

# SUBSPACE-BASED BLIND CHANNEL ESTIMATION IN DS-CDMA SYSTEMS WITH UNKNOWN WIDE-SENSE STATIONARY INTERFERENCE

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## ABSTRACT

A new blind subspace-based channel and signature waveform estimation technique is proposed for DS-CDMA communication systems operating in the presence of unknown wide-sense stationary interference. Unlike the existing algorithms, our technique requires single receive antenna and is applicable to the general case of arbitrary transmitted symbol constellations. Necessary and sufficient conditions for identifiability of the proposed technique are derived. Closed-form expressions for the mean-squared error (MSE) of the estimated channel are obtained and verified by means of simulations.

## 1. INTRODUCTION

Attaining the performance gains promised by multiuser detection schemes depends critically on accurate knowledge of the signature waveform of the user-of-interest at the receiver side [1]. The main challenge is that due to the spread spectrum nature of the DS-CDMA signals, the transmission channel may be subject to frequency selective fading [2], [3]. This may cause a signature waveform distortion, and, consequently, an unknown mismatch between the presumed and the actual received signature waveforms. Therefore, signature waveform estimation is an important prerequisite of any multiuser detection procedure.

Among numerous signature estimation techniques, blind subspace-based methods [3]-[5] constitute a prominent trend. The usual assumption made in the conventional subspace-based signature estimation techniques is that the additive ambient noise is temporally white, and, hence, the signal subspace can be obtained using eigen-decomposition of the received data covariance matrix. However, this assumption may be violated due to, for instance, the presence of narrow-band interferers [4], [5]. It is well-known that in the latter case, the signal subspace can not be identified from the subspace spanned by the eigenvectors associated with the largest eigenvalues of the received data covariance matrix, and, hence, some alternative means to identify the signal subspace should be sought.

Assuming that the receiver is equipped with two well-separated antennas such that the interference is spatially white between them, Wang and Poor have proposed to identify the signal subspace from the cross-correlation between the received data of these antennas [4]. As deploying well-separated antennas in the current mobile transceivers may be practically infeasible, a single-antenna blind technique has been proposed in [5] to identify the signal subspace. This technique is based on the assumptions that interference is a circular Gaussian random process and the user transmitted symbols are drawn from the binary phase-shift keying (BPSK) constellation. Note that most of the leading standard proposals for the third generation (3G) of wireless communication systems recommend sym-

metric constellation such as quadrature phase-shift keying (QPSK). Therefore, practical applications of the latter technique may be rather limited.

In this paper, we propose an alternative approach to the problem of blind subspace-based signature waveform estimation in the presence of unknown interference. Our technique can be applied to a single-antenna receiver and to the general case of an arbitrary transmitted symbol constellation. We only assume that the unknown interference is wide-sense stationary. Note that this assumption has been frequently used in various interference rejection schemes for CDMA communication systems [5], [6] and includes several more particular interference models such as multi-tone and autoregressive (AR) interference models. Exploiting the idea presented in [7] in the context of array processing, we obtain a subspace which is orthogonal to the subspace spanned by the user signals. We then use the so-obtained subspace along with the known spreading sequence of the user-of-interest to identify the channel vector, and, subsequently, the signature vector of this user. We also derive the necessary and sufficient conditions which warrant the identifiability of the proposed technique.

In the case when the data covariance matrix is estimated from a finite number of data samples, the first-order perturbation theory [3] is used to derive a closed-form analytical expression for the mean-squared error (MSE) of the estimated channel vector. Using some mild and physically justifiable assumptions, a simplified version of this expression is also presented for the high signal-to-interference (SIR) regime. From the latter expression, an impact of the key parameters (such as the number of data samples, the received power of the user of interest, and the received interference power) on the performance of the proposed algorithm is studied.

The rest of our paper is organized as follows. In Section 2, we present the signal model. Our technique is presented in Section 3, where the necessary and sufficient identifiability conditions are also derived. The finite-sample performance of the proposed algorithm is analyzed in Section 4. Section 5 contains computer simulation results. Conclusions are drawn in Section 6.

