DMT TIME DOMAIN EQUALIZATION BASED ON ARMA MODELS

Kyriakos Alexopoulos, Stelios Perissakis, Nikolaos Zervos
Ellemedia Technologies
kalexhol@hol.gr, sper@intracom.gr, nzervos @ellemedia.com

ABSTRACT

This paper describes a novel approach for the design of a time domain equalizer for a Discrete Multi Tone (DMT) Digital Subscriber Loop (DSL) modem. The key role of the equalizer is to act upon the equivalent digital channel impulse response at the receiving end to yield a combined impulse response with a linear phase characteristic. Using two-port channel models from the ANSI VDSL standard [1] it is shown how group delay characteristics of typical channels can be modeled by small order polynomials that fall into one of the following categories; all-zero mixed phase, all-pole mixed phase, ARMA minimum phase, and finally ARMA maximum phase systems. A discussion follows to suggest a suitable algorithm based on Prony's method for finding appropriate minimum phase channel models which can be inverted to give equalizer blocks.

1. INTRODUCTION

Digital Subscriber Line (DSL) technology has made possible the transmission of very high data rates over the common telephone twisted pair. Although different modulation methods have been proposed for DSL transmission, the most widely accepted method is the Discrete Multi Tone (DMT) modulation, currently used for ADSL and VDSL, and recently the new VDSL2 standard. The basic building blocks of a DMT system are at the Tx-side the inverse transform (IFFT) that groups multiple modulated system sub-carriers into a single time-domain symbol vector, and at the Rx-side the forward transform (FFT) that decomposes a time domain vector into a set of orthogonal sub-carriers. Assuming that the signal appears unchanged at the input of the Rx FFT block, one could achieve perfect reconstruction of the individual sub-carriers between the two transceiver endpoints. However, a number of factors contribute to the intermingling of sub-carrier content, within a symbol itself, as well as between neighboring symbols that necessitate additional signal processing mechanisms to safeguard system performance.

One of the contributing factors to the intricacy in sub-carrier separation is the analog channel over which the signal has to travel before it reaches the receiving end. The channel introduces frequency dependent distortions both in terms of the energy content and the temporal character of the transmitted signal. To accommodate the channel alterations DMT implementations apply symbol cyclic extension (CE), as well as a frequency domain equalizer (FEQ) at the receiver side. The role of the cyclic extension is to add a redundant buffer zone between successive symbol frames so as to ensure that the pulse spreading due to the channel temporal distortion does not cause one frame to intrude upon its neighbors. The FEQ on the other hand acts as the inverse of the channel impulse response, effectively removing the energy and temporal distortions introduced by the channel, assuming that these are contained within the span of an extended symbol frame.

The use of the cyclic extension is not the panacea to the problem of Inter Symbol Interference (ISI). It always comes at the cost of a reduction in the overall data throughput, and it's allowed duration does not guarantee the isolation of neighboring DMT symbols for all channels. Channels that display significant variations in their group delay characteristic will result in transmitted carriers being shifted relative to each other by different amounts to an extent that energy for certain frequencies would leak into adjacent frames and cause ISI.

The area of developing alternative equalizer structures to complement or even replace the single tap FEQ, has been under investigation and several implementations do exist with a time-domain equalizer FIR filter [2] [3], or even multi-tap frequency domain equalizer structures [4]. This work is different in that we do not attempt to simply map the equalization problem to an algebraic one; rather, we start by investigating the response for some typical channels and based on the intuition developed, we suggest appropriate channel dependent models from which efficient equalizers can be implemented.

2. CHANNEL GROUP DELAY AND INTERSYMBOL INTERFERENCE

The ANSI VDSL standard includes typical channels based on some typical topologies of different cable types described in terms of two-port models [1]. These topologies are summarized in Figure 1, with vertical cells indicating bridged tap sections.

All the above channels demonstrated increased group delay variation at low frequencies ($f<100$kHz) as the physical length (L) of the cable increased. This is demonstrated by Figure 2 for the case of VDSL1a. Also, bridged-taps result in nulls in the frequency spectrum with the group delay showing significant change at those areas as is depicted for VDSL5 in Figure 4.

