TRAINING DESIGN FOR CHANNEL AND CFO ESTIMATION IN MIMO SYSTEMS

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ABSTRACT

For MIMO systems operating over frequency-selective channels, we establish the Cramer-Rao Bound (CRB) for the CFO and channel parameters. We derive training sequences so that the resulting CRB on the CFO is independent of the channel. We show that these designs lead to simple implementation of the maximum likelihood estimators of the CFO and channel parameters, Simulation results illustrate the performance of the proposed designs.

1. INTRODUCTION

Reliable coherent communication systems require accurate synchronization (timing and frequency) and channel estimation, particularly when the data rate or bandwidth is large, and in multiple antenna systems. Blind estimation techniques have been proposed in order to save bandwidth; however, the tradeoff with power and complexity is ambiguous. Practical systems typically use some form of training which may be prepended, appended, or embedded in the data packet. Training typically leads to low complexity receivers and good performance at the expense of a typically small increase in bandwidth [1]-[4].

Devoting resources to the pilots improves channel estimation, but the tradeoff is with degraded BER (total fixed power) or reduced rate (fixed bandwidth). Optimal training for the CFO-free frequency-selective MIMO channel has been recently addressed by many authors e.g., [6]-[9]. In the presence of CFO, training design has received relatively little attention. Single antenna systems were considered in [4] using the worst-case asymptotic Cramér-Rao Bounds (CRB), leading to a white Gaussian preamble. Here, we use the *exact* CRB as a metric for training design, an idea also considered in [10] for CFO estimation, and in [8] for frequency-selective channel estimation. We tackle the challenging problem of training design for *both CFO and channel estimation*. Our findings are novel in both single and multiple antenna scenarios. Our preamble designs lead to low complexity estimators, and are applicable to both serial and block (either cyclic-prefixed or zero-padded) transmissions. Proofs of the results established in this paper can be found in [12].

Notation: Superscripts ^{H}, ^{T} and † denote Hermitian, transpose and pseudo-inverse operators. The trace, statistical expectation and the Kronecker product are denoted by $\mathrm{Tr}\{\cdot\}$, $E\{\cdot\}$ and $\otimes$ respectively. The L_2 norm of a vector $\boldsymbol{a}$ is denoted by $\|\boldsymbol{a}\|_2$. $\mathbf{I}_N$ denotes the $(N \times N)$ identity matrix and $\mathbf{1_Q}$ denotes the $(Q \times Q)$ matrix of ones. Finally, $\mathrm{diag}\,(a_1, ..., a_N)$ is the $(N \times N)$ diagonal matrix whose nth diagonal entry is a_n. Subscripts may be dropped if there is no ambiguity.

2. SIGNAL MODEL

We consider a MIMO system with M_t transmit and M_r receive antennas. The baud-sampled discrete-time channel impulse response (CIR) between the ith transmit and the jth receive antennas is denoted by $\boldsymbol{h}_{i,j} = [h_{i,j}(0), ..., h_{i,j}(L-1)]^T$ where L is an upper bound on the length of the longest CIR, We assume that all the transmit (resp. receive) antennas are driven by the same local oscillator (as in a collocated system). This implies that a single CFO characterizes the frequency mismatch. In order to estimate the CFO and the CIRs, the transmit antennas send possibly different $N + L - 1$-point preambles[1], $a_i(-L+1) \cdots a_i(N-1)$, $i = 1 \cdots M_t$, simultaneously. The received signal block at the jth receive antenna is modeled as

$$\boldsymbol{x}_j = \boldsymbol{\Gamma}(\omega_0) \sum_{i=1}^{M_t} \mathbf{A}_i \boldsymbol{h}_{i,j} + \boldsymbol{w}_j, \qquad j = 1, 2, ..., M_r \tag{1}$$

where $\boldsymbol{x}_j = [x_j(0)...x_j(N-1)]^T$, $\boldsymbol{w}_j = [w_j(0)...w_j(N-1)]^T$, ω_0 is the normalized angular CFO, with

$$\begin{aligned} \boldsymbol{\Gamma}(\omega_0) &= \mathrm{diag}\left(1, e^{j\omega_0}, ..., e^{j\omega_0(N-1)}\right) , \\ \mathbf{A}_i(k, \ell) &= a_i(k-\ell), \quad k = 0, ..., N-1, \ \ell = 0, ..., L-1 . \end{aligned}$$

We assume the $\boldsymbol{w}_j$'s to be mutually independent circularly symmetric complex white Gaussian noise vectors, with variance σ^2. We also assume that no a priori information about the statistics of the channels is available.