## 2. SIGNAL MODEL

Consider a  $K$ -user synchronous DS-CDMA system<sup>1</sup> operating in the presence of an external interference. The received continuous-time baseband signal can be modeled as [3]

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=1}^K A_k b_k(m) w_k(t - mT_s) + i(t) \quad (1)$$

<sup>1</sup>The synchronous case is mainly considered for the sake of notational brevity. Extending our analysis to the asynchronous systems is direct [4].

where  $T_s$  is the symbol period,  $A_k$ ,  $b_k(m)$ , and  $w_k(t)$  denote the received amplitude, the  $m$ th zero-mean i.i.d. data symbol with variance  $\sigma_s^2$ , and the signature waveform of the  $k$ th user, respectively, and  $i(t)$  is the zero-mean, wide-sense stationary interference with an unknown arbitrary correlation function which also includes the white Gaussian ambient noise.

Let  $L_c$  be the spreading factor and  $\mathbf{c}_k = [c_k[1], c_k[2], \dots, c_k[L_c]]^T$  denote the spreading sequence associated with the  $k$ th user where  $(\cdot)^T$  stands for the transpose. Assuming that the spreading code is short (i.e., the chip sequence period is the same as the symbol period) and the user channel impulse responses are quasi-static (i.e., fixed during the observation period), the signature waveform of this user is given by [3]

$$w_k(t) = \sum_{l=1}^{L_c} c_k[l] h_k(t - lT_c) \quad (2)$$

where  $h_k(t)$  is the channel impulse response of the  $k$ th user, and  $T_c = T_s/L_c$  is the chip period.

Let  $h_k(t)$  has a finite support of  $[0, \alpha T_c]$ , where  $L-1 \leq \alpha < L$  and  $L$  is a positive integer. Let us also assume that  $L \ll L_c$ , so that the effect of inter-symbol-interference (ISI) can be neglected [3]. Sampling (1) in the interval corresponding to the  $n$ th transmitted symbol of each user and neglecting the first  $L-1$  ISI-contaminated samples, the ISI-free received sampled data vector can be written as [3]

$$\mathbf{x}(n) = \sum_{k=1}^K A_k b_k(n) \mathbf{w}_k + \mathbf{i}(n) \quad (3)$$

where  $\mathbf{x}(n) = [x(nT_s + LT_c), x(nT_s + (L+1)T_c), \dots, x(nT_s + L_c T_c)]^T$ ,  $\mathbf{w}_k = [w_k(LT_c), w_k((L+1)T_c), \dots, w_k(L_c T_c)]^T$ , and  $\mathbf{i}(n) = [i(nT_s + LT_c), i(nT_s + (L+1)T_c), \dots, i(nT_s + L_c T_c)]^T$ . Using (2), the signature vector  $\mathbf{w}_k$  can be expressed as [3]

$$\mathbf{w}_k = \begin{bmatrix} c_k[L] & \dots & c_k[1] \\ c_k[L+1] & \dots & c_k[2] \\ \vdots & \ddots & \vdots \\ c_k[L_c] & \dots & c_k[L_c - L + 1] \end{bmatrix} \mathbf{h}_k \triangleq \mathbf{C}_k \mathbf{h}_k \quad (4)$$

where  $\mathbf{h}_k = [h_k(0), h_k(T_c), \dots, h_k((L-1)T_c)]^T$ . According to (4), if the spreading code of the user-of-interest is known at the receiver, then, provided that the channel vector  $\mathbf{h}_k$  is estimated, obtaining the signature vector  $\mathbf{w}_k$  is straightforward [3]. Hence, throughout this paper we consider the problem of channel vector estimation rather than the signature vector estimation. For the sake of consistency, we also assume without any loss of generality that  $\mathbf{h}_k$  is a unit Euclidean norm vector ( $\|\mathbf{h}_k\| = 1$ ) [3], that is, the normalization factor is absorbed in  $A_k$ . One can rewrite (3) in a more compact form as [3]

$$\mathbf{x}(n) = \mathbf{W} \mathbf{b}(n) + \mathbf{i}(n) \quad (5)$$

where  $\mathbf{W} = [A_1 \mathbf{w}_1, \dots, A_K \mathbf{w}_K]$  and  $\mathbf{b}(n) = [b_1(n), \dots, b_K(n)]^T$ . From equation (5), it follows that

