ROBUST DESIGN OF LINEAR MIMO TRANSCEIVERS UNDER CHANNEL UNCERTAINTY

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ABSTRACT

This paper considers the robust design of a linear transceiver with imperfect channel state information (CSI) at the transmitter of a MIMO link. The framework embraces the design problem when CSI at the transmitter consists of the channel mean and covariance matrix or, equivalently, the channel estimate and the estimation error covariance matrix. The design of the linear MIMO transceiver is based on a general cost function covering several well known performance criteria. In particular, two families are considered in detail: Schur-convex and Schur-concave functions. Approximations are used in the low SNR and high SNR regimes separately to obtain simple optimization problems that can be readily solved. Numerical examples show gains compared to other suboptimal methods.

1. INTRODUCTION

The design of a MIMO communication system depends on the degree of knowledge of the Channel State Information (CSI). For a given communication channel, the best spectral efficiency and/or performance is obviously achieved when perfect CSI is available at both sides of the link. Optimal linear transceiver design has been extensively studied in this case [1, 2]. In practical communication systems, imperfect CSI may arise from a variety of sources such as channel estimation errors, quantization of the channel estimate in the feedback channel, outdated channel estimates, etc [3]. These effects are of paramount importance in practical implementations. By modeling such imperfections and taking them into account in the transceiver design, a robust communication system is obtained.

While imperfection of CSI at the receiver (CSIR) can be assumed to be sufficiently small in many cases, CSI at the transmitter (CSIT) will be far from perfect in many realistic situations. Hence, it is reasonable to assume perfect CSIR and imperfect CSIT, as is considered in this paper.

There are different ways to design a system that is robust to imperfect CSIT. In [3, 4], worst-case designs are considered. This guarantees a certain system performance for any possible channel sufficiently close to the estimated one. This approach leads to conservative designs, which may translate into a significant increase of the required transmit power. Alternatively, the CSI uncertainty can be modeled statistically. This guarantees a certain system performance averaged over the channels that could have caused the current estimated channel [5, 6, 7]. The latter statistical modeling approach is used in the sequel.

For the stochastic approach, different types and amount of CSIT determine the transceiver design. Previous work has considered channel mean CSIT or channel covariance CSIT. The case of mean CSIT is addressed in [8] for minimizing the average MSE, and in [9] (ML receiver) for maximizing the mutual information by beamforming. The case when the channel covariance is the only CSIT is addressed in [10] (ML receiver) for minimizing an upper bound of the average pairwise error probability (PEP) by eigenbeamforming, and in [9] (ML receiver) for maximizing the mutual information by beamforming. When both mean and covariance CSIT are available, a robust design is more involved. The problem of minimizing the average MSE and maximizing the average SNR in MISO channels is considered in [5], and in [6] for minimizing the sum MSE and minimizing an upper bound of PEP with an equivalent channel based on conditional channel mean and linear transceivers. In [7] the problem of minimizing average MSE with transmitters diagonalizing the covariance matrix is addressed. Linear precoders used with orthogonal STBC to minimize an upper bound of PEP are discussed in [11].

Herein, we will consider robust linear transceiver design with imperfect CSIT and perfect CSIR for MIMO systems. The framework proposed embraces the cases of mean and/or covariance feedback CSIT. Two very general classes of cost functions, Schur-convex and Schur-concave functions [12], are considered instead of choosing a specific criterion.

The following notations are used in the paper. Uppercase and lowercase boldface denote matrices and vectors respectively. The operators $(\cdot)^*$ and $\otimes$ are Hermitian transpose and Kronecker product respectively. The operator $\mathbf{d}[\cdot]$ is a vector consisting of the diagonal elements of the matrix argument, and $\operatorname{diag}[\cdot]$ is the diagonal matrix with the vector argument as diagonal elements.

2. SYSTEM MODEL

Consider a narrowband MIMO channel with n transmit and m receive antennas. The corresponding discrete-time signal model can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where the transmitted signal vector is $\mathbf{s} \in \mathbb{C}^{n\times 1}$, the channel matrix is $\mathbf{H} \in \mathbb{C}^{m\times n}$, the received signal vector is $\mathbf{y} \in \mathbb{C}^{m\times 1}$, and the noise vector, $\mathbf{n} \in \mathbb{C}^{m\times 1}$, is zero-mean circular symmetric complex Gaussian interference-plus-noise with arbitrary covariance matrix $\mathbf{R}_n$, *i.e.* $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_n)$, which is assumed to be perfectly known at both sides of the link.

