels, using channel inversion, is considered. While in the flat-fading case, the solution with minimal noise enhancement is readily found and is automatically optimal in terms of energy norm, the problem becomes intricate when intersymbol interference is present. In order to analyze this behavior, a framework, based on the semidefinite programming technique, is employed in the paper. Additionally, connections with possible analytical solutions based on the common system block matrix representation are established. The proposed framework is easily adopted for other linear equalization methods, pre-equalization, and can also be used for the feedforward filter construction in successive interference cancellation strategies.

1. INTRODUCTION

When linear equalization techniques are concerned, if no explicite knowledge of transmit data and receive noise statistics is assumed, channel inversion (zero-forcing) is to be applied. As common in communications theory, minimization of the noise enhancement remains to be the principal goal. Practical application requires also that the solution is causal, and both bounded-input bounded-output (BIBO) and internally stable. On the other hand, the control theory research introduces another concept: energetic stability. Although the data and noise signals in communications theory are often not considered as energy signals (signals of finite energy), there are beneficial aspects of having filters with low energy norm, especially in terms of robustness, which will be shown in the sequel.

If a multicarrier approach for spatial and intersymbol interference (ISI) removal in multiple-input multiple-output (MIMO) ISI systems is applied, the equalization has to cope only with flat fading channels [1]. In this case, the problem of channel inversion is relatively simple. The solution, based on the Moore-Penrose pseudoinverse [2], is immediately stable and causal, and has the lowest energy norm.

However, the idea of inverting the channel becomes intricate when spatial and temporal equalization are to be performed jointly under the constraints of practical realizability (e.g., [3–6] and the references therein). Only the general expression for the pseudoinverse, which yields a non-causal filter, can be considered optimal in an equivalent way as the matrix pseudoinverse is for the flat fading channels [3]. The introduction of the causality constraint brings complex dependencies of the filter energy norm on the system dimensions [3, 4], entangled with the fact that left inverses, if they exist at all, are not unique. These issues are addressed in this paper by employing and extending semidefinite programming (SDP) techniques [7–9], in order to accommodate equalization of wireless MIMO channels. Furthermore, the used framework provides a computationally efficient method to analyze how important filter properties such as noise enhancement, energy norm (robustness) and possible upper-bounded error with respect to perfect zero-forcing (ZF), depend on the system parameters like delay, filter and channel orders, antenna array size, etc. Additionally, relations with expressions based on the conventional system block matrix representation are explored.

As far as notation is concerned, all vectors and matrices are in boldface. The trace of a matrix is denoted by $\mathrm{tr}(\cdot)$, while $\sigma_{max}(\cdot)$ stands for the largest singular value of a matrix. $\boldsymbol{I}_M$ is the identity $M \times M$ matrix. Hermitian and transpose operation, and Moore-Penrose pseudoinverse are denoted by $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\ddagger$, respectively. Frobenius norm of $\boldsymbol{A}$ [2] is written as $\|\boldsymbol{A}\|_F$. $\boldsymbol{X} \preceq \boldsymbol{Y}$ means that $\boldsymbol{Y} - \boldsymbol{X}$ is positive semidefinite. The constraint of the form $\boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{F}_0 + \sum_{i=1}^{m} x_i \boldsymbol{F}_i \succeq \boldsymbol{0}$, where $\boldsymbol{x} \in \mathbb{R}^m$ is the vector of variables, and $\boldsymbol{F}_i = \boldsymbol{F}_i^* \in \mathbb{C}^{M\times M}$ are given, is called linear matrix inequality (LMI) [7]. $L^2(\mathbb{C}^M)$ denotes the space of complex vector functions defined on $[-\pi, \pi)$.

2. SYSTEM MODEL AND PROBLEM STATEMENT

Frequency selective MIMO system with K transmit and R receive antennas ($R \geq K$) is considered. The channel is modeled as a $R\times K$ causal matrix polynomial (matrix transfer function) $\boldsymbol{H}(z)$ (in z^{-1}, standard z-transform used). The equalization is performed using a $K \times R$ causal matrix polynomial filter $\boldsymbol{G}(z)$. $\boldsymbol{G}(z)$ is said to be a left inverse of $\boldsymbol{H}(z)$, if $\boldsymbol{G}(z)\boldsymbol{H}(z) = \boldsymbol{I}_K$.