$$\mathbf{R} \triangleq E\{\mathbf{x}(n)\mathbf{x}(n)^H\} = \sigma_s^2 \mathbf{W} \mathbf{W}^H + \mathbf{\Sigma} \quad (6)$$

where  $\mathbf{\Sigma} \triangleq E\{\mathbf{i}(n)\mathbf{i}(n)^H\}$  and  $(\cdot)^H$  stands for the Hermitian transpose. Since  $\mathbf{i}(n)$  is wide-sense stationary, the entries of  $\mathbf{\Sigma}$  depend only on the difference between the observation times. Hence,  $\mathbf{\Sigma}$  is a Hermitian Toeplitz matrix, and, therefore, it is centro-Hermitian [7], that is,

$$\mathbf{J} \mathbf{\Sigma}^* \mathbf{J} = \mathbf{\Sigma} \quad (7)$$

where  $\mathbf{J}$  is the permutation matrix with ones on the main anti-diagonal and zeros elsewhere, and  $(\cdot)^*$  denotes complex conjugate.

### 3. THE PROPOSED TECHNIQUE

Exploiting the idea presented in [7] for direction-of-arrival (DOA) estimation, we use (7) to facilitate estimation of the user signature waveforms without knowing the correlation matrix  $\mathbf{\Sigma}$ . Let us form the covariance difference matrix [7] as  $\mathbf{R}_d \triangleq \mathbf{R} - \mathbf{J} \mathbf{R}^* \mathbf{J}$ . From (6) and (7), it follows that  $\mathbf{R}_d = \sigma_s^2 \mathbf{W} \mathbf{W}^H - \sigma_s^2 \mathbf{J} \mathbf{W}^* \mathbf{W}^T \mathbf{J}$ . Note that  $\mathbf{R}_d$  depends on the user transmitted signals while it is independent from the unknown interference covariance matrix  $\mathbf{\Sigma}$ . We can rewrite the latter matrix as [7]

$$\mathbf{R}_d = \sigma_s^2 \begin{bmatrix} \mathbf{W} & \mathbf{J} \mathbf{W}^* \end{bmatrix} \begin{bmatrix} \mathbf{I}_K & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_K \end{bmatrix} \begin{bmatrix} \mathbf{W} & \mathbf{J} \mathbf{W}^* \end{bmatrix}^H \quad (8)$$

where  $\mathbf{I}_K$  is the  $K \times K$  identity matrix. Since  $\begin{bmatrix} \mathbf{W} & \mathbf{J} \mathbf{W}^* \end{bmatrix}$  is an  $(L_c - L + 1) \times 2K$  matrix, we have that if  $L_c > 2K + L - 1$ , then  $\mathbf{R}_d$  is rank-deficient. Considering hereafter such a case, the matrix  $\mathbf{R}_d$  can be eigendecomposed as

$$\mathbf{R}_d = \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_n^H \end{bmatrix} \quad (9)$$

where  $\mathbf{\Lambda}_s$  is the  $2K \times 2K$  diagonal matrix whose diagonal elements are the non-zero eigenvalues of  $\mathbf{R}_d$ , and  $\mathbf{U}_s$  is the  $(L_c - L + 1) \times 2K$  matrix whose columns are the eigenvectors associated with these eigenvalues. It should be noted that if  $\lambda > 0$  is an eigenvalue of  $\mathbf{R}_d$ , then  $-\lambda$  is also an eigenvalue of this matrix [7]. From (8) and (9) it follows that  $\text{range}(\mathbf{R}_d) = \text{range}(\begin{bmatrix} \mathbf{W} & \mathbf{J} \mathbf{W}^* \end{bmatrix}) = \text{range}(\mathbf{U}_s)$ . Since all columns of  $\mathbf{U}_n$  are orthogonal to all vectors in  $\text{range}(\mathbf{U}_s)$ , we have  $\mathbf{U}_n^H \begin{bmatrix} \mathbf{W} & \mathbf{J} \mathbf{W}^* \end{bmatrix} = \mathbf{0}$ . Let us assume without any loss of generality that the first user is the user of interest. From the latter equation it follows that