Consider the use of a linear precoder $\mathbf{B} \in \mathbb{C}^{n\times l}$ at the transmitter and a linear equalizer $\mathbf{A} \in \mathbb{C}^{m\times l}$ at the receiver:

$$\mathbf{s} = \mathbf{B}\mathbf{x}, \quad \hat{\mathbf{x}} = \mathbf{A}^*\mathbf{y} \tag{2}$$

where the data symbol vector, $\mathbf{x} \in \mathbb{C}^{l\times 1}$, is zero-mean with unit-energy uncorrelated symbols, *i.e.*, $\mathbb{E}[\mathbf{x}\mathbf{x}^*] = \mathbf{I}$.

The total average transmitted power is $P_T = \mathbb{E}[\mathbf{s}^*\mathbf{s}] = \text{tr}[\mathbf{B}\mathbf{B}^*]$. It is also convenient to define the mean square error (MSE) matrix as:

$$\begin{aligned}\mathbf{E}(\mathbf{A},\mathbf{B}) &\triangleq \mathbb{E}\left[(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})^*\right] \\ &= (\mathbf{A}^*\mathbf{H}\mathbf{B}-\mathbf{I})(\mathbf{B}^*\mathbf{H}^*\mathbf{A}-\mathbf{I}) + \mathbf{A}^*\mathbf{R}_n\mathbf{A}\end{aligned} \quad (3)$$

and its diagonal elements $\mathbf{d}[\mathbf{E}]$ coincide with the MSEs.

3. PROBLEM FORMULATION

3.1. Robust Design Formulation with Imperfect CSI

To be general, we will consider that the performance of the MIMO system is measured by an arbitrary function $\mathcal{F}_0$ of the MSEs, SINRs, or BERs of the established substreams. Interestingly, as proved in [13], BER and SINR can both be re-mapped to functions of MSE under certain mild conditions. Therefore, it suffices to consider functions of the MSEs, $\mathcal{F}_0(\mathbf{d}[\mathbf{E}])$, as a performance measure.

The main purpose of this paper is to optimize the linear transceiver $(\mathbf{A},\mathbf{B})$ including robustness against channel estimation errors. We take a stochastic design method following a Bayesian philosophy that considers the distribution function of the actual channel conditioned on the obtained estimation.

The following model is used to describe channel uncertainty:

$$\hat{\mathbf{H}} = \mathbf{H} + \mathbf{H}_\triangle \quad (4)$$

where $\mathbf{H}$ is the actual channel, $\hat{\mathbf{H}}$ is the channel estimate and $\mathbf{H}_\triangle$ is the estimation error ($\mathbf{H}$ and $\mathbf{H}_\triangle$ are modeled as jointly Gaussian distributed [14]). As a consequence, the average performance as a function of MSEs given the channel estimate $\hat{\mathbf{H}}$ is

$$\mathbb{E}_{\mathbf{H}|\hat{\mathbf{H}}}[\mathcal{F}_0(\mathbf{d}[\mathbf{E}])] = \mathbb{E}_{\mathbf{H}_\triangle}[\mathcal{F}_0(\mathbf{d}[\mathbf{E}])]. \quad (5)$$

The robust problem formulation that minimizes the average cost function given the channel estimate can therefore be formulated as

$$\begin{aligned}\min_{\mathbf{A},\mathbf{B}} \quad & \mathbb{E}_{\mathbf{H}_\triangle}\left[\mathcal{F}_0\left(\mathbf{d}\left[\mathbf{E}(\mathbf{A},\mathbf{B})\right]\right)\right] \\ \text{s.t.} \quad & \text{tr}\left[\mathbf{B}\mathbf{B}^*\right] \le P_T.\end{aligned} \quad (6)$$

Of course, obtaining the expected value of any arbitrary function $\mathcal{F}_0$ in a convenient form is in general not possible. We should then focus on some simple choices such as when $\mathcal{F}_0$ is linear and interchange the order of expectation:

$$\begin{aligned}\min_{\mathbf{A},\mathbf{B}} \quad & \mathcal{F}_0\left(\mathbf{d}\left[\mathbb{E}\left[\mathbf{E}(\mathbf{A},\mathbf{B})\right]\right]\right) \\ \text{s.t.} \quad & \text{tr}\left[\mathbf{B}\mathbf{B}^*\right] \le P_T.\end{aligned} \quad (7)$$