For the description of system performances, two system norms will be used. The H_2 and H_∞ norm of a causal matrix polynomial $\boldsymbol{W}(z)$ are defined as [8]:

$$\|\boldsymbol{W}\|_2 = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \mathrm{tr}\big(\boldsymbol{W}(e^{j\omega})\boldsymbol{W}(e^{j\omega})^*\big)d\omega}, \qquad (1)$$

$$\|\boldsymbol{W}\|_\infty = \sup_{|z|>1} \sigma_{max}(\boldsymbol{W}(z)). \qquad (2)$$

*This work has been supported (in part) by the *Bundesministerium für Bildung und Forschung (BMBF)* under grant 01BU350.

If an arbitrary (non-causal) frequency response $\boldsymbol{W}(e^{j\omega})$ is considered as a linear operator $\boldsymbol{W} : L^2(\mathbb{C}^K) \rightarrow L^2(\mathbb{C}^M)$, the induced operator norm

$$\|\boldsymbol{W}\|_E = \sup_{\boldsymbol{u}\in L^2(\mathbb{C}^K),\boldsymbol{u}\neq\boldsymbol{0}} \frac{\|\boldsymbol{W}\boldsymbol{u}\|_2}{\|\boldsymbol{u}\|_2}, \tag{3}$$

where $\|\boldsymbol{u}\|_2 = \sqrt{\boldsymbol{u}\cdot\boldsymbol{u}}$, $\boldsymbol{u}\cdot\boldsymbol{v} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\boldsymbol{u}(e^{j\omega})^*\boldsymbol{v}(e^{j\omega})d\omega$, is called the energy norm. It can be shown that for all causal matrix transfer functions (MTF) $\boldsymbol{W}(z)$, $\|\boldsymbol{W}\|_E = \|\boldsymbol{W}\|_\infty$ [3].

Let $\boldsymbol{H}(z)$ be a causal and energy stable (finite energy norm) $R\times K$ system. Its (non-causal) pseudoniverse is given as [3]

$$\boldsymbol{H}(z)^\ddagger = \left(\boldsymbol{H}(z)^*\boldsymbol{H}(z)\right)^{-1}\boldsymbol{H}(z)^*. \tag{4}$$

Using the theory of Banach algebras [4], it can be shown that a causal left inverse of $\boldsymbol{H}(z)$ exists if and only if exists $\delta\in\mathbb{R}$ such that

$$(\forall|z|>1)\quad \boldsymbol{H}(z)^*\boldsymbol{H}(z)\succeq\delta^2\boldsymbol{I}_K. \tag{5}$$

It is known that (4) gives the (non-causal) filter with the smallest possible energy norm among all left inverses $\|\boldsymbol{H}^\ddagger\|_E = \delta^{-1}$, where δ is the largest constant so that (5) is valid, and which is independent of system dimensions [4]. While the causality constraint brings no significant difficulties if the system is symmetric ($R = K$), it is interesting that the energy norm behavior of the causal pseudoinverse seems unpredictable when $R > K$, with the only certain property of being larger than the energy norm of the optimal, non-causal filter (4) [3].

Now, we can formulate two fundamental problems of interest for practical MIMO detection. One direction might be to minimize the equalization filter H_2 norm, $\|\boldsymbol{G}\|_2$, which presents the measure of the noise enhancement in the system, subject to an upper bound on $\|\varphi\boldsymbol{I}_K - \boldsymbol{G}\boldsymbol{H}\|_\infty$ (the allowed error with respect to perfect ZF, to support cases when no inverse exists), with $\varphi(z) = z^{-d}$ being the delay function, or determine that the problem is infeasible.