$$\mathbf{U}_n^H \mathbf{w}_1 = \mathbf{0} \quad (10)$$

$$\mathbf{U}_n^H \mathbf{J} \mathbf{w}_1^* = \mathbf{0}. \quad (11)$$

It can be shown that there exists a unitary matrix  $\mathbf{\Omega}$  such that  $\mathbf{U}_n \mathbf{\Omega} = \mathbf{J} \mathbf{U}_n^*$  [8]. From the latter fact it readily follows that (10) and (11) are equivalent, i.e., from either of them the other can be obtained [8]. Hence, either (10) or (11) can be exploited to identify the channel vector  $\mathbf{h}_1$ . From (10) along with (4), it follows that  $\mathbf{U}_n^H \mathbf{C}_1 \mathbf{h}_1 = \mathbf{0}$ , and, therefore,  $\mathbf{h}_1$  is a nontrivial solution to

$$\mathbf{T} \mathbf{h} = \mathbf{0} \quad (12)$$

where  $\mathbf{T} = \mathbf{U}_n^H \mathbf{C}_1$ . It is easy to verify that (12) is a linear system with  $L_c - L + 1 - 2K$  equations and  $L$  unknowns. To have a unique nontrivial solution for (12), it is necessary that the number of equations is greater than or equal to the number of unknowns, that is,

$$L_c + 1 \geq 2K + 2L. \quad (13)$$

Condition (13) restricts both the maximum admissible number of active users and the channel length. However, if (13) does not hold, one can resort to the temporal oversampling technique to facilitate identification of lengthier channels in more heavily loaded environments [8].

Equation (13) represents only the necessary condition for uniqueness of the solution of (12). The *necessary and sufficient* conditions for unique identifiability of  $\mathbf{h}_1$  can be obtained as follows. As  $\mathbf{h}_1$  is a nontrivial solution to (12), it follows that  $\mathbf{w}_1 = \mathbf{C}_1 \mathbf{h}_1$  lies

in both  $\text{range}(\mathbf{C}_1)$  and  $\text{null}(\mathbf{U}_n) = \text{range}([\mathbf{W} \quad \mathbf{J}\mathbf{W}^*])$ . Therefore,  $\mathbf{w}_1$  is in the intersection between the two latter subspaces and can be uniquely identified from (12) if and only if

$$\dim\{\text{range}(\mathbf{C}_1) \cap \text{range}([\mathbf{W} \quad \mathbf{J}\mathbf{W}^*])\} = 1 \quad (14)$$

where  $\dim\{\cdot\}$  stands for the dimension of a subspace. If we further assume that  $\mathbf{C}_1$  is a full column-rank matrix, then  $\mathbf{w}_1$  corresponds to a unique channel vector  $\mathbf{h}_1$ . Hence,  $\mathbf{h}_1$  can be uniquely identified from (12) if and only if (14) holds true and  $\mathbf{C}_1$  is full column-rank.

In practice,  $\mathbf{R}$  is estimated using  $\hat{\mathbf{R}} = \frac{1}{N} \sum_{n=1}^N \mathbf{x}(n)\mathbf{x}^H(n)$  and the proposed method can be formulated as follows.

1. Compute the eigendecomposition of  $\hat{\mathbf{R}}_d \triangleq \hat{\mathbf{R}} - \mathbf{J}\hat{\mathbf{R}}^*\mathbf{J}$  as

$$\hat{\mathbf{R}}_d = [\hat{\mathbf{U}}_s \quad \hat{\mathbf{U}}_n] \begin{bmatrix} \hat{\mathbf{\Lambda}}_s & \mathbf{0} \\ \mathbf{0} & \hat{\mathbf{\Lambda}}_n \end{bmatrix} \begin{bmatrix} \hat{\mathbf{U}}_s^H \\ \hat{\mathbf{U}}_n^H \end{bmatrix} \quad (15)$$

where the matrices  $\hat{\mathbf{U}}_s$ ,  $\hat{\mathbf{U}}_n$ , and  $\hat{\mathbf{\Lambda}}_s$  are the finite-sample estimates of the matrices  $\mathbf{U}_s$ ,  $\mathbf{U}_n$ , and  $\mathbf{\Lambda}_s$ , respectively, and  $\hat{\mathbf{\Lambda}}_n$  is the diagonal matrix whose diagonal elements are the  $L_c - L + 1 - 2K$  eigenvalues of  $\hat{\mathbf{R}}_d$  with the least absolute values.

