On the Outage Properties of Adaptive Network Coded Cooperation (ANCC) in Large Wireless Networks

Xingkai Bao and Jing Li (Tiffany)
Department of Electrical and Computer Engineering, Lehigh University, Bethlehem, PA 18015
Email: {xib3, jingli}@ece.lehigh.edu

***Abstract*— Adaptive-network-coded-cooperation (ANCC) is an efficient user cooperation scheme proposed for large wireless networks comprising a host of terminals communicating with a common destination. By matching code graphs with instantaneous network graphs in a distributed and adaptive manner, the protocol enables network coding to be exploited in networks with unreliable channels and changing topologies. This paper analyzes the outage behavior of ANCC when the number of terminals trends to infinity. A threshold phenomenon is revealed which demonstrates that an arbitrarily small outage can be achieved with a sufficiently large network as long as the channel conditions are above a certain threshold. Comparison with the existing cooperation schemes shows that ANCC achieves a substantial gain of 30 dB over repetition, and falls only 1 dB short of space-time-coded-cooperation (but obviating the need for stringent inter-user synchronization).**

***Index Terms*— User cooperation, outage, network coding.**

I. Introduction

User cooperation in large wireless networks to combat (slow) channel fading is gaining increasing interests [1][2][3]. Consider a relay network comprising a host of transmitters communicate wirelessly to a common destination. A straightforward way to achieve cooperative diversity is to adapt the cooperation schemes developed for the basic three-node scenario, such as repetition-cooperation and space-time-coded-cooperation (STCC), to the multiple-node scenario [2]. However, these schemes do not scale well, and are therefore inefficient, expensive or wasteful to operate in a large network. For example, the repetition scheme discussed in [2] consumes a large bandwidth overhead and becomes increasingly bandwidth inefficient as the network size grows. Space-time-coded-cooperation requires coordinated control and stringent inter-user synchronization at the symbol level [2], which is technology challenging especially among a large number of distributed terminals.

Recently, a practical and efficient user cooperation scheme, termed *adaptive-network-coded-cooperation* (ANCC), is proposed for large relay networks [3]. The protocol imports the concept of network coding [4][5], a technique originally developed for the computer routing problem, into wireless user cooperation. The network coding literature uses the prevailing assumption of *lossless* networks. However, wireless networks consisting of randomly faded channels are inherently unreliable. In cases when a relay fails to retrieve a packet (known as *symbol* in network coding literatures) that is needed for its designated coding operation, *fixed* coding schemes can not be performed. ANCC solves this problem by coupling *networks-on-graphs*, i.e. instantaneous network topologies described in graphs, with the well-known class of *codes-on-graphs*, i.e. low-density parity-check (LDPC) codes[6] and LDPC-like codes. [3] details the key idea of ANCC by demonstrating how a directed network graph can be efficiently transformed to a bipartite code graph. Below we brief the same idea using matrices. For convenience, we use "packet" and "symbol" interchangeably.

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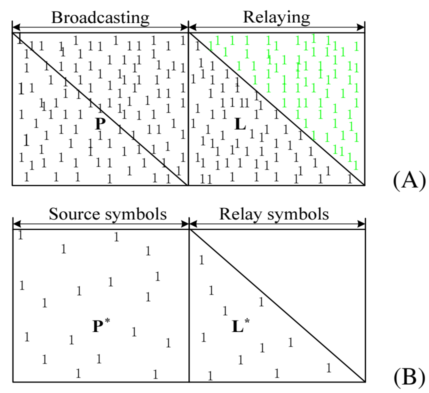

Fig. 1. (A) Matrix representing link connectivity between transmitters. Right upper triangular of $\mathbf{L}$ is marked in green to denote its irrelevance to code construction. (B) Parity check matrix of the LT-LDPC network code thinned from (A).

Consider m terminals communicating to a common destination using time-division multiplexing. The wireless transmission can be divided into two phases: the broadcast phase and the relay phase, and the symbols transmitted at these phases are referred to as source-symbols and relay-symbols, respectively. Let binary matrix $\mathbf{P}_{m\times m} = \{p_{i,j}\}$ describe the link connectivity among the senders at the broadcast phase, where $p_{i,j} = 1$ represents a successful reception of user j's source-symbol by user i. Without loss of generality, $p_{i,i} = 1$, since a node knows its own source-symbol. Similarly, let lower-triangular matrix $\mathbf{L}_{m\times m} = \{l_{i,j}\}$ describe the link connectivity at the relay phase, where $l_{i,j} = 1$, $i \geq j$, represents a successful reception of node j's relay symbol by node i. (How relay-symbols are computed should become clear shortly.) We let $l_{i,j} = 0$ for all $i < j$, since a relay-symbol from user j makes relevance to user i only if it arrives before i performs its relaying operation. Interestingly, these two matrices describing the instantaneous network topology, when placed together, can be naturally taken as the parity check matrix $\mathbf{H}$ of a $(2m, m)$ linear network code. As illustrated in Figure 1(A), m source-symbols correspond to the m columns in the left matrix $\mathbf{P}$, m relay-symbols correspond to the m columns in the right matrix $\mathbf{L}$, and the ith relay-symbol forms the check sum of the source-symbols and relay-symbols user i receives prior to its turn of relaying. In Figure 1(A), we marked the upper right triangular in $\mathbf{L}$ in light green to denote the fact that despite the existence of link connectivities therein, they are not of interest to network code construction. The random code resulted directly from the network topology is decodable using the message-passing algorithm, but the high density of "1"s and the existence of many short cycles will make decoding rather inefficient and suboptimal. To rectify this, $\mathbf{H}$ is required to be thinned before serving as the network code. Thinning can be accomplished in a distributed and random manner by asking each terminal randomly to drop some or most of the symbols it receives and use only a few to compute its parity-symbol. The resulting code, shown in Figure 1(B), now becomes an instance from an ensemble of random lower-triangular LDPC (LT-LDPC) codes, which enjoy low encoding and decoding complexity and which offer high performances through message-passing decoding. We note that the order of the terminals may make a difference to the resulting network code at a specific time instance where the network assumes a specific instantaneous topology, but the impact is minimal, if at all, on the average. Finally, for ANCC to be practical, a bit map needs to be included in each relay-symbol to signal to the destination how the checks in $\mathbf{H}$ are formed, and an adaptive decoder implemented using, for example, software defined radio (SDR) is required at the destination.

This paper studies the outage behavior of ANCC when the number of terminals trends to infinity, which provides a theoretical support to the excellent performance first discovered in [3] by simulations. We reveal a threshold phenomenon that an arbitrarily small outage can be achieved with a sufficiently large network as long as the channel conditions are above a certain threshold. For comparison purpose, we also analyze the repetition-cooperation and space-time-coded-cooperation schemes.

II. System Model

We consider all the channels in this paper follow frequency non-selective slow fading model with channel fading coefficient α and addictive channel noise Z. The channel fading coefficient α captures the effects of path loss, shadowing and frequency non-selective fading. We model it as zero-mean, independent, circularly symmetric complex Gaussian random variables with variances $1/\lambda$, so that the magnitude $|\alpha|$ is Rayleigh distributed, and the channel power $|\alpha|^2$ is exponentially distributed with parameter λ, mean $1/\lambda$ and probability density function (pdf):

$$p_{|\alpha|^2}(y) = \lambda e^{-\lambda y}. \qquad (y > 0) \tag{1}$$

The additive channel noise Z captures the receiver noise and other interference in the system. We model it as a complex Gaussian random variable with variance N_0. Furthermore, assume the same transmit power constraint, P, for all the terminals. The transmit signal-to-noise ratio (SNR) γ is defined as

$$\gamma = \frac{P}{N_0}. \tag{2}$$

III. Mutual Information

In this section, we analyze and compare the mutual information for repetition-cooperation, space-time-coded-cooperation and adaptive-network-coded-cooperation. We assume that the channel fading coefficient is only known by the receiver, but not by the transmitter. Let *sent-set*, $S(i)$, be the set of terminals having successful receptions of terminal i's source symbol. Let *received-set*, $R(i)$, be the set of broadcasting symbols successfully received by terminal i. We use subscript (i, j) to denote the channel from terminal i to terminal j, and use (i, d) to represent the channel from terminal i to the destination, where $i, j \in \{1, 2, ..., m\}$. For convenience, the analysis below considers equal power allocation between the broadcast and the relay phase and equal time duration for all the symbols. Nevertheless, since different cooperation protocols require different numbers of time slots and different relaying strategies, caution must be taken in normalizing the transmit SNR and mutual information per unit bandwidth per unit time.

A. Repetition-Based Cooperations

We consider two types of repetition in this paper: full-repetition-cooperation (FREP) and partial-repetition-cooperation (PREP). In FREP, each terminal r is given $m-1$ time slots to repeat all the other terminals' symbols at the relay phase. In effect, it can at the best relay for $|R(r)|$ terminals (using equal transmit power)[1], and stay idle in other time slots. On the other hand, in PREP, each terminal r helps only one other terminal i, randomly selected from $R(i)$. Hence, on average, each source symbol is repeated by one relay.

The mutual information of FREP between terminal i and the destination can be computed using the classic Shannon formula whose equivalent SNR is the sum of the instantaneous SNRs of the home channel (i, d) and all the relay channels (r, d), where $r \in S(i)$. Mathematically, it can be written as [2]

$$I_{frep} = \frac{1}{m} \log \left(1 + \frac{\gamma}{2}|\alpha_{i,d}|^2 + \sum_{r \in S(i)} \frac{\gamma}{2|R(r)|}|\alpha_{r,d}|^2 \right). \tag{3}$$

The mutual information is divided by m because m^2 time slots are assigned to m terminals, and the transmit SNR γ is divided by $|R(r)|$ because terminal r needs to share its relay power among all the symbols in its received-set.

Similar, the (average) mutual information of PREP can be computed using the Shannon formula with a sum SNR for the home channel and a single relay channel:

$$I_{prep} = \frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2+\frac{\gamma}{2}|\alpha_{r,d}|^2\right). \qquad (r \in S(i)) \tag{4}$$

Since a complete round of cooperation consists of a total of $2m$ time slots, the mutual information and the transmit SNR γ is factored by $1/2$.

B. Space-Time-Coded-Cooperation (STCC)

For space-time-coded-cooperation, at the relay phase, terminal i's date packet is re-encoded using an optimal $|S(i)|$-by-1 space-time code (STC) and relayed simultaneously by all the terminals r where $r \in S(i)$ in one time slot. Since $2m$ time slots are spent altogether for m terminals, and the mutual information should be adjusted by 1/2.

[1]For a set, $|.|$ denotes its cardinality, this should not be confused with the usual notation for an absolute value.

[2]Unless mentioned specially, all the $\log(.)$ functions in this paper have base 2.

As pointed in [2], because of space-time transmission, the mutual information achieved in the relay phase can be considered as the Shannon capacity with equivalent SNR being the sum of the instantaneous SNRs from terminal r for all $r \in S(i)$. Since a relay terminal $r \in S(i)$ needs to participate in $|R(r)|$ space-time transmissions, it shares its relay power among all $|R(i)|$ space-time symbols. Assuming there exists capacity-approaching $|S(i)|$-by-1 STC for all values of $|S(i)| \leq m$, we have:

$$I_{stc} = \frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big)+\frac{1}{2}\log\left(1+\frac{\gamma}{2}\sum_{r\in S(i)}\frac{|\alpha_{r,d}|^2}{|R(r)|}\right). \quad (5)$$

C. Adaptive-Network-Coded-Cooperation (ANCC)

For ANCC, at the relay phase, all the source symbols are coded into a single codeword of some network code, and each terminal relays a different parity symbol of the codeword. Thus, $2m$ time slots are used for m terminals, and the mutual information and the transmit SNR γ need be divided by 2.

We assume that the random network code in use is optimal (LT-LDPC codes are one class of near-optimal codes which can perform very closely to the capacities [7]). Thus, the total mutual information got at the second phase should be the sum of the Shannon formula with all the terminals' instantaneous SNR. Since all the terminals share the total mutual information in the second phase, for each terminal, the mutual information should be divided by m. Therefore, we arrive at the following result:

$$I_{ancc}=\frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big)+\frac{1}{2m}\sum_{r=1}^{m}\log\Big(1+\frac{\gamma}{2}|\alpha_{r,d}|^2\Big). \quad (6)$$

IV. Outage probability

Outage probability, $\Gamma(R)$, is defined as the probability that a system fails to support an instantaneous information rate of R. The mutual information formulated in the previous subsection can be used to evaluate the outage performance for (large) wireless networks, but a closed-form expression is intractable, and the numerical results depend on the size of the network m and the conditions of the inter-user channels (i.e. $S(i)$ and $R(r)$). Below we focus on the asymptotic case when the network size increases without bound. Under the assumption of $m \to \infty$, $|S(i)|$ and $|R(r)|$ tend to be equal for all $i, r \in \{1, 2, ...m\}$, and they both trend to infinity. The outage computation for different schemes is therefore greatly simplified and can eventually arrive at a closed form. Such asymptotic analysis is useful since it sheds insight into the scalability of each scheme and forecasts the performance for large networks.

A. Repetition based protocol

When $m \to \infty$, $|S(i)| = |R(r)|$ and the mutual information of full-repetition-cooperation becomes

$$\begin{aligned}\lim_{m\to\infty} I_{frep} &= \lim_{m\to\infty}\frac{1}{m}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2+\sum_{r\in S(i)}\frac{\gamma}{2|R(r)|}|\alpha_{r,d}|^2\right),\\ &\leq \lim_{m\to\infty}\frac{1}{m}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2+\frac{\gamma}{2}\max_{r\neq i}|\alpha_{r,d}|^2\right)=0. \quad (7)\end{aligned}$$

It follows that the asymptotic outage probability for FREP is

$$\Gamma_{frep}(R) = \lim_{m\to\infty} Pr[I_{frep} < R] = 1, \quad (8)$$

The mutual information of partial-repetition-cooperation when $m \to \infty$ can be written as

$$\lim_{m\to\infty} I_{prep}=\frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2+\frac{\gamma}{2}|\alpha_{r,d}|^2\Big), \qquad (r \in S(i)). \quad (9)$$

The asymptotic outage probability can then be written as

$$\begin{aligned}\Gamma_{prep}(R) &= \lim_{m\to\infty} Pr[I_{prep} < R]\\ &= Pr\left[\frac{1}{2}\log(1+\frac{\gamma}{2}|\alpha_{i,d}|^2+\frac{\gamma}{2}|\alpha_{r,d}|^2) < R\right],\\ &= Pr\left[|\alpha_{i,d}|^2+|\alpha_{r,d}|^2 < \theta_1\right], \quad (10)\end{aligned}$$

where

$$\theta_1 = \frac{2(2^{2R}-1)}{\gamma}. \quad (11)$$

The above equation can be further simplified to

$$\begin{aligned}\Gamma_{prep}(R) &= \int_0^{\theta_1}\int_0^{\theta_1-y_2}\lambda_{r,d}e^{-\lambda_{r,d}y_1}dy_1\lambda_{r,d}e^{-\lambda_{i,d}y_2}dy_2,\\ &= \begin{cases}1-\exp(-\lambda\theta_1)-\theta_1\lambda\exp(-\lambda\theta_1), & (\lambda_{r,d}=\lambda_{i,d}=\lambda)\\ 1-\frac{\lambda_{r,d}\exp(-\lambda_{i,d}\theta_1)-\lambda_{i,d}\exp(-\lambda_{r,d}\theta_1)}{\lambda_{r,d}-\lambda_{i,d}}. & (\lambda_{r,d}\neq\lambda_{i,d})\end{cases} \quad (12)\end{aligned}$$

where $\lambda_{i,d}$ is the parameter of the exponentially distributed channel power $|\alpha_{i,d}|^2$.

B. Space-time-coded-Cooperation (STCC)

For STCC, we can write the asymptotic mutual information as

$$\lim_{m\to\infty} I_{stc}=\frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big)+\frac{1}{2}\log\left(1+\frac{\gamma}{2}\lim_{m\to\infty}\sum_{r\in S(i)}\frac{|\alpha_{r,d}|^2}{|R(r)|}\right).$$

When m trends to infinity, we have $|S(i)| = |R(r)| \to \infty$.

If we assume that all the user-destination channels are independent and identically distributed (i.i.d.) with channel power $|\alpha|^2$ following the pdf in (1), then

$$\lim_{m\to\infty}\sum_{r\in S(i)}\frac{|\alpha_{r,d}|^2}{|R(r)|} = E\Big[|\alpha|^2\Big] = \int_0^\infty y\,\lambda\,e^{-\lambda y}\,dy = \frac{1}{\lambda}. \quad (13)$$

Thus we have the asymptotic mutual information

$$\lim_{m\to\infty} I_{stc} = \frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\right)+\frac{1}{2}\log\left(1+\frac{\gamma}{2\lambda}\right), \quad (14)$$

and the outage probability

$$\begin{aligned}\Gamma_{stc}(R) &= \lim_{m\to\infty} Pr[I_{stc} < R]\\ &= Pr\left[\frac{1}{2}\log\left(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\right)+\frac{1}{2}\log\left(1+\frac{\gamma}{2\lambda}\right) < R\right],\\ &= Pr[\,|\alpha_{i,d}|^2 < \theta_2], \quad (15)\end{aligned}$$

where

$$\theta_2 = \frac{2^{2R-\log\left(1+\frac{\gamma}{2\lambda}\right)}-1}{\gamma/2}. \quad (16)$$

Therefore,

$$\Gamma_{stc}(R) = \int_0^{\theta_2}\lambda_{i,d}e^{-\lambda_{i,d}y}dy = 1-e^{-\lambda_{i,d}\theta_2}. \quad (17)$$

C. Adaptive-Network-Coded-Cooperation (ANCC)

When $m \to \infty$, the mutual information of ANCC is

$$\lim_{m\to\infty} I_{ancc} = \frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big) + \lim_{m\to\infty}\frac{1}{2m}\sum_{r=1}^{m}\log\Big(1+\frac{\gamma}{2}|\alpha_{r,d}|^2\Big).$$

Assuming i.i.d. user-destination channels and following the same line of thinking as STCC, we have

$$\begin{aligned}
&\lim_{m\to\infty} I_{ancc}\\
&= \frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big) + \frac{1}{2}E\Big[\log(1+\frac{\gamma}{2}|\alpha|^2)\Big],\\
&= \frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big) + \frac{1}{2}\int_0^\infty \log(1+\frac{\gamma}{2}y)\lambda e^{-\lambda y}\,dy,\\
&= \frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big) + \frac{1}{2\ln(2)}\exp\Big(\frac{2\lambda}{\gamma}\Big)\,\mathrm{Ei}\Big(\frac{2\lambda}{\gamma}\Big), \quad (18)
\end{aligned}$$

where Ei(.) is the *exponential-integral function* defined as

$$\mathrm{Ei}(x) \triangleq \int_x^\infty \frac{e^{-t}}{t}dt. \qquad (x>0) \qquad (19)$$

Hence, the asymptotic outage probability of ANCC is given by

$$\begin{aligned}
\Gamma_{ancc}(R) &= \lim_{m\to\infty} Pr[I_{ancc} < R]\\
&= Pr\Big[\frac{1}{2}\log\Big(1+\frac{\gamma}{2}|\alpha_{i,d}|^2\Big) + \frac{e^{2\lambda/\gamma}}{2\ln(2)}\mathrm{Ei}\Big(\frac{2\lambda}{\gamma}\Big) < R\Big],\\
&= Pr\left[|\alpha_{i,d}|^2 < \theta_3\right] = \int_0^{\theta_3} \lambda_{i,d}\exp\left(-\lambda_{i,d}\, y\right) dy,\\
&= 1-\exp\left(-\lambda_{i,d}\,\theta_3\right). \qquad (20)
\end{aligned}$$

where

$$\theta_3 = \frac{2^{2R-\exp(\frac{2\lambda}{\gamma})\mathrm{Ei}(\frac{2\lambda}{\gamma})/2\ln(2)}-1}{\gamma/2}. \qquad (21)$$

V. Numerical Results

Following the closed-form outage expressions of full-repetition-cooperation, partial-repetition-cooperation, space-time-coded-cooperation and adaptive-network-coded-cooperation in (8), (12), (17) and (20), we evaluate numerically their respective asymptotic outage probabilities. Figure 2 presents a case where the spectral efficiency $R = 0.5$ and all the user-destination channels are i.i.d. with parameter $\lambda_{i,d} = 1$. For comparison purpose, we also plot the outage for direct transmission (i.e. no cooperation), which is given by

$$\Gamma_{direct} = 1-\exp(-\lambda_{i,d}\theta_4), \qquad \text{where } \theta_4 = \frac{2^{\mathrm{R}}-1}{\gamma}. \qquad (22)$$

Due to the poor bandwidth efficiency, full-repetition-cooperation gets an asