SUBOPTIMUM SPACE MULTIPLEXING STRUCTURE COMBINING DIRTY PAPER CODING AND RECEIVE BEAMFORMING

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ABSTRACT[1]

We consider the MIMO broadcast channel where a base station with multiple antennas transmits simultaneously to many users. The optimum strategy with Channel State Information (CSI) at the transmitter is based on Dirty Paper Coding (DPC) principles. In this paper, we propose a suboptimum strategy suitable when terminals are equipped with multiple antennas. This approach combines DPC and receive beamforming and is shown to provide near-optimum performance with very reasonable complexity. Beamforming design is based on the maximization of the sum rate while precoder design is based on Zero Forcing (ZF) criteria. We have to remark that it is not required iterative processes or interaction between transmitter and receivers to find the suboptimum solution. It is also important to mention that each terminal just needs to know its own channel to perform the optimization while the base station requires full knowledge of all users' channels.

1. INTRODUCTION

Multiple Input – Multiple Output (MIMO) communications have challenged the research community with promising capacity increases with respect the standard single-antenna systems [1, 2]. Most of the original work was motivated by the point-to-point link trying to provide structures achieving the theoretical capacity predicted by the logdet formula. Probably, the most well studied strategy is BLAST with many variants sharing in common the space-time layered structure.

More recently, there has been an interest in MIMO systems where a multi-antenna node transmits to multiple terminals, or where multiple nodes transmit to a single multi-antenna terminal. The former case is known as the broadcast channel (BC) while the later is known as the multiple access channel (MAC).

Available literature shows many different approaches dealing with this topic. One of the most representative works are [3, 4] where theoretical analysis is provided through the duality principle between the downlink and the uplink case. The main point is that the achievable rate region of the BC is equivalent to the rate region of its dual MAC with a sum power constraint. This duality allows the characterization of the BC rate region to be performed in the dual MAC context where well-known optimization algorithms can be used. The achievable rate region will be considered as an upper bound for all the suboptimum approaches.

Transmit beamforming is the most immediate approach using the background of decades in array processing. It is well known that with only one antenna per user, it turns out that an analogous uplink-downlink duality result exists, as is shown in [5,6]. However, with multiple receive antennas per user, this duality does not hold any longer, and so it is necessary to solve the downlink sum rate maximization directly.

In [7], several suboptimum approaches for communicating on the BC are suggested. One of them is the design of the precoder according to the SVD decomposition (it is known to be optimum in the point-to-point link) and apply MMSE beamformers at receivers to cope with residual interference. The main advantage of this scheme is its simplicity although we have to remark that received covariance matrices must be estimated or fed back. Another proposal in [7], known as Maximum Transmit SINR, maximizes an upperbound of the SINR.

JADE (Joint Approximated Diagonalization of Eigenstructures) refers to a well-known result coming from blind source separation [8]. This approach establishes that for a set of complex hermitian matrices associated to the MIMO transmission, it is possible to find a unitary matrix that minimizes MAI. If the matrices do not have a common eigenstructure, the algorithm provides a kind of 'average eigenstructure'. Unfortunately, performance depends very much on the specific structure of involved channels.

Reference [9] proposes another approach known as Orthogonal Space Division Multiplex (OSDM). This scheme is a zero-forcing strategy where optimization is performed iteratively through a recursive procedure between the transmitter and receivers. The main disadvantage of this scheme is that it requires multiple iterations of communication between both ends before reaching the desired solution. However, performance improves considerably in comparison with other suboptimum schemes.

Our approach is based on [10] where a suboptimum precoder of a DPC structure is proposed using the QR decomposition of the channel matrix. However, this technique was discussed for the case of single-antenna terminals. We have extended this procedure where there could be several antennas at the receiver. Our contribution optimizes the receive beam-

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forming strategy for any terminal assuming that its channel is known but that no other terminal's channel is known. On the other hand, the transmitter has full knowledge of every channel. We propose a suboptimum precoder using a combination of DPC and zero-forcing criteria whose performance in terms of mutual information is close to the optimum case but with reduced complexity.

2. SYSTEM MODEL

We consider a MIMO broadcast channel where a base station with N_t antennas transmits to N_u users, each with N_r receive antennas. We will further assume that we have the same number of transmit antennas as the number of user. Figure 1 shows the corresponding block diagram of the system, where $\mathbf{T}$ is the linear precoder (or transmit beamformer) (size N_txN_u), $\mathbf{H}_k$ represents the kth user's flat MIMO channel (size N_rxN_t), $\mathbf{r}_k$ represents the kth user's linear receiver (or receive beamformer) (size N_rx1), and $\mathbf{n}_k$ is the AWGN at receiver k. The dirty paper coding block jointly encodes users so that the interference among users is reduced or eliminated at the output of their decoders.