2. Compute  $\hat{\mathbf{T}} = \hat{\mathbf{U}}_n^H \mathbf{C}_1$  and find the least square (LS) estimate of the channel  $\mathbf{h}_1$  as

$$\hat{\mathbf{h}}_1 = \mathcal{M}\{\hat{\mathbf{T}}^H \hat{\mathbf{T}}\} \quad (16)$$

where  $\mathcal{M}\{\cdot\}$  stands for the normalized eigenvector associated with the smallest (minor) eigenvalue.

#### 4. PERFORMANCE ANALYSIS

In this section, we use the first-order perturbation analysis to derive an approximate expression for the MSE of the channel vector estimate  $\hat{\mathbf{h}}_1$ . A simplified version of this expression will also be derived for the high SIR regime. For the sake of simplicity, we assume that  $\mathbf{i}(n)$  has a circular Gaussian distribution. Note that the circular Gaussian interference has been frequently considered in the literature on interference rejection for CDMA systems [5]. The approximate expressions for the MSE of the channel estimate can be derived as follows.

*Theorem:* Assume that  $\mathbf{i}(n)$  is a circular Gaussian random vector and  $\mathbf{h}_1$  is estimated using (16). Then, the MSE of the estimation error  $\delta\mathbf{h}_1 = \hat{\mathbf{h}}_1 - \mathbf{h}_1$  is approximately given by [8]

$$\begin{aligned} \mathbb{E}\{\|\delta\mathbf{h}_1\|^2\} &\approx \frac{\text{tr}(\mathbf{\Sigma}\mathbf{\Psi})}{N} \mathbf{w}_1^H \mathbf{R}_d^\dagger (\mathbf{R} + \mathbf{J}\mathbf{R}^T\mathbf{J}) \mathbf{R}_d^\dagger \mathbf{w}_1 \\ &\quad - \frac{2}{N} \mathbf{w}_1^H \mathbf{R}_d^\dagger \mathbf{J}(\mathbf{\Sigma}\mathbf{\Psi}\mathbf{\Sigma})^T \mathbf{J}\mathbf{R}_d^\dagger \mathbf{w}_1 \end{aligned} \quad (17)$$

where  $\mathbf{R}_d^\dagger$  is the pseudo-inverse of  $\mathbf{R}_d$  and  $\mathbf{\Psi} = \mathbf{U}_n \mathbf{T}^\dagger \mathbf{T}^H \mathbf{U}_n^H$ . Moreover, if the following three conditions hold

$$\mathbf{w}_i^H \mathbf{w}_j = \delta_{ij} \|\mathbf{w}_i\|^2 \quad (18)$$

$$\mathbf{w}_i^H \mathbf{J}\mathbf{w}_j^* = 0 \quad (19)$$

$$\lambda_{\max}(\mathbf{\Sigma}) \ll \frac{1}{2} \sigma_s^2 A_1^2 \|\mathbf{w}_1\|^2 \quad (20)$$

then (17) is simplified to

$$\mathbb{E}\{\|\delta\mathbf{h}_1\|^2\} \approx \frac{\text{tr}(\mathbf{\Sigma}\mathbf{\Psi})}{N\sigma_s^2 A_1^2} \quad (21)$$

where  $\lambda_{\max}(\cdot)$  stands for the maximum eigenvalue of a matrix, and  $\delta_{ij}$  is the Kronecker delta.

Note that although (18) and (19) do not perfectly hold, they appear to be reasonable practical approximations. In practice, CDMA spreading codes are deliberately designed so that even after passing through a frequency selective channel, the resulting spread-spectrum signature waveforms occupy a wide frequency band and behave as almost white pseudo-random signals (see, e.g., [9]). Hence, in most practical scenarios (18) and (19) should hold approximately because the signature vectors are the sampled versions of almost white pseudo-random signature waveforms. The accuracy of these approximations will be validated in the simulation section.