On the other hand, in terms of robustness, it is beneficial to minimize the filter H_∞ norm (or equivalently the energy norm, because of the causality assumption) under the same constraints on the equalization error. To explain this, suppose that the channel is estimated erroneously at the receiver with $\boldsymbol{H}(z) = \boldsymbol{H}_C(z) + \boldsymbol{\Delta H}(z)$, where $\boldsymbol{H}_C(z)$ is the exact channel and $\boldsymbol{\Delta H}(z)$ is the estimation error, with $\|\boldsymbol{\Delta H}\|_E \leq \mu$. In this case, if channel inversion is applied, it can be shown that the equalization error is tightly upper bounded with $\mu\|\boldsymbol{G}\|_E$ [3].

These problems motivate the search for the tools which will be able to handle both H_2 and H_∞ system norm constraints in the general case ($R\geq K$).

In the sequel, it will be assumed that the channel $\boldsymbol{H}(z)$ and the equalization filter $\boldsymbol{G}(z)$ have finite impulse responses (FIR) with N_C and N_F taps, respectively:

$$\boldsymbol{H}(z) = \sum_{l=0}^{N_C-1}\boldsymbol{H}(l)z^{-l},\quad \boldsymbol{G}(z) = \sum_{l=0}^{N_F-1}\boldsymbol{G}(l)z^{-l}. \tag{6}$$

The process of signal transmission and detection is described with the following equations (zero delay assumed for convenience):

$$\boldsymbol{r}(k) = \sum_{l=0}^{k}\boldsymbol{H}(l)\boldsymbol{s}(k-l) + \boldsymbol{n}(k), \tag{7}$$

$$\hat{\boldsymbol{s}}(k) = \sum_{l=0}^{k}\boldsymbol{G}(l)\boldsymbol{r}(k-l), \tag{8}$$

where $\boldsymbol{s}(k)$ and $\boldsymbol{r}(k)$ are transmit and receive vector data streams, $\boldsymbol{n}(k)$ is the receiver noise, and $\hat{\boldsymbol{s}}(k)$ is the input for hard detection.

3. SDP-BASED FRAMEWORK

Both norm constraints in the main problem formulation (Section 2) contain certain matrix polynomials. In principle, both of these polynomials can be represented in the following form:

$$\boldsymbol{W}(z,\boldsymbol{g}) = \boldsymbol{W}_0 + \sum_{i=1}^{\mathcal{G}} g_i\boldsymbol{W}_i(z), \tag{9}$$

where vector $\boldsymbol{g} = [g_1, g_2, \cdots, g_{\mathcal{G}}]$ contains all coefficients of the equalization filter $\boldsymbol{G}(z)$. Therefore, matrix polynomial $\boldsymbol{W}(z,\boldsymbol{g})$ depends affinely on $\boldsymbol{g}$.

A state-space realization $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D})$ of a MTF $\boldsymbol{W}(z)$ is [10]:

$$\boldsymbol{x}(i+1) = \boldsymbol{A}\boldsymbol{x}(i) + \boldsymbol{B}\boldsymbol{u}(i),\qquad \boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{x}(i) + \boldsymbol{D}\boldsymbol{u}(i), \tag{10}$$

with

$$\boldsymbol{W}(z) = \boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}. \tag{11}$$

For a $\boldsymbol{W}(z,\boldsymbol{g})$ that affinely depends on $\boldsymbol{g}$, there exists a state-space realization $(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}(\boldsymbol{g}),\boldsymbol{D}(\boldsymbol{g}))$, with $\boldsymbol{C}$ and $\boldsymbol{D}$ depending affinely on $\boldsymbol{g}$.

In order to handle H_2 norm constraints, by using results from [9], the following theorem can be proved:

Theorem 1 (Bounded H_2 Norm and LMIs) *The constraint*

$$\|\boldsymbol{W}(\boldsymbol{g})\|_2 \leq \eta, \tag{12}$$

where $\boldsymbol{W}(z,\boldsymbol{g})$ depends affinely on $\boldsymbol{g}$, is equivalent to the following LMI (in $\boldsymbol{g}$ and $\boldsymbol{X}$):

$$\mathrm{tr}(\boldsymbol{X}) \leq \eta^2,\qquad \begin{bmatrix} \boldsymbol{X} & \boldsymbol{C}(\boldsymbol{g}) & \boldsymbol{D}(\boldsymbol{g}) \\ \boldsymbol{C}(\boldsymbol{g})^* & \boldsymbol{P}^{-1} & \boldsymbol{0} \\ \boldsymbol{D}(\boldsymbol{g})^* & \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \succeq 0, \tag{13}$$

where $\boldsymbol{X} = \boldsymbol{X}^*$ is the slack matrix variable. $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}(\boldsymbol{g}), \boldsymbol{D}(\boldsymbol{g}))$ is the state-space description of $\boldsymbol{W}(z,\boldsymbol{g})$, and $\boldsymbol{C}(\boldsymbol{g})$, $\boldsymbol{D}(\boldsymbol{g})$ depend affinely on $\boldsymbol{g}$. $\boldsymbol{P}$ is the solution of the Lyapunov equation

$$\boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^* - \boldsymbol{P} + \boldsymbol{B}\boldsymbol{B}^* = \boldsymbol{0}. \tag{14}$$

The H_∞ norm can be controlled using the following theorem, which is a consequence of the discrete-time bounded real lemma [10], using the Schur complement lemma [11]:

Theorem 2 (Bounded H_∞ Norm and LMIs) *The constraint*

$$\|\boldsymbol{W}(\boldsymbol{g})\|_\infty \leq \varepsilon, \tag{15}$$

where $\boldsymbol{W}(z,\boldsymbol{g})$ depends affinely on $\boldsymbol{g}$, is equivalent to the following LMI in $\boldsymbol{X}$ and $\boldsymbol{g}$:

$$\boldsymbol{X}\succeq 0,\quad \begin{bmatrix} -\boldsymbol{A}^*\boldsymbol{X}\boldsymbol{A}+\boldsymbol{X} & -\boldsymbol{A}^*\boldsymbol{X}\boldsymbol{B} & \boldsymbol{C}(\boldsymbol{g})^* \\ -\boldsymbol{B}^*\boldsymbol{X}\boldsymbol{A} & \varepsilon^2\boldsymbol{I}-\boldsymbol{B}^*\boldsymbol{X}\boldsymbol{B} & \boldsymbol{D}(\boldsymbol{g})^* \\ \boldsymbol{C}(\boldsymbol{g}) & \boldsymbol{D}(\boldsymbol{g}) & \boldsymbol{I} \end{bmatrix} \succeq 0, \tag{16}$$

where $(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}(\boldsymbol{g}), \boldsymbol{D}(\boldsymbol{g}))$ is the state-space description of $\boldsymbol{W}(z,\boldsymbol{g})$, and $\boldsymbol{C}(\boldsymbol{g})$, $\boldsymbol{D}(\boldsymbol{g})$ depend affinely on $\boldsymbol{g}$.

Consider an arbitrary $M\times L$, FIR matrix polynomial

$$\boldsymbol{W}(z) = \boldsymbol{W}(0) + \sum_{l=1}^{N_W-1}\boldsymbol{W}(l)z^{-l}. \tag{17}$$

State-space description of an arbitrary $\boldsymbol{W}(z)$ is not unique. For the purpose of equalization filters construction using LMIs, we propose the following state-space realization (10)-(11):

$$\begin{aligned}
\boldsymbol{A} &= \begin{bmatrix} \mathbf{0}_{L\times L(N_W-2)} & \mathbf{0}_{L\times L} \\ \boldsymbol{I}_{L(N_W-2)} & \mathbf{0}_{L(N_W-2)\times L} \end{bmatrix}, \\
\boldsymbol{B} &= \begin{bmatrix} \boldsymbol{I}_L \\ \mathbf{0}_{L(N_W-2)\times L} \end{bmatrix}, \\
\boldsymbol{C} &= \begin{bmatrix} \boldsymbol{W}(1) & \boldsymbol{W}(2) & \dots & \boldsymbol{W}(N_W-1) \end{bmatrix}, \\
\boldsymbol{D} &= \boldsymbol{W}(0),
\end{aligned} \qquad (18)$$

where $\boldsymbol{A} = \boldsymbol{B} = \boldsymbol{C} = \mathbf{0}$ in the trivial case $N_W = 1$. It can be shown that the state-space realization (18) has the following properties:

- $\boldsymbol{C}$ and $\boldsymbol{D}$ depend affinely on polynomial coefficients (the elements of $\boldsymbol{W}(i)$, $i = 0 \dots N_W - 1$).
- The solution of Lyapunov equation (14) is unique, positive definite and has a simple form: $\boldsymbol{P} = \boldsymbol{I}_{L(N_W-1)}$.