To associate the group delay characteristics for channel physical models with discrete filters we looked at the group delay of simple first and second order polynomials, examples of which are shown in Figure 3. The group delay of a first order polynomial within the unit circle is "bell" shaped with the peak located at the frequency where the root of the polynomial lies. The proximity of the root to the unit circle determines the peak value and bandwidth of the "bell"; the closer the root to the unit circle, the highest the amplitude, and the narrower the bandwidth.

By observing the group delay characteristics for several channels and the group delay characteristic of simple polynomials, we construe that most group delay distortions are localized in frequency and can be efficiently described by a pair of zeros or poles, each zero or pole on opposite sides of the unit circle, or alternatively a zero-pole pair with both roots either inside or outside the unit-circle. The closeness of the pair to the unit circle determines the width of the group delay distortion, while the proximity of the roots to each other determines the severity of the group delay distortion. Using zero-pole pairs that lie on the inside of the unit-circle would result in a minimum phase model, which would be ideal for the implementation of a practical equalizer block because its inverse can be used directly to give a stable IIR filter.

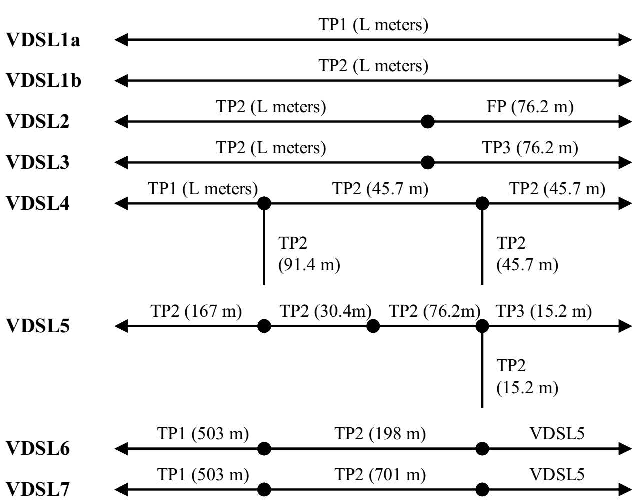

Figure 1: Schematic description of ANSI VDSL typical channel topologies.

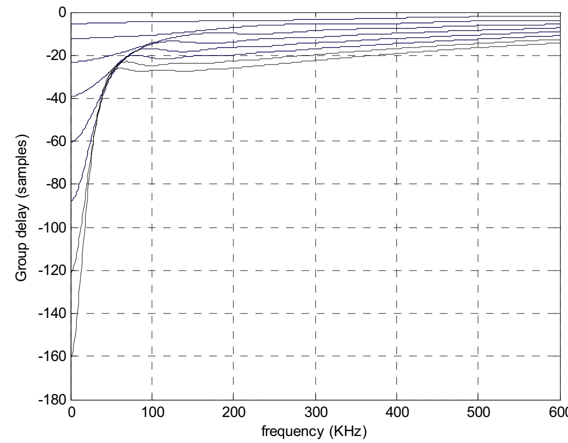

Figure 2: Group delay characteristic for VDSL1a with L increasing from 200m to 1600m in steps of 200m (mean group delay has been removed for all cases).

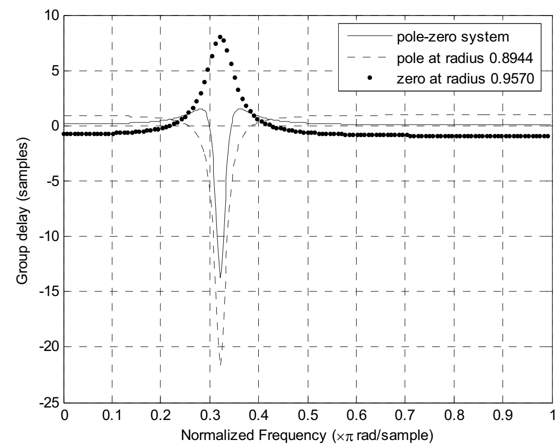

Figure 3: Examples of group delay characteristics of real second order polynomials.