It should also be noted that the received power of the user of interest is equal to  $\sigma_s^2 A_1^2 \|\mathbf{w}_1\|^2$ , while the interference power is lower bounded by the left-hand side of (20) because  $\mathbb{E}\{\|\mathbf{i}(n)\|^2\} = \text{tr}(\mathbf{\Sigma}) \geq \lambda_{\max}(\mathbf{\Sigma})$ . Hence, if SIR is reasonably high, it is guaranteed that (20) holds. Based on this observation, one can consider (21) as a simple approximation of (17) in the high SIR regime that explicitly clarifies the MSE of the estimated channel vector in terms of the number of data samples, variance of the transmitted symbols, the received amplitude of the user of interest, and  $\text{tr}(\mathbf{\Sigma}\mathbf{\Psi})$ . Note that the latter quantity can be viewed as a weighted interference power where the weighting factor  $\mathbf{\Psi}$  depends on the matrix  $\mathbf{C}_1$  and the principal angles between  $\text{range}(\mathbf{C}_1)$  and  $\text{range}(\mathbf{U}_n)$  [3].

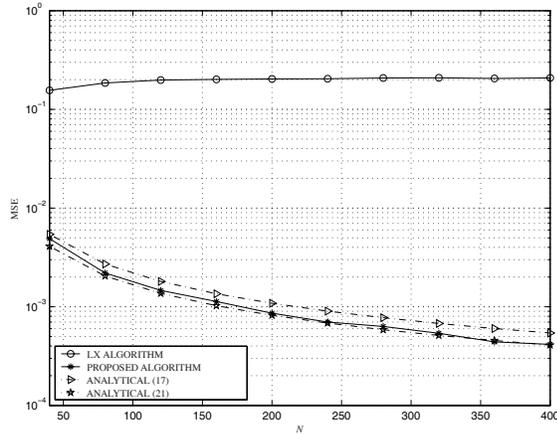
Assuming that  $\text{tr}(\mathbf{\Sigma}) = P_o$  (i.e., the interference power is equal to  $P_o$ ), and  $\mathbf{h}_1$  is the unique nontrivial solution to (12), one can also obtain an upper-bound for the MSE in (21) as follows. As (12) has a unique nontrivial solution, it directly follows that  $\text{rank}(\mathbf{T}) = L-1$ . Let us denote the positive singular values of  $\mathbf{T}$  as  $\xi_1 \geq \xi_2 \geq \dots \geq \xi_{L-1} > 0$ . Note that the positive eigenvalues of  $\mathbf{\Psi}$  and those of  $\mathbf{T}^\dagger \mathbf{T}^\dagger$  are equal to  $\xi_{L-1}^{-2} \geq \xi_{L-2}^{-2} \geq \dots \geq \xi_1^{-2}$ . Since  $\mathbf{\Sigma}$  and  $\mathbf{\Psi}$  are positive (semi-)definite matrices, we have

$$\frac{\text{tr}(\mathbf{\Sigma}\mathbf{\Psi})}{N\sigma_s^2 A_1^2} \leq \frac{\text{tr}(\mathbf{\Sigma})\lambda_{\max}(\mathbf{\Psi})}{N\sigma_s^2 A_1^2} = \frac{P_o}{N\sigma_s^2 A_1^2 \xi_{L-1}^2}. \quad (22)$$

It should be noted that if the largest eigenvalue of  $\mathbf{\Psi}$  is unique, i.e.,  $\xi_{L-1}^{-2} > \xi_{L-2}^{-2}$ , then (22) holds with equality if and only if  $\mathbf{\Sigma} = P_o \mathbf{s}\mathbf{s}^H$  where  $\mathbf{s}$  is the eigenvector of  $\mathbf{\Psi}$  associated with  $\xi_{L-1}^{-2}$ . It follows from (22) that the MSE of the estimated channel vector can become very large if  $\xi_{L-1}$  goes to zero. This is an expected result since if  $\xi_{L-1} = 0$ , then  $\text{rank}(\mathbf{T}) = L-2$ , and, therefore, (12) has a nontrivial solution other than  $\mathbf{h}_1$ .

#### 5. SIMULATIONS

Computer simulations have been conducted to evaluate the performance of the proposed algorithm and validate the obtained theoretical results. In the numerical examples,  $L_c = 40$  and the spreading sequence associated with each user has been randomly drawn from the binary set of  $\{-1, +1\}$  and then fixed throughout all examples. Similarly, the entries of the channel vectors of the length  $L = 4$  have been randomly and independently drawn from a zero-mean complex Gaussian process and then have been normalized so that  $\|\mathbf{h}_k\| = 1$  ( $k = 1, \dots, K$ ) and fixed throughout all examples. The transmitted symbols have been drawn from the QPSK constellation with the variance  $\sigma_s^2 = 1$ . Interference vector  $\mathbf{i}(n)$  is considered as a circular Gaussian random vector such that the  $(l, k)$ -th entry of the correlation matrix  $\mathbf{\Sigma}$  is  $[\mathbf{\Sigma}]_{lk} = 0.98^{|l-k|}$ . Throughout the simulations  $K = 5$  is selected and we assume that all users have identical powers. Each point of the simulation curves is the result of averaging



**Fig. 1.** MSEs of the estimated channel versus the number of data samples  $N$  for the first interference model.

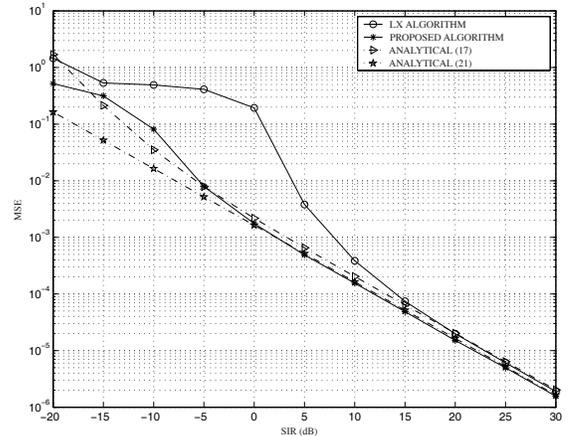
over 1000 Monte-Carlo realizations of interference and the transmitted data sequences.

Fig. 1 shows the experimental MSE of the proposed algorithm as well as the theoretical MSEs (17) and (21) versus  $N$  for  $SIR = 0$  dB. For the sake of comparison, the MSE curve of the conventional (i.e., white noise assumption-based) Liu and Xu (LX) algorithm [3] is also drawn. It can be observed from Fig. 1 that the analytical MSE curves obtained from (17) and (21) follow the experimental MSE curve with quite a good accuracy. As predicted in Section 4, these curves converge to zero with the rate  $1/N$ . From Fig. 1, it also follows that the MSE of the estimated channel using the LX algorithm is constantly high and does not converge to zero. Note that the LX algorithm is based on the mismatched assumption that the signal subspace is spanned by the eigenvectors associated with the  $K$  largest eigenvalues of  $\mathbf{R}$ .

Fig. 2 shows the experimental and analytical MSEs versus  $SIR$  for  $N = 100$ . A substantial performance improvement can be observed from this figure with respect to the LX algorithm. Note that the effect of interference is negligible in high  $SIR$ s where, as Fig. 2 demonstrates, the conventional LX algorithm can also be used to obtain a reliable channel vector estimate. Note also that the MSE expressions (17) and (21) are derived using the first-order perturbation theory, and, hence, they cannot accurately predict the MSE values at very low  $SIR$ s, where the MSE of the channel vector estimate is quite large.

## 6. CONCLUSIONS

In this paper, we have proposed a new subspace-based blind channel and signature waveform estimation technique for DS-CDMA communication systems operating in the presence of unknown wide-sense stationary interference. Using the centro-Hermitian property of the interference covariance matrix along with the idea of covariance differencing [7], we derive a new algorithm for blind identification of user signatures. In contrast to the existing algorithms [4], [5], the proposed technique can be implemented using a single receiving antenna and is applicable to arbitrary constellations of transmitted symbols. Necessary and sufficient conditions for identifiability of the proposed technique have also been derived. Using the first-order perturbation theory, closed-form expressions for the mean-squared error of the estimated channel vector have been obtained and the



**Fig. 2.** MSEs of the estimated channel versus  $SIR$  for the first interference model.

effects of different parameters on the performance of the proposed algorithm have been studied.

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