The problems from Section 2 can now be solved by minimizing one slack variable t, subject to $\|\boldsymbol{G}\|_2 \leq t$ (or $\|\boldsymbol{G}\|_\infty \leq t$) and $\|\varphi \boldsymbol{I}_K - \boldsymbol{G}\boldsymbol{H}\|_\infty \leq \varepsilon$ (the upper bounded error). In the latter case, the state-space realization (18) is applied on the FIR function $\boldsymbol{W}(z) = z^{-d}\boldsymbol{I}_K - \boldsymbol{G}(z)\boldsymbol{H}(z)$, with $M = L = K$, and the equivalent LMI is obtained using Theorem 2. The constraints $\|\boldsymbol{G}\|_2 \leq t$ ($\|\boldsymbol{G}\|_\infty \leq t$) are transformed into LMIs by applying the Theorem 1 (Theorem 2) directly, with $\boldsymbol{W}(z) = \boldsymbol{G}(z)$, $M = K$, $L = R$.

Therefore, by transforming the constraints into LMI conditions, the original problem formulation is shown to be equivalent to standard SDP problems that can be solved efficiently with several recently developed numerical algorithms [7].

4. BLOCK MATRIX REPRESENTATION

Differently from the previous approach, consider equalization (7)-(8) using the block matrix representation. The desired relation (zero delay assumed again for convenience) $\boldsymbol{G}(z)\boldsymbol{H}(z) = \boldsymbol{I}$ can be written in the following form:

$$\tilde{\boldsymbol{G}}_{K\times RN_F}\tilde{\boldsymbol{H}}_{RN_F\times K(N_C+N_F-1)} = \tilde{\boldsymbol{I}}_{K\times K(N_C+N_F-1)}, \qquad (19)$$

where

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} \boldsymbol{G}(0) & \boldsymbol{G}(1) & \cdots & \boldsymbol{G}(N_F-1) \end{bmatrix}, \qquad (20)$$

$$\tilde{\boldsymbol{I}} = \begin{bmatrix} \boldsymbol{I}_K & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}, \qquad (21)$$

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \boldsymbol{H}(0) & \cdots & \boldsymbol{H}(N_C-1) & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \boldsymbol{H}(0) & \dots & \boldsymbol{H}(N_C-1) \end{bmatrix}. \qquad (22)$$

The matrix equation (19) can be studied as a system of linear equations where the filter coefficients are unknowns. In the sequel, we examine the properties of the following expression for the equalization filter coefficients

$$\boldsymbol{G}^{\#} = \tilde{\boldsymbol{I}}\tilde{\boldsymbol{H}}^{\ddagger}. \qquad (23)$$

When (19) is consistent, one or more solutions exist for the equalizing filter. If more solutions exist, then "the best one" should be found. Let the quality of one solution be determined by $\|\boldsymbol{G}\|_2$, which should be as low as possible. It can be seen that the solution (23) is the best in this sense by noticing first the following relation

$$\|\boldsymbol{G}\|_2 = \|\tilde{\boldsymbol{G}}\|_F, \qquad (24)$$

which is obtained by the direct calculation from (1) and the definition of the matrix Frobenius norm [2]. However, it stems from the properties of the Moore-Penrose pseudoinverse that (23) is the solution of (19) with minimum Frobenius norm [2]. Therefore, we conclude that when (19) is consistent, the solution (23) is the one with the smallest H_2 norm of $\boldsymbol{G}(z)$. Since the pseudoinverse gives the unique solution with minimum Frobenius norm [2], (only) in the case when perfect ZF is possible and desired ($\varepsilon = 0$ in (15)), the approach based on the equation (23) solves the noise enhancement minimization variation of the principal problem posed in Section 2, and it is equivalent to the LMI-based method from Section 3.

The inconsistency of (19) brings several additional aspects into consideration. The expression (23) still exists and, similarly to (24), it can be shown that in this case the following relation is valid

$$\|\tilde{\boldsymbol{I}} - \tilde{\boldsymbol{G}}\tilde{\boldsymbol{H}}\|_F = \|\boldsymbol{I} - \boldsymbol{G}\boldsymbol{H}\|_2. \qquad (25)$$

Therefore, using the properties of the pseudoinverse [2], it can be concluded that (23) gives the minimum feasible $\|\boldsymbol{I} - \boldsymbol{G}\boldsymbol{H}\|_2$, and the corresponding minimum $\|\boldsymbol{G}\|_2$. This might provide insight if $\|\boldsymbol{I} - \boldsymbol{G}\boldsymbol{H}\|_2$ is used as an error measure with respect to perfect ZF, instead of $\|\boldsymbol{I} - \boldsymbol{G}\boldsymbol{H}\|_\infty$, which is addressed in [6].

5. NUMERICAL EXAMPLES

For numerical analysis, two fixed, randomly chosen, 3-tap MIMO channels, with dimensions 2×2 and 3×2, respectively, are observed. SDP-based solutions were implemented using SeDuMi [12].

In the case of 2×2 MIMO, the filter is assumed to have 3 taps. It is known that, in principle, symmetric MIMO channel is not perfectly invertible with any FIR filter [13]. This is verified in Fig. 1, using the proposed SDP-based framework, which shows that the feasibility of the problem is guaranteed only for $\varepsilon > 0.04$ ($d = 0$ for Fig. 1). On the other side, when $R > K$, ideal ZF is possible, in principle. The necessary number of equalization filter taps in that case is $N_F = (N_C - 1)K/(R - K)$ [13], which yields $N_F = 4$ for the studied 3×2 system. Therefore, filter with 4 taps was used for the equalization of the 3×2 channel, with results in Fig. 1 confirming the theoretical prediction. Interestingly, when $\|\boldsymbol{G}\|_\infty$ was minimized, in the case of perfect ZF, the equalization filter seems to converge to the one obtained by the minimization of $\|\boldsymbol{G}\|_2$, which indicates good properties in terms of robustness.

In Fig. 2, the characteristic points obtained by the pseudoinverse solution (23), denoted by PINV, and the general, non-causal pseudoinverse (GPINV) (4) are shown for the 3×2 channel. In the case of GPINV, delayed version is assumed, in order for the causality to be guaranteed. One can observe the starting (highest) points in the tradeoffs given by the PINV solution and the effect of lowering the tradeoff curves with increasing the filter order. However, the performances of a system with no delay are clearly limited, and only the introduction of an infinite delay leads to the properties of the ideal, non-causal filter. Interestingly, even a relatively coarse approximation of GPINV, with only 5 taps, gives a good estimate of the final limit when minimization of $\|\boldsymbol{G}\|_2$ is concerned.

Finally, in Fig. 3, raw bit error rates (BER) as a function of the transmit signal (4-QAMs, uncorrelated) to the noise (Gaussian, white) power ratio (SNR), for the 3×2 MIMO channel, are plotted ($d = 0$). The improvements obtained by introducing larger equalization filter orders are easily noticed, which justifies the studying of not only the necessary and sufficient ZF conditions. Also, one can observe that the introduction of an upper bound on the equalization error, instead of perfect ZF, might be beneficial, as expected.