If the function $\mathcal{F}_0$ is not linear, we can linearize the function by a Taylor expansion around the point $\mathbb{E}[\mathbf{d}[\mathbf{E}]]$ resulting a similar cost function depending on $\mathbb{E}[\mathbf{E}]$ and its higher moments:

$$\mathbb{E}\left[\mathcal{F}_0\left(\mathbf{d}\left[\mathbf{E}\right]\right)\right] = \mathcal{F}_0(\mathbf{d}\left[\mathbb{E}\left[\mathbf{E}\right]\right]) + o\left(\|\mathbf{d}\left[\mathbf{E}\right] - \mathbb{E}\left[\mathbf{d}\left[\mathbf{E}\right]\right]\|^2\right).$$

3.2. Mean and Covariance CSIT

If only the channel covariance is available as CSIT, weighted space-time block coding may be applied [14, 15]. Mean feedback (possibly with covariance feedback) CSIT is a more relevant situation where the linear transceiver signal model may be adopted.

We model the mean and covariance CSIT as

$$\mathbf{H} \sim \mathcal{CN}(\hat{\mathbf{H}}, \mathbf{R}_\triangle \otimes \mathbf{I}_m) \quad (8)$$

where $\hat{\mathbf{H}}$ is the channel estimate (or equivalently the channel mean $\bar{\mathbf{H}}$) and $\mathbf{R}_\triangle$ is the channel estimation error covariance (or equivalently the channel covariance matrix). The case of only mean CSIT is implicitly considered here if $\mathbf{R}_\triangle$ is an identity matrix.

4. ROBUST DESIGN WITH IMPERFECT CSIT

When the receiver has perfect knowledge of the channel state, it can always optimize the linear equalizer $\mathbf{A}$ for each channel realization $\mathbf{H}$. The linear MMSE receiver $\mathbf{A}$ minimizing the MSE matrix in (3) is the well known Wiener filter (*c.f.* [2])

$$\mathbf{A} = \left(\mathbf{H}\mathbf{B}\mathbf{B}^*\mathbf{H}^* + \mathbf{R}_n\right)^{-1}\mathbf{H}\mathbf{B} \quad (9)$$

and without loss of generality we assume $\mathbf{R}_n = \mathbf{I}$, so the resulting MMSE matrix is

$$\mathbf{E}(\mathbf{B}) = \left(\mathbf{I} + \mathbf{B}^*\mathbf{H}^*\mathbf{H}\mathbf{B}\right)^{-1} = \left(\mathbf{I} + \mathbf{W}\right)^{-1} \quad (10)$$

where $\mathbf{W} \triangleq \mathbf{B}^*\mathbf{H}^*\mathbf{H}\mathbf{B}$. Thus, the robust design problem (7) can be rewritten as

$$\begin{aligned}\min_{\mathbf{B}} \quad & \mathcal{F}_0\left(\mathbf{d}\left[\mathbb{E}\left[\mathbf{E}(\mathbf{B})\right]\right]\right) \\ \text{s.t.} \quad & \text{tr}\left[\mathbf{B}\mathbf{B}^*\right] \le P_T.\end{aligned} \quad (11)$$

In order to solve this optimization problem with respect to $\mathbf{B}$, we need a closed-form expression for $\mathbb{E}[\mathbf{E}(\mathbf{B})]$, which is very difficult to obtain in general. Expressing $\mathbf{E}(\mathbf{B})$ in terms of $\mathbf{W}$, the problem can be more conveniently rewritten as

$$\begin{aligned}\min_{\mathbf{B}} \quad & \mathcal{F}_0\left(\mathbf{d}\left[\mathbb{E}_{\mathbf{W}}\left[(\mathbf{I}+\mathbf{W})^{-1}\right]\right]\right) \\ \text{s.t.} \quad & \text{tr}\left[\mathbf{B}\mathbf{B}^*\right] \le P_T.\end{aligned} \quad (12)$$

which requires the distribution of $\mathbf{W}$. Unfortunately $\mathbf{W}$ follows a *non-central* complex Wishart distribution [16] that is very involved in computation:

$$\mathbf{W} \sim \mathcal{CW}_l(m, \mathbf{B}^*\mathbf{R}_\triangle\mathbf{B}, (\mathbf{B}^*\mathbf{R}_\triangle\mathbf{B})^{-1}\mathbf{B}^*\hat{\mathbf{H}}^*\hat{\mathbf{H}}\mathbf{B}). \quad (13)$$