Let $\boldsymbol{x} = [\boldsymbol{x}_1^T \cdots \boldsymbol{x}_{M_r}^T]^T$, $\mathbf{A} = [\mathbf{A}_1 ... \mathbf{A}_{M_t}]$, $\boldsymbol{h}_j = [\boldsymbol{h}_{1,j}^T ... \boldsymbol{h}_{M_t,j}^T]^T$ and $\boldsymbol{h} = [\boldsymbol{h}_1^T \cdots \boldsymbol{h}_{M_r}^T]^T$. The signal model in eq. (1) can then be rewritten as

$$\begin{aligned} \boldsymbol{x}_j &= \boldsymbol{\Gamma}(\omega_o) \mathbf{A} \boldsymbol{h}_j + \boldsymbol{w}_j , & (2) \\ \boldsymbol{x} &= [\mathbf{I}_{M_r} \otimes \boldsymbol{\Gamma}(\omega_0) \mathbf{A}] \boldsymbol{h} + \boldsymbol{w} , & (3) \end{aligned}$$

where the vector $\boldsymbol{w}$ is defined similar to $\boldsymbol{x}$.

3. CRAMÉR-RAO BOUNDS

In the CFO-free case, $\omega_o = 0$, and $\Gamma(\omega_o) = \mathbf{I}_N$. It is easy to show that a necessary and sufficient condition for channel identifiability is that $\mathbf{A}$ have full column rank. The CRB for $\boldsymbol{h}$ is obtained as

$$CRB(\boldsymbol{h})|_{\text{no-CFO}} = \sigma^2 \mathbf{I}_{M_r} \otimes (\mathbf{A}^H \mathbf{A})^{-1} . \tag{4}$$

The CRBs for the $\boldsymbol{h}_j$'s, the channels to receiver j, are identical and mutually decoupled. For the same receive antenna, the CRBs for

[1] We consider preambles rather than midambles or postfixes for simplicity.

the CIRs associated with different transmit antennas are in general coupled; they can be decoupled if the training sequences satisfy $\mathbf{A}_k^H \mathbf{A}_\ell = \mathbf{0}$ if $k \neq \ell$. Optimal design of the training sequences is investigated in Section 4.

In the general case where the received signals are corrupted by CFO, the CRB for ω_0 and $\boldsymbol{h}$ are obtained as ($\mathbf{A}$ is assumed tall in this case)

$$\begin{aligned}\mathrm{CRB}(\omega_0) &= \frac{\sigma^2}{2}\left[\boldsymbol{h}^H(\mathbf{I}_{M_r}\otimes\mathbf{A}^H\mathbf{D}\Pi_{\mathbf{A}}^{\perp}\mathbf{D}\mathbf{A})\boldsymbol{h}\right]^{-1}, \quad (5)\\ \mathrm{CRB}(\boldsymbol{h}) &= \sigma^2\mathbf{I}_{M_r}\otimes(\mathbf{A}^H\mathbf{A})^{-1} + \\ &\quad \mathrm{CRB}(\omega_0)(\mathbf{I}_{M_r}\otimes\mathbf{C})\boldsymbol{h}\boldsymbol{h}^H(\mathbf{I}_{M_r}\otimes\mathbf{C}^H) \quad (6)\end{aligned}$$

$$\begin{aligned}\text{where } \mathbf{D} &= \mathrm{diag}\,(0, 1, \cdots N-1)\\ \Pi_{\mathbf{A}}^{\perp} &= \mathbf{I} - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\,,\\ \xi &= \boldsymbol{h}^H(\mathbf{I}_{M_r}\otimes\mathbf{A}^H\mathbf{D}\Pi_{\mathbf{A}}^{\perp}\mathbf{D}\mathbf{A})\boldsymbol{h}\,,\\ \text{and } \mathbf{C} &= (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{D}\mathbf{A}\,.\end{aligned}$$

Notice that the CRB for $\boldsymbol{h}$ is equal to that in the CFO-free case plus an extra term which takes into account the effect of CFO estimation error. In the case of single antenna systems, the preceding CRB expressions reduce to the ones given in [4]. In contrast with the CFO-free case, the CRBs for different $\boldsymbol{h}_j$'s are now coupled. Further, the CRBs for the CFO and the CIRs are now channel-dependent. Next, we design training sequences which make the CRB of ω_0 *channel-zeros-independent*, i.e., for a fixed $\|\boldsymbol{h}\|_2$, $CRB(\omega_0)$ is independent of the channel realizations (or channel zeros).