All the schemes that we have outlined in the introduction section are based on the following view: Determine ($\mathbf{T}$, $\mathbf{r}_1$, $\mathbf{r}_2$, …, $\mathbf{r}_{Nu}$) to optimize the performance, either in terms of maximizing the sum rate or maximizing a common SINR achievable by all users. In order to make a fair comparison, all schemes in the simulation results will be measured under a common figure of merit, the sum rate. The specific description of all the different approaches following this model is straightforward according to the original papers, but the detailed description is out of the scope of this paper due to the limited length of the contribution.

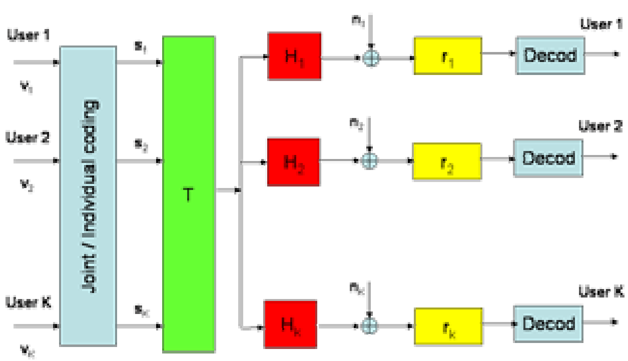

Figure 1. Block diagram

3. OUR PROPOSAL ZF-DPC WITH RECEIVE BEAMFORMING

Our proposal intends to mix the concept of DPC with its simpler ZF implementation with some beamforming design that provides an acceptable loss of performance. Received signal stacking all users in a single column-vector becomes,

$$\mathbf{y} = \mathbf{R}^H \mathbf{H}\mathbf{T}\mathbf{s} + \mathbf{R}^H \mathbf{n} \tag{1}$$

where $\mathbf{R}$ is a diagonal block matrix collecting individual array processing at every receiver, and $\mathbf{0}$ vectors mean that receivers do not cooperate

$$\mathbf{R} = diag\{\mathbf{r}_1 \quad \mathbf{r}_2 \quad \dots \quad \mathbf{r}_{N_u}\} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{r}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{r}_{N_u} \end{bmatrix} \tag{2}$$

$\mathbf{H}$ is the channel matrix (N_uN_r x N_t), that can be expressed as the stacking of the per user matrices $\mathbf{H}_i$ (N_r x N_t):

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \mathbf{H}_2 \\ \vdots \\ \mathbf{H}_{N_u} \end{bmatrix} \tag{3}$$

For the case of single-antenna receivers, the optimum precoding matrix $\mathbf{T}$ is obtained using the duality principle [3]. This strategy is also remarked in [10] where a tight suboptimum design is proposed where $\mathbf{T}$ is the unitary matrix associated to the QR-decomposition of the channel matrix $\mathbf{H}$. In this contribution we are going to follow this suboptimum design where the determination of $\mathbf{T}$ is straightforward for the transmitter if it knows all channels and also the beamforming processing to be done at receivers,

$$\mathbf{H}_{eq} = \mathbf{R}^H \mathbf{H} = \mathbf{L}\mathbf{Q} \tag{4}$$

where $\mathbf{L}$ is the upper triangular matrix and $\mathbf{Q}$ is the orthogonal matrix associated with the QR decomposition of matrix $\mathbf{H}_{eq}$. According to this idea, in our scenario we force $\mathbf{T} = \mathbf{Q}^H$.

Therefore, the optimization procedure in this case is based on the design of the beamformers constrained to $\mathbf{r}_i = f(\mathbf{H}_i)$ remarking that its own channel is the only available information at each receiver. Applying these ideas to eq. (1), we reach an equivalent model as follows,

$$\mathbf{y} = \mathbf{R}^H \mathbf{H}\mathbf{T}\mathbf{s} + \mathbf{R}^H \mathbf{n} = \mathbf{L}\mathbf{Q}\mathbf{T}\mathbf{s} + \mathbf{R}^H \mathbf{n} = \tag{5}$$
$$= \mathbf{L}\mathbf{Q}\mathbf{Q}^H \mathbf{s} + \mathbf{R}^H \mathbf{n} = \mathbf{L}\mathbf{s} + \mathbf{R}^H \mathbf{n}$$

Assuming that DPC is working properly (recall that due to the triangular structure of $\mathbf{L}$, DPC is able to cancel all the interference, this is why it is labeled as ZF-DPC), the instantaneous sum rate R is given by

$$R = \sum_{k=1}^{N_u} \log_2\left(1 + snr\left|l_k\right|^2\right) \tag{6}$$

where l_k are the diagonal elements of the triangular $\mathbf{L}$ matrix and *snr* is the common Signal to Noise Ratio. Therefore, the optimization problem can be expressed as:

Optimization criteria

Find $\mathbf{R}$ constrained to be block diagonal with $\mathbf{R}^H\mathbf{R}$=$\mathbf{I}$ and $\mathbf{r}_i = f(\mathbf{H}_i)$, that maximizes

$$R = \sum_{k=1}^{N_u} \log_2\left(1 + snr\left|l_k\right|^2\right) \tag{7}$$

where l_k are the diagonal elements of the triangular **L** associated to the QR decomposition of $\mathbf{H}_{eq} = \mathbf{R}^H\mathbf{H} = \mathbf{LQ}$.