The exact distribution of the non-central Wishart distribution is derived in [17] in terms of Zonal polynomials and in [18] in terms of generalized Laguerre polynomials. Both of them are too complicated to be applicable in practice. A very convenient alternative is to approximate the above non-central Wishart distribution with a central distribution (which is much easier to manipulate) in terms of a moment approximation [19], so the non-central Wishart distribution in (13) can be approximated as:

$$\mathbf{W} \sim \mathcal{CW}_l\left(m, \mathbf{B}^*\mathbf{\Psi}_H\mathbf{B}\right), \quad \mathbf{\Psi}_H = (\mathbf{R}_\triangle + \hat{\mathbf{H}}^*\hat{\mathbf{H}}/m) \quad (14)$$

which gives a second order moment accurate to the order of $1/m$.

4.1. Low SNR Formulation

With the above CSI modeling and non-central Wishart approximation, the robust design problem (12), and eventually the original problem (7), can be approximated and solved efficiently. Under low SNR conditions[1], results are presented in the companion paper [20].

Result 1: When the cost function $\mathcal{F}_0$ is Schur-concave, the specific structure of the optimal transmitter $\mathbf{B}$ actually diagonalizes the equivalent channel covariance matrix $\mathbf{\Psi}_H$. The whole optimization problem is reduced to a power loading problem, which depends on the specific cost function.

Result 2: When the cost function $\mathcal{F}_0$ is Schur-convex, the optimal transmitter $\mathbf{B}$ is independent of the choice of the function $\mathcal{F}_0$. The optimal $\mathbf{B}$ does not fully diagonalize $\mathbf{\Psi}_H$, but produces equal diagonal elements of $\mathbb{E}[\mathbf{E}(\mathbf{B})]$. The optimization of $\mathbf{B}$ is a convex quadratic program (QP) and can be solved very efficiently.

[1] It is convenient to define SNR vector as $\mathbf{SNR} = \mathbf{d}[\mathbf{B}^*\mathbf{\Psi}_H\mathbf{B}]$. By low SNR, we mean $\max \mathbf{SNR} < 1/(2m+l)$.

4.2. High SNR Formulation

Under high SNR conditions[2], the robust design problem (7) is solved by the following theorem. Due to the limited space, please refer to [21] for a detailed proof, which is based on majorization theory [12].

Lemma 1 *The expected value of the MMSE matrix in* (10)*, where* $\mathbf{W}$ *is Wishart-distributed as in* (14)*, can be approximated (assuming* $n \geq l$ *and* $m > l+1$*) as*

$$\mathbb{E}\left[\mathbf{E}(\mathbf{B})\right] = \mathbb{E}\left[(\mathbf{I}+\mathbf{W})^{-1}\right]$$
$$\approx \frac{(\mathbf{B}^*\mathbf{\Psi}_H\mathbf{B})^{-1}}{m-l}\left[\mathbf{I} - \frac{(m-l)(\mathbf{B}^*\mathbf{\Psi}_H\mathbf{B})^{-1} + \mathrm{tr}\left[(\mathbf{B}^*\mathbf{\Psi}_H\mathbf{B})^{-1}\right]}{(m-l)^2-1}\right].$$