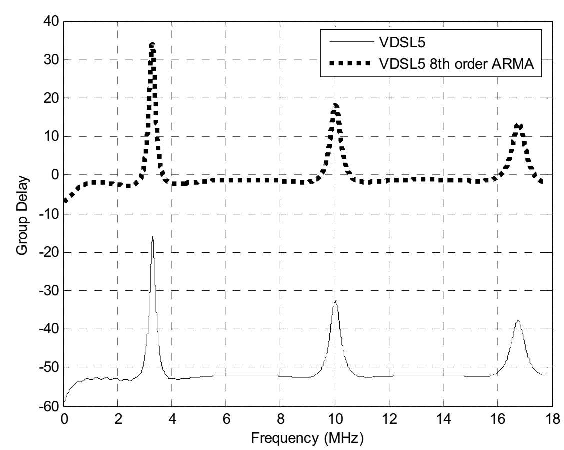

Figure 4: Group delay characteristic of VDSL5 and corresponding 8^{th} order ARMA model.

3. DESIGN OF GROUP DELAY EQUALIZER

Several approaches have been considered to achieve the design of phase equalizers using ARMA models. However early attempts suffered in terms of algorithm stability and poor convergence, especially in the case that zeros and poles were placed near the unit-circle. In brief, we considered the design of allpass filters to match the prescribed channel phase based on the work of M. Lang and T. Laakso [5], as well as direct minimization of the L_∞ norm, of the absolute error between the model and the actual group delay characteristics using numerical methods.

The proposed approach for the design of the group delay equalizer relies on the application of ARMA modeling to a modified channel response that is minimum phase and has the same group delay characteristic (ignoring the mean) as the prescribed channel response. The channel impulse response pre-conditioning assists the ARMA modeling procedure to produce minimum phase models which can be used as practical equalizer IIR filters, by simple inversion of the quotient of the two polynomials.

There are four steps involved in the design process of this equalizer; channel estimation, computation of the channel phase spectrum, conversion of channel estimate to a minimum phase system that retains the original phase spectrum, and finally the derivation of an ARMA model that matches the spectrum of the minimum phase system. The preferred implementation for each stage of the design process is outlined and related to practical implementation issues.

3.1. Channel estimation

In practice, the channel impulse response is not available in advance but needs to be estimated by the modem-Rx on each side of the communication link. This computation is based on averaging the output of the FFT block for a series of successive symbol frames with known content. In terms of resource requirements, it does not add extra system complexity to the existing DMT modem, since channel estimation is a function performed during the transceiver training phase in order to derive the taps of the FEQ. As part of the evaluation of the proposed equalizer design, channel estimation has been omitted, and the channel description has been taken directly from two-port models.

3.2. Phase spectrum computation

The purpose is to find the phase spectrum of the channel and remove any linear phase distortion. The computation of the phase spectrum was implemented based on group delay estimate which can be accomplished through eq. 1. This way the use of inverse trigonometric functions is avoided and linear delay can be effectively computed by taking the mean of the resulting group delay vector.

$$G(k) = \frac{H'_{\mathrm{Im}}(k)H_{\mathrm{Re}}(k) - H'_{\mathrm{Re}}(k)H_{\mathrm{Im}}(k)}{\left\|H(k)\right\|} \qquad \text{(eq.1)}$$

In practice, the channel frequency response is given as a discrete signal with samples at equally spaced frequencies, and derivatives of the real and imaginary parts are approximated by ratios of finite differences, i.e. $H'_{\mathrm{Im}}(k) = \dfrac{H_{\mathrm{Im}}(k+1) - H_{\mathrm{Im}}(k)}{\Delta\omega}$. Having performed the computation for all k, the mean value of the group delay vector is subtracted and the phase spectrum is reconstructed.

3.3. Minimum phase system estimation

Based on the phase spectrum resulting from the process in section (3.2), we wish to design a system with minimum phase impulse response. This can be achieved by the iterative method proposed by Yegnanarayana et.al [6]. We kickoff with a channel frequency response estimate with the linear phase distortion removed. An inverse N-point DFT is performed on the signal to yield an N-point time sequence. The second half of the time samples are zeroed before the modified time vector goes through an N-point DFT to return to a frequency-domain signal representation. The signal amplitude spectrum is retained, but the phase spectrum is substituted with the desired phase spectrum. This completes the first iteration, while the process continues until, after the IDFT step, the energy of the second half of the time samples becomes less than a predefined fraction of the energy of the first half.

For all the channels in Figure 1 the iterations showed quick convergence giving good results after 10 steps. In terms of resource requirements, because the process relies on Fourier Transforms, it makes it suitable for DMT modems that have strong DFT and IDFT engines, provided that these resources could be allocated during this part of the training phase to the purpose of the equalizer design.

3.4. ARMA Model Estimation

The final step in the design of our time domain pre-equalizer is to model the channel impulse response with an IIR filter structure. Prony's method [7] was found to give good results and because it involves the solution of a system of linear equations it is also suited for efficient practical implementation.

4. PERFORMANCE EVALUATION

In Figure 5, the impulse responses of the equalized channels are plotted against the original channel impulse responses to show how an 8^{th} order IIR filter can be used as an equalizer to shorten the channel. In addition, performance evaluation of the equalizer blocks is done in terms of the received Signal to Noise Ratio (SNR) per carrier achieved at the DMT receiver. The SNR measurements were carried out on a simulation platform developed in Matlab as part of the VIPNet Eureka project [8].

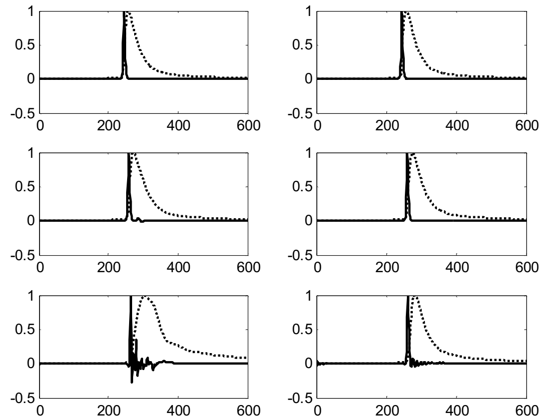

Figure 5: Channel impulse responses and equalized responses using 8th order IIR TEQ, for channels VDSL1a, VDSL1b, VDSL2, VDSL3, VDSL4, VDSL7 (L=1200m).

In our experiments the full VDSL bandwidth (17.764 MHz) has been loaded using 4096 sub-carriers with a uniform power spectral density (PSD) of -61dBm, each modulated by a 4-QAM constellation. The received SNR per carrier at the output of the FEQ was measured for all channels assuming that AWGN is injected in the channel at -140dBm/Hz, while the resolution for the ADC and the DAC were set to 12 bits. The cyclic extension was set according to ANSI standard to 640 samples and the frame windowed overlap-add was used between successive frames at the Tx. Also, Rx windowing has been used.

Results showed that for long channels at low frequencies, where the group delay showed significant variation, the presence of a TEQ is useful; an example of the SNR improvement achieved is shown in Figure 6 for channel VDSL1b (L=1200m).

5. CONCLUSIONS AND FUTURE WORK

By looking at some standard VDSL channel two-port models, we have used the group-delay characteristic to generate minimum phase ARMA models and consequently invert those to use as IIR equalizers. The algorithmic complexity for the training and run-time execution of the proposed approach can be accommodated in a practical DMT system.

Future work should include further evaluation of ARMA channel modeling for longer channels relevant to ADSL and the emerging VDSL2 standard. Also, it is in our plans to tackle implementation issues and also to compare the performance of the proposed TEQ against existing solutions.

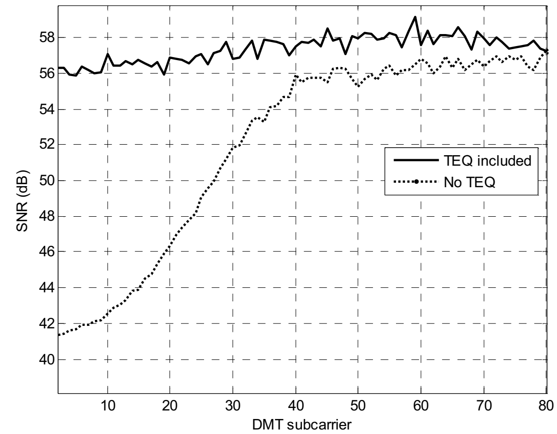

Figure 6: SNR per sub-carrier for channel VDSL1b (L=1200m) with and without TEQ.

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