4. TRAINING SEQUENCE DESIGN

In the absence of a priori information about the channels, the total transmit energy allocated to training, $\mathcal{P}$, is split equally between the transmit antennas, i.e.,

$$(\mathbf{C0}) \qquad \|\boldsymbol{a}_1\|_2^2 = \cdots \|\boldsymbol{a}_{M_t}\|_2^2 = \frac{\mathcal{P}}{M_t}$$

where $\|\boldsymbol{a}_i\|_2^2 = \sum_{n=-L+1}^{N-1}|a_i(n)|^2$. If the Peak-to-Average Power Ratio (PAPR) is an issue, the more stringent constant-modulus (CM) Condition (C1) will be imposed:

$$(\mathbf{C1}) \qquad |a_i(n)|^2 = \frac{\mathcal{P}}{M_t(N+L-1)},$$

for $n = -L+1, \cdots, N-1$; $i = 1 \cdots M_t$. The CM property is desirable if nonlinear distortions due to power amplifiers are of concern. Since (C0) is less restrictive than (C1), the former will lead to better estimation performance at the expense of a larger PAPR.

From eqs. (5) and (6) we note that the training sequences that minimize $CRB(\omega_0)$ and $\mathrm{Tr}\,\{CRB(\boldsymbol{h})\}$ are channel-dependent. To circumvent this problem, the asymptotic CRB and the worst-channel scenario were used in [4]. Here, we take a different approach. We investigate training designs that make $CRB(\omega_0)$ channel-independent, i.e., independent of the channel zeros. Since we cannot make both $CRB(\omega_0)$ and $\mathrm{Tr}\,\{CRB(\boldsymbol{h})\}$ channel-independent, we derive a channel-independent upper bound on $\mathrm{Tr}\,\{CRB(\boldsymbol{h})\}$. Although the channel-independent CRBs are not the minimum CRBs, which are channel-dependent, they have the nice property of being constant for all channel realizations provided the norm of $\boldsymbol{h}$ is constant.

$CRB(\omega_0)$ in eq. (5) is channel-independent if the training sequences satisfy

$$\mathbf{A}^H\mathbf{D}\Pi_{\mathbf{A}}^{\perp}\mathbf{D}\mathbf{A} = \rho\mathbf{I} \qquad (7)$$

where ρ is a positive real scalar. Such sequences will be referred to as Channel-Independent Performance Training (CIPT) sequences. Finding all CIPT sequences seems to be a difficult task. Here, we give examples of such sequences.

Consider sequences of size $N+L-1$ where $N = LM_tQ$ with $Q(\geq 1)$ being an integer, which have a cyclic prefix structure, i.e., $a_i(-n) = a_i(N-n)$ for $n = 1 \cdots L-1$, and such that their N-point DFTs, i.e., DFT of $[a_i(0)\cdots a_i(N-1)]^T$, satisfy

$$\tilde{a}_i(m) = \sqrt{\frac{\kappa\mathcal{P}N}{M_tL}}e^{j\varphi_i(m)}\sum_{k=0}^{L-1}\delta(m+(i-1)Q-kM_tQ), \quad (8)$$

for $m = 0\cdots N-1$ and $i = 1, \cdots, M_t$, where the $\varphi_i(m)$'s are arbitrary phase values and $\kappa = (M_t/\mathcal{P})\sum_{n=0}^{N-1}|a_i(n)|^2$. Note that $\kappa = 1$ under (C0). In the frequency-domain, each such training sequence is a L-tooth comb, with a spacing of M_tQ bins; the combs for adjacent transmitters are offset by Q frequency bins (see [6] for related designs). Under (C0), $\kappa = 1$, $\varphi_i(m) = \phi_i$, $\forall m$, where the ϕ_i's are arbitrary phases. This implies that under (C0), the time-domain training sequence consists of an impulse train, spaced L apart, amplitude modulating a different tone. For the sequences in (8), $\mathbf{A}^H\mathbf{A} = \kappa\mathcal{P}/M_t\mathbf{I}$, and

$$\mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H = \frac{1}{Q}\,\mathbf{1}_Q\otimes\mathbf{I}_{N/Q}\,. \qquad (9)$$