Theorem 1 *Let* $\mathcal{F}_0 : \mathbb{R}^l \mapsto \mathbb{R}$ *be a linear cost function increasing in each argument and minimized when the arguments in are decreasing order. Then the following nonconvex constrained optimization problem*

$$\begin{array}{ll} \min\limits_{\mathbf{B}} & \mathcal{F}_0\left(\mathbf{d}\left[\mathbb{E}\left[\mathbf{E}(\mathbf{B})\right]\right]\right) \\ \text{s.t.} & \mathrm{tr}\left[\mathbf{B}\mathbf{B}^*\right] \leq P_T \end{array} \tag{15}$$

can be approximated in the high SNR regime by the following simpler problem:

$$\begin{array}{ll} \min\limits_{\mathbf{p},\boldsymbol{\rho}} & \mathcal{F}_0(\rho_1, \rho_2, \cdots, \rho_l) \\ \textit{s.t.} & \sum\limits_{j=i}^{l}\left\{\frac{(p_j\lambda_j)^{-1}}{m-l}\left[1 - \frac{(m-l)(p_j\lambda_j)^{-1} + \sum_{k=1}^{l}(p_k\lambda_k)^{-1}}{(m-l)^2-1}\right]\right\} \\ & \qquad\qquad \leq \sum\limits_{j=i}^{l}\rho_j, \;\; 1 \leq i \leq l \\ & \sum\limits_{j=1}^{l} p_j \leq P_T, \;\; p_j \geq 0 \\ & \rho_i \geq \rho_{i+1}, \;\; 1 \leq i < l \end{array} \tag{16}$$

where λ_i *is the* i*-th largest eigenvalue of* $\mathbf{\Psi}_H$ *in increasing order. If* $\mathcal{F}_0$ *is convex, the constraint* $\rho_i \geq \rho_{i+1}$ *is not necessary. The mapping from* (16) *to* (15) *is given by*

$$\mathbf{B} = \mathbf{U}_{H,1}\mathbf{\Sigma}_B\mathbf{Q}$$

where the matrix $\mathbf{U}_{H,1}$ *consists of the eigenvectors of* $\mathbf{\Psi}_H$ *corresponding to the* l *largest eigenvalues in increasing order,* $\mathbf{\Sigma}_B = \mathrm{diag}\left[\{\sqrt{p_i}\}\right]$ *contains the power allocation obtained by the simplified problem* (16) *and* $\mathbf{Q}$ *is a unitary matrix such that* $\mathbf{d}\left[\mathbb{E}\left[\mathbf{E}(\mathbf{B})\right]\right] = \boldsymbol{\rho}$ *(see [22] for a practical method to obtain* $\mathbf{Q}$*).*

The above scalarized optimization problem (16) can be greatly simplified when the cost function $\mathcal{F}_0$ is Schur-concave or Schur-convex.

Theorem 2 *If* $\mathcal{F}_0$ *is Schur-concave, the optimal solution is*

$$\mathbf{B} = \mathbf{U}_{H,1}\mathbf{\Sigma}_B$$

and the scalarized problem (16) *can be further simplified:*

$$\begin{array}{ll} \min\limits_{\mathbf{p},\boldsymbol{\rho}} & \mathcal{F}_0(\rho_1, \rho_2, \cdots, \rho_l) \\ \textit{s.t.} & \frac{(p_j\lambda_j)^{-1}}{m-l}\left[1 - \frac{(m-l)(p_j\lambda_j)^{-1} + \sum_{k=1}^{l}(p_k\lambda_k)^{-1}}{(m-l)^2-1}\right] \\ & \qquad\qquad \leq \rho_j, \;\; 1 \leq i \leq l \\ & \sum\limits_{j=1}^{l} p_j \leq P_T, \;\; p_j \geq 0 \\ & \rho_i \geq \rho_{i+1}, \;\; 1 \leq i < l. \end{array} \tag{17}$$

[2]By high SNR, we mean $\min \textbf{SNR} \geq (2m-l)/[(m-l+1)(m-l-1)]$.

If $\mathcal{F}_0$ *is Schur-convex, the optimal solution to Theorem 1 is*

$$\mathbf{B} = \mathbf{U}_{H,1}\mathbf{\Sigma}_B\mathbf{Q}$$

where $\mathbf{Q}$ *is a unitary matrix such that* $\mathbb{E}\left[\mathbf{E}(\mathbf{B})\right]$ *has identical diagonal elements (see [22] for a practical method to obtain* $\mathbf{Q}$*). The scalarized optimization* (16) *can be further simplified to*

$$\begin{array}{ll} \min\limits_{\mathbf{p}} & \sum\limits_{j=1}^{l}\left\{\frac{(p_j\lambda_j)^{-1}}{m-l}\left[1 - \frac{(m-l)(p_j\lambda_j)^{-1} + \sum_{k=1}^{l}(p_k\lambda_k)^{-1}}{(m-l)^2-1}\right]\right\} \\ \textit{s.t.} & \sum\limits_{j=1}^{l} p_j \leq P_T \\ & p_j \geq 0. \end{array} \tag{18}$$

In general the problems (16) - (18) are nonconvex, so the optimization relies on a grid and gradient search method, which may give local minima. Surprisingly, it can be shown that the problem (18) approaches a convex function for sufficiently large $(m-l)P_T$, which makes implementation feasible.