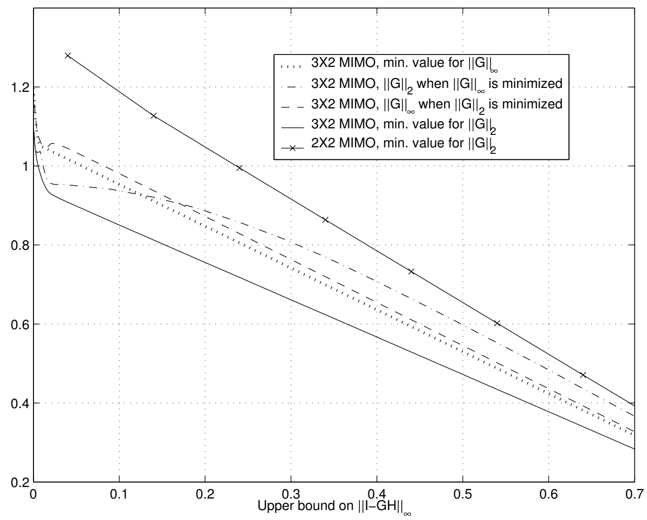

Fig. 1. Tradeoff curves for 2×2 and 3×2 MIMO channel.

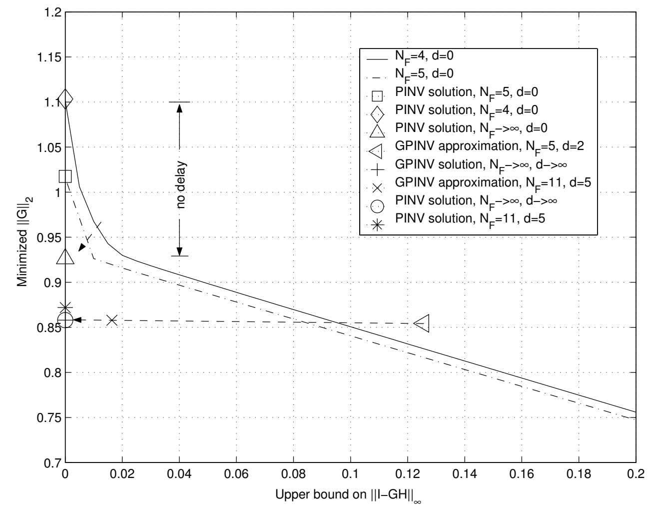

Fig. 2. Characterization of the tradeoffs by pseudoinverse solutions.

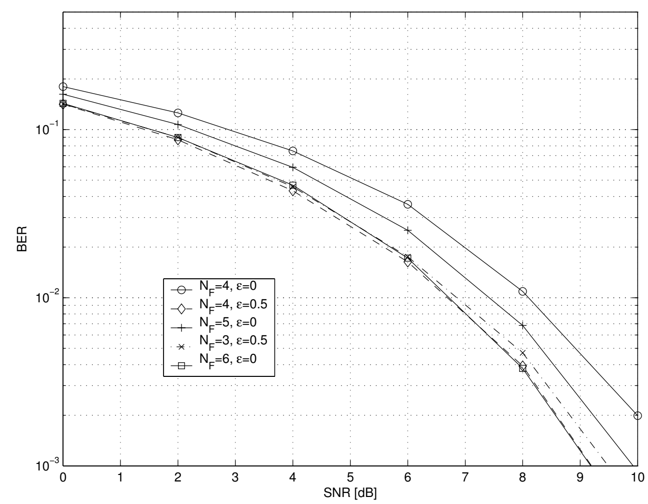

Fig. 3. BERs vs. SNR for 3×2 MIMO channel.

6. CONCLUSION

The problem of channel inversion becomes involved in frequency selective MIMO equalization because the system parameters have intricate impacts on overall performances. Furthermore, if more solutions exist, qualitatively the best one should be found, with respect to the filter H_2 and H_∞ norm. Equalization can be enabled and also improved even in the cases when a certain error is tolerated and upper-bounded. The proposed SDP-based framework is shown to be suitable for the analysis of the mentioned effects and for obtaining the equalization filters under the corresponding constraints. Certain characteristic points of the gained tradeoff curves are proved to be equivalent to the solutions based on the common block-matrix system representation, which is otherwise not suitable for handling the imposed constraints in general. Finally, the presented results enable future investigation of a large spectrum of constraints that can be based on the system H_∞ and H_2 norms.

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