We can show that the above sequences satisfy the condition in eq. (7) with ρ given by

$$\rho = \frac{\kappa\mathcal{P}N^2\left[1-(M_tL/N)^2\right]}{12M_t}\,.$$

Next, we use eq. (6) to obtain

$$\begin{aligned}\mathrm{Tr}\,\{CRB(\boldsymbol{h})\} &= \frac{\sigma^2}{2}\Big[2M_r\mathrm{Tr}\left\{(\mathbf{A}^H\mathbf{A})^{-1}\right\}\\ &+ \xi^{-1}[\boldsymbol{h}^H(\mathbf{I}_{M_r}\otimes\mathbf{C}^H\mathbf{C})\boldsymbol{h}]\Big] \qquad (10)\end{aligned}$$

where $\mathbf{C}$ is given in eq. (7). For the CIPT design in eq. (8), we have that $\xi = \rho\|\boldsymbol{h}\|_2$ and (using eq. (9))

$$\begin{aligned}\mathrm{Tr}\,\{CRB(\boldsymbol{h})\} &= \frac{LM_t^2M_r\sigma^2}{2\kappa\mathcal{P}}\\ &+ \frac{6\sigma^2M_t}{\kappa\mathcal{P}N^2\left[1-(M_tL/N)^2\right]} \qquad (11)\\ &\times \frac{\boldsymbol{h}^H(\mathbf{I}_{M_r}\otimes\bar{\mathbf{A}}^H\mathbf{D}(Q^{-1}\mathbf{1}_Q\otimes\mathbf{I}_{N/Q})\mathbf{D}\bar{\mathbf{A}})\boldsymbol{h}}{\|\boldsymbol{h}\|_2}\end{aligned}$$

where $\bar{\mathbf{A}} = [\kappa\mathcal{P}/M_t]^{-1/2}\mathbf{A}$; so $\bar{\mathbf{A}}^H\bar{\mathbf{A}} = \mathbf{I}$. A channel-independent upper bound is given by

$$\mathrm{Tr}\,\{CRB(\boldsymbol{h})\} < \frac{LM_t^2M_r\sigma^2}{\kappa\mathcal{P}} + \frac{6\sigma^2M_t\lambda_{max}}{\kappa\mathcal{P}N^2\left[1-(M_tL/N)^2\right]} \quad (12)$$

where λ_{max} is the maximum eigenvalue of $\bar{\mathbf{A}}^H\mathbf{D}(Q^{-1}\mathbf{1}_Q\otimes\mathbf{I}_{N/Q})\mathbf{D}\bar{\mathbf{A}}$. The upper bound is equal to $\mathrm{Tr}\,\{CRB(\boldsymbol{h})\}$ in the CFO-free case plus an extra term which depends on how the $\varphi_i(m)$'s in eq. (8) are chosen. One can also design these angles so that the upper bound is minimized. However, maximum energy efficiency (resp. unit PAPR) imposes constraints on the $\varphi_i(m)$'s. Note that the above upper bound is *strictly* channel-independent: it does not even depend on the norm of $\boldsymbol{h}$. We summarize these results next:

Proposition 1 If $N = LM_tQ$ and the training sequences are designed as in eq. (8), then the CRB for ω_0 is channel-independent and is given by

$$\mathrm{CRB}(\omega_0) = \frac{6\sigma^2}{\kappa\mathcal{P}\|\boldsymbol{h}\|_2 N^2\left[1-(M_tL/N)^2\right]} \tag{13}$$

where $\kappa = (M_t/\mathcal{P})\sum_{n=0}^{N-1}|a_i(n)|^2$. Further, the trace of $CRB(\boldsymbol{h})$ is upper bounded by the channel-independent quantity in eq. (12).

When $N >> L$, the above channel-independent CRB for ω_0 is identical to the asymptotic CRB developed in [4]. Further, in the case of single antenna systems, periodic training was used for CFO estimation in [1]; but the CRB derived there was channel-dependent because the SNR was channel-dependent. Finally for the interesting case where $N = 2LM_t$ (see Sec 5), eq. (13) becomes proportional to N^{-2}, $\mathrm{CRB}(\omega_0) = \frac{8\sigma^2}{\kappa\mathcal{P}\|\boldsymbol{h}\|^2N^2}$.

5. MAXIMUM LIKELIHOOD ESTIMATION

The ML estimate of ω_0 and $\boldsymbol{h}$ are found to be

$$\hat{\omega}_0 = \arg\min_{\omega}\sum_{j=1}^{M_r}\|\boldsymbol{x}_j^H\Gamma(\omega)\Pi_{\mathbf{A}}^{\perp}\Gamma(-\omega)\boldsymbol{x}_j\|_2 \tag{14}$$

$$\hat{\boldsymbol{h}}_j = \mathbf{A}^{\dagger}\Gamma(-\hat{\omega}_0)\boldsymbol{x}_j\ . \tag{15}$$

When $N = M_tLQ$, and the training sequences satisfy the CIPT design condition in eq. (8), we obtain

$$\Pi_{\mathbf{A}}^{\perp} = \mathbf{I} - \frac{1}{Q}\,\mathbf{1}_Q \otimes \mathbf{I}_{N/Q}\ .$$

The ML estimates greatly simplify to

$$\hat{\omega_0} = \arg\max_{\omega}\sum_{m=1}^{N/(M_tL)-1}\mathcal{R}\left[r(mM_tL)e^{-jmM_tL\omega}\right] \tag{16}$$

$$\hat{\boldsymbol{h}}_{i,j} = \frac{M_t}{\kappa\mathcal{P}}\mathbf{A}_i^H\Gamma(-\hat{\omega}_0)\boldsymbol{x}_j \tag{17}$$

where $\mathcal{R}\left[x\right]$ indicates the real part of x and $r(\tau)$ is the correlation

$$r(\tau) = \sum_{j=1}^{M_r}\sum_{k=0}^{N-\tau-1}x_j^*(k)x_j(k+\tau)\ .$$

In the case of single antenna systems, the ML estimate of ω_0 is identical to the repetitive-slot-based non-linear least squares estimate in [1, 3, 11]. This holds true only for sequences satisfying eq. (8), i.e., if the number of repetitive slots in the training sequence after removing the cyclic-prefix is not equal to N/L, then the repetitive-slot-based nonlinear least squares estimates are not ML. The acquisition range of the above ML estimator of ω_0 is $[-\pi/M_tL, \pi/M_tL)$, which decreases with M_t and L. Thus, for large M_t or/and L, the proposed training design is viable for fine CFO estimation only, i.e., for small values of ω_0.

If $N = 2M_tL$, the ML estimator in eq (16) is given in closed-form as

$$\hat{\omega}_0 = \frac{1}{M_tL}\arg\{r(M_tL)\}\ . \tag{18}$$

In the case of single antenna systems, this estimator reduces to the one proposed in [2]. If $N > 2M_tL$, the MLE cannot be written in closed-form.

The mean-square errors (MSE's) of the ML estimates asymptotically (i.e., large N) achieve the CRB. In the CFO-free scenario, the ML estimator of $\boldsymbol{h}$ is obtained from eq. (15) after replacing $\hat{\omega}_0$ by zero. In this case, the MSE of the ML estimate achieves the CRB for *any* number of samples N.

Finally, if we use the CIPT training design in eq. (8) with $\varphi_i(m) = 0$, $m = 0, \cdots, N-1$ and $i = 1\cdots M_t$, which satisfies (C0), the MLE simplifies further to

$$\hat{h}_{i,j}(\ell) = \frac{M_t}{\kappa\mathcal{P}}\sum_{k=0}^{N/L-1}x_j(\ell+kL)e^{-j2\pi(i-1)\frac{\ell+kL}{LM_t}}e^{-j\hat{\omega}_0(\ell+kL)} \tag{19}$$

6. SIMULATION RESULTS

Consider a system with 1 receive and 2 transmit antennas. The channels are assumed independent with $L = 4$ taps each. The $h_i(\ell)$'s are uncorrelated zero-mean Gaussian random variable with exponential power delay profile $E\left\{|h_i(\ell)|^2\right\} = C\exp(-0.2\ell)$ where scaling factor C ensures that $\sum_{\ell=0}^{L-1}E\left\{|h_i(\ell)|^2\right\} = 1$. The length of the training sequence is set to 23, i.e., $N = 16$. In each Monte-Carlo simulation, a new CFO is randomly drawn from the interval $[-\pi/10, \pi/10]$ and a new channel realization is generated. Two types of training sequences are used to estimate the CFO and CIR: i) independent PN sequences, as recommended in [4] for single-input systems, and ii) our CIPT sequence design in eq. (8). Figure 1 displays the CRBs for ω_0 for different realizations of the normalized channel, i.e., with $\|\boldsymbol{h}\|_2$ kept constant. It can be seen that for the CIPT design $CRB(\omega_0)$ is the same for all realizations of the normalized channel, but it can significantly vary in the case of PN-based training design. In what follows, we do not normalize the channels. Figures 2-3 display the MSE of the ML CFO and channel estimates vs. the average SNR $\frac{\mathcal{P}}{N\sigma^2}$. These MSEs are obtained using 500 Monte-Carlo runs. For the CIPT design, the closed-form CFO estimator in eq. (18) was used since $N = 2M_tL$, and the channel coefficients were estimated using the simple expression in eq. (19). For the PN design, a gradient-type numerical optimization was used to estimate ω_0. The optimal CIPT design offers a 3 dB SNR gain over the PN training design.

Figures 2-3 suggest that the proposed CIPT design not only leads to simple estimation algorithms but also provides a better averaged (over the channel realizations) CFO and channel estimation performance than does the PN training design. Furthermore, Figure 3 shows that the channel-independent upper bound on $CRB(\boldsymbol{h})$ given in eq. (12) is quite tight.

Figures 4-5 display performance vs. N for a fixed SNR: the CIPT design outperforms the PN-based design for all values of N. The gap in performance decreases when N increases.

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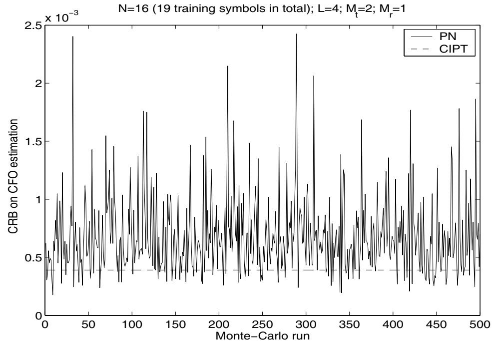

Fig. 1. CRBs on CFO estimation for different channel realizations

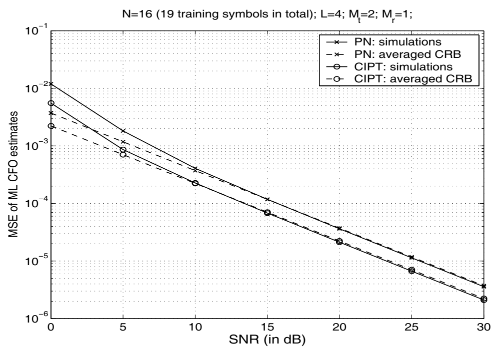

Fig. 2. MSEs and (averaged) CRBs for CFO estimation vs. SNR

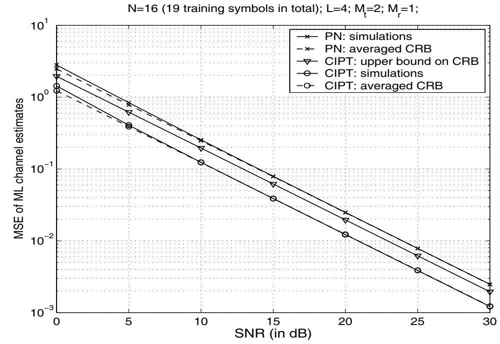

Fig. 3. MSEs and (averaged) CRBs for channel estimation vs SNR

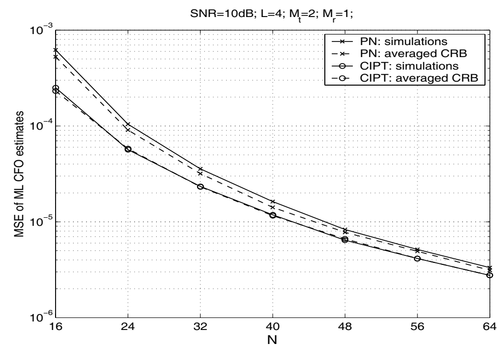

Fig. 4. MSEs and avg. CRBs for CFO estimation vs. N

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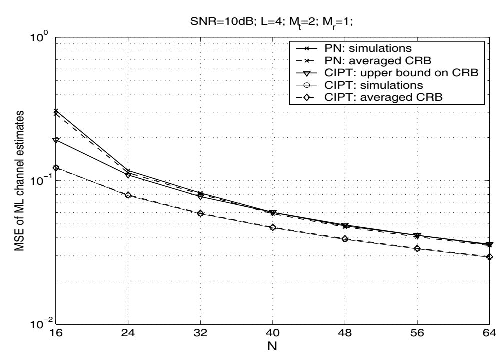

Fig. 5. MSEs and (averaged) CRBs for channel estimation vs. N