5. NUMERICAL EXAMPLES

Here some numerical results based on Monte Carlo simulations are presented to evaluate our robust designs for high SNR formulation.

As design criterion we choose the minimization of the worst BER with a given set of equal constellations (which is a Schur-convex function [2]). Three methods are compared: i) naive solution (which assumes the channel estimate as perfect, so the optimal transceiver in [2] is used); ii) the proposed robust solution; and iii) the ideal solution assuming an instantaneous and exact CSIT. Observe that for all these approaches the receivers are always given by the Wiener filter in (9).

In the high SNR formulation, the simplified problem is nonconvex and is solved by grid and gradient search method. As pointed out in Section 4, the search converges very quickly when $(m-l)P_T$ is large because the problem approaches a convex function.

The channel estimate $\hat{\mathbf{H}}$ is modeled as (4) where the true channel matrix $\mathbf{H} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_n \otimes \mathbf{I}_m\right)$ and $\mathbf{H}_\Delta \sim \mathcal{CN}\left(\mathbf{0}, \varepsilon\mathbf{R}_\Delta \otimes \mathbf{I}_m\right)$. The positive scalar ε measures the accuracy of the channel estimate $\hat{\mathbf{H}}$. The covariance matrix $\mathbf{R}_\Delta$ is Toeplitz with the first column defined by the correlation coefficient ρ_t as $[1 \; \rho_t \; \rho_t^2 \; \cdots \; \rho_t^{n-1}]^T$. Simulation results are averaged over $\mathbf{H}$ and $\mathbf{H}_\Delta$.

Fig. 1 shows the numerical comparison of the three methods. The robust design is always better than the naive case, with a gain of about 2-10dB in terms of transmit power. When the SNR increases, the performance difference between the robust design and the naive design increases as well.

6. CONCLUSION

We have addressed the robust linear transceiver design problem for MIMO systems in the case of imperfect CSIT and perfect CSIR. The proposed framework solves the design problem when the CSIT takes the form of the channel mean and/or the covariance matrix. Two very general classes of cost functions, Schur-convex and Schur-concave, are considered in detail. Approximations of the Wishart distribution, which are valid for the low SNR and the high SNR regime separately, are used to obtain simple optimization problems. For low SNR, the Schur-convex cost function may always be solved by a convex quadratic program, while certain Schur-concave cost functions are also solved as convex problems. For high SNR, Schur-convex

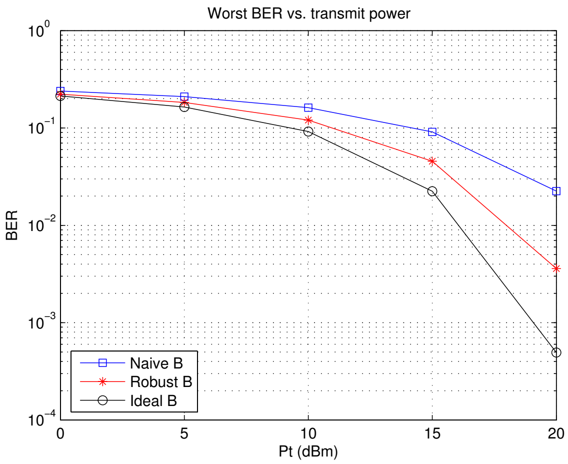

Fig. 1. Worst case BER for high SNR with mean and covariance feedback CSIT using 64QAM for all substreams ($m = 12, n = 10, l = 4, \rho_t = 0.75, \varepsilon = 0.6$).

cost functions may be solved by a convex quadratic program under certain mild conditions. Summarizing, the complicated nonconvex robust linear MIMO transceiver design problem can be well approximated as simpler scalar optimization problems that can be readily solved in practice. Numerical examples of the design framework showed promising gains compared to other suboptimal methods.

7. REFERENCES

[1] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Transactions on Signal Processing*, vol. 50, no. 5, pp. 1051 – 1064, May 2002.

[2] D. P. Palomar, J. M. Cioffi, and M. A. Lagunas, "Joint Tx-Rx beamforming design for multicarrier MIMO channels: a unified framework for convex optimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 9, pp. 2381 – 2401, Sept. 2003.

[3] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*. CRC Press, Aug. 2001.

[4] S.A. Vorobyov, A.B. Gershman, and Zhi-Quan Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Transactions on Signal Processing*, vol. 51, no. 2, pp. 313 – 324, Feb. 2003.

[5] A. Pascual Iserte, A.I. Perez-Neira, and M.A. Lagunas Hernandez, "Exploiting transmission spatial diversity in frequency selective systems with feedback channel," in *IEEE International Conference on Acoustics, Speech, and Signal Processing*, April 2003, vol. 4, pp. IV – 85–8.

[6] F. Rey, M. Lamarca, and G. Vazquez, "Robust power allocation algorithms for MIMO OFDM systems with imperfect CSI," *IEEE Transactions on Signal Processing*, vol. 53, no. 3, pp. 1070 – 1085, March 2005.

[7] M. Kiessling and J Speidel, "Statistical prefilter design for MIMO ZF and MMSE receivers based on majorization theory," in *Proceedings IEEE International Conference on Acoustics, Speech, and Signal Processing*, May 2004, vol. 2, pp. 313–16.

[8] E.A. Jorswieck, A. Sezgin, H. Boche, and E. Costa, "Optimal transmit strategies in MIMO ricean channels with MMSE receiver," in *Proceedings IEEE Vehicular Technologe Conconcerence fall*, Los Angeles, USA, Sept. 2004.

[9] S.A. Jafar and A. Goldsmith, "Transmitter optimization and optimality of beamforming for multiple antenna systems," *IEEE Transactions on Wireless Communications*, vol. 3, no. 4, pp. 1165 – 1175, July 2004.

[10] H. Sampath and A. Paulraj, "Linear precoding for space-time coded systems with known fading correlations," *IEEE Communications Letters*, vol. 6, no. 6, pp. 239–241, June 2002.

[11] M. Vu and A. Paulraj, "Optimal linear precoders for MIMO wireless correlated channels with non-zero mean in space-time coded systems," *IEEE Transactions on Signal Processing*, 2005, to appear.

[12] A. W. Marshall and I. Olkin, *Inequalities: Theory of Majorization and Its Applications*, Academic Press, New York, 1979.

[13] D. P. Palomar, M. A. Lagunas, and J. M. Cioffi, "Optimum linear joint transmit-receive processing for MIMO channels with QoS constraints," *IEEE Transactions on Signal Processing*, vol. 52, no. 5, pp. 1179 – 1197, May 2004.

[14] G. Jöngren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Transactions on Information Theory*, vol. 48, no. 3, pp. 611 – 627, March 2002.

[15] S. Zhou and G. B. Giannakis, "Optimal transmitter eigenbeamforming and space-time block coding based on channel mean feedback," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2599–2613, Oct. 2002.

[16] C. G. Khatri, "Classical statistical analysis based on a cetrain multivariate complex Gaussian distribution," *Annals of Mathematical Statistics*, vol. 36, no. 1, pp. 98–144, Feb 1965.

[17] A.T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Annals Mathematical Statistics*, vol. 35, pp. 475 – 501, 1964.

[18] J.J.J. Roux and E.L. Raath, "Generalized Laguerre series forms of Wishart distributions," *South African Statistical Journal*, vol. 7, pp. 23 – 34, 1973.

[19] H.S. Steyn and J.J.J. Roux, "Approximations for the noncentral Wishart distribution," *South African Statistical Journal*, vol. 6, pp. 165 – 173, 1972.

[20] X. Zhang, D. P. Palomar, and B. Ottersten, "Robust design of linear MIMO transceiver for low SNR," in *Proceedings Asilomar Conference on Signals, Systems & Computers*, Oct. 2005.

[21] X. Zhang, D. P. Palomar, and B. Ottersten, "Robust design of linear MIMO transceivers," *IEEE Journal on Selected Areas in Communications*, Sept. 2005, Submitted.

[22] P. Viswanath and V. Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1984 – 1991, Sept. 1999.