Let us remark that previous relationship $\mathbf{R}^H\mathbf{R}=\mathbf{I}$ transforms eq. (6) in the standard problem with spatially uncorrelated noise:

$$\mathbf{y} = \mathbf{Ls} + \mathbf{R}^H\mathbf{n} = \mathbf{Ls} + \mathbf{n} \tag{8}$$

Solution

The set of beamformers $\mathbf{r}_i$ that maximizes the rate in (6) for high SNR are the eigenvectors related to the maximum eigenvalue of matrix $\mathbf{H}_i^H\mathbf{H}_i$:

$$\mathbf{H}_i\mathbf{H}_i^H\mathbf{r}_i = \lambda_{\max}\mathbf{r}_i \tag{9}$$

Proof

For high SNR, the sum rate can be expressed as

$$R = \sum_{k=1}^{N_u}\log_2\left(1 + snr\left|l_k\right|^2\right) \approx \tag{10}$$

$$\approx \sum_{k=1}^{N_u}\log_2\left(snr\left|l_k\right|^2\right) =$$

$$= N_u\log_2 snr + \log_2\left(\prod_{k=1}^{N_u}\left|l_k\right|^2\right)$$

Therefore, we just need to maximize the last term that can be expressed as a determinant

$$\log_2\left(\prod_{k=1}^{N_u}\left|l_k\right|^2\right) = \log_2\left(\det\mathbf{LL}^H\right) =$$

$$= \log_2\left(\det\mathbf{LQQ}^H\mathbf{L}^H\right) = \log_2\left(\det\mathbf{H}_{eq}\mathbf{H}_{eq}^H\right) \tag{11}$$

As the logarithm function is a monotonic increasing function, the optimization criteria may be expressed as follows

$$J(\mathbf{R}) = \max_{\mathbf{R}}\left(\det\mathbf{R}^H\mathbf{H}^H\mathbf{HR}\right)$$

Constrained to **R** block diagonal,

$$\mathbf{R}^H\mathbf{R} = \mathbf{I},\ \ \mathbf{r}_i = f(\mathbf{H}_i) \tag{12}$$

Recall that **R** is constrained to be block diagonal because the users perform linear combining separately from one another. This constraint limits the determinant to the main diagonal of the desired matrix. This point is critical because if we consider the whole matrix, crossed terms depending on two different terminals' beamformers appear. The constraint $\mathbf{r}_i = f(\mathbf{H}_i)$ imposes that the optimization criteria just depends on the main diagonal elements.

Therefore, the original constrained optimization must be expressed as follows:

$$J(R) = \max_{\mathbf{R}}\left(\det\left(diag\left(\mathbf{R}^H\mathbf{H}^H\mathbf{HR}\right)\right)\right) =$$

$$= \max_{\mathbf{r}_i}\prod_{i=1}^{N_u}\mathbf{r}_i^H\mathbf{H}_i\mathbf{H}_i^H\mathbf{r}_i \tag{13}$$

Constrained to $\mathbf{r}_i^H\mathbf{r}_i = 1, \forall i \in (1, N_u)$

The solution to this problem is very well known as the set of eigenvectors associated with the maximum eigenvalues of the corresponding matrices, as proposed in (9).

It is very important to remark that this approach is non-iterative and that receivers just need to estimate their own channels. The main disadvantage could be the implementation of DPC which is currently unknown and could be very complex. In this sense, we propose substitute the DPC structure by a vectorized Tomlinson-Harashima precoder as is described in [11, 12]. However, the performance of this structure is out of the scope of this paper. The results that we are going to show will assume perfect DPC behavior and must be considered as an upperbound of more realistic implementations.

4. ANALYTICAL RESULTS AND SIMULATIONS

We have calculated through simulations the sum rate for all the methods described in the introduction section. Figure 2 shows the performance of these methods for three users (and three transmit antennas) and two antennas per terminal. It is important to remark that we have not included multiuser diversity in this analysis.

It can be observed that JADE and SVD perform quite badly because in fact they are nor able to reduce the MAI effect. They were defined for point to point communications, therefore this performance is expected. Maximum transmit SINR (labelled as MTx SINR) performs a little bit better but the most interesting behaviour is provided by the OSDM. It is remarkable that our proposal labelled by ZF-DPC + BF has similar performance. It is important to have in mind that OSDM is iterative between receivers and transmitter which makes this contribution quite impractical. Obviously, DPC and optimum beamforming show the best performance. Our method is shown to be as good as optimum beamforming for high SNR.

$$R_{DPC} = \log_2\det\left|\mathbf{S}_n + \sum_{k=1}^{N_u}\mathbf{H}_k^H\mathbf{Q}_k\mathbf{H}_k\right| \tag{14}$$

Equation 14 presents the sum rate for DPC where $\mathbf{S}_n$ is the noise covariance matrix and $\mathbf{Q}_k$ is the optimized covariance matrices of different users. The sum rate for the beamforming structure is given by equation 15

$$R_{BF} = \sum_{k=1}^{N_u}R_k = \sum_{k=1}^{N_u}\log_2\frac{\det\left|\mathbf{S}_n + \sum_{j=1}^{N_u}\mathbf{H}_k^H\mathbf{Q}_j\mathbf{H}_k\right|}{\det\left|\mathbf{S}_n + \sum_{j=1, j\neq k}^{N_u}\mathbf{H}_k^H\mathbf{Q}_j\mathbf{H}_k\right|} \tag{15}$$

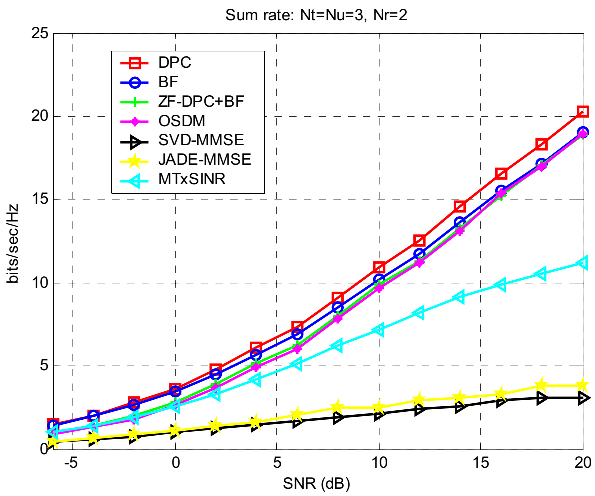

Figure 2. Comparison of some existing algorithms for multi-user multiantennas schemes

Figure 3 shows for 6 transmit antennas and 6 users and different number of receive antennas the ratio of our suboptimum proposal in front of optimum DPC. The performance is similar for different number of antennas and is close to optimal for low and high signal to noise ratios (SNR). For high SNR, it is observed a saturation behavior stating that there will be a gap between both approaches, as is seen in figure 2. Unfortunately, for the most realistic interval of SNR between 0 and 10 dB, the ratio decreases, but it is always upper the 85 %. Clearly the loss of performance increases with the number of antennas, but in most mobile applications with 2 at the terminals, the behavior will reach more than 90%.

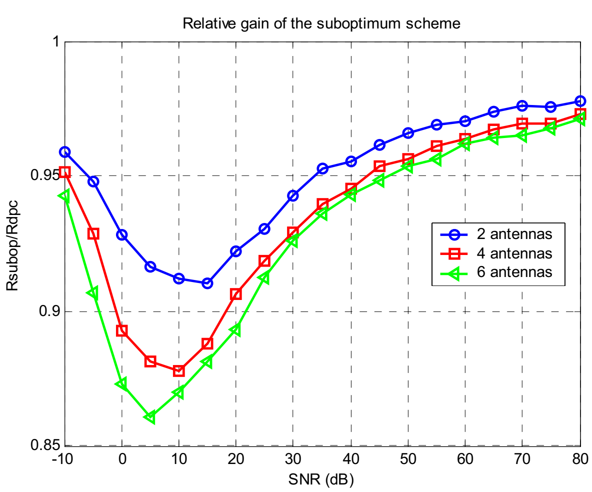

Fig.3. Relative gain of the suboptimum approach

5. CONCLUSIONS

This paper presents a new approach to the broadcast channel problem where the main motivation is to provide a suboptimum solution combining DPC with Zero Forcing precoder and optimum beamforming design. The receiver design just relies on the corresponding channel matrix (and not on the other users channels) while the common precoder uses all the available information of all the involved users. No iterative process between transmitter and receiver is needed in order to reach the solution of the optimization process. We have shown that this approach provides near-optimum performance in terms of sum rate but with reduced complexity. An open point is the practical implementation of DPC. We are currently evaluating the performance of vectorized Tomlinson – Harashima [11, 12] in combination with the beamforming design. In this lattercase, beamformers are implemented adaptively through the Generalized Side Lobes Canceller (GSLC) in order to cope with mismatched channel estimation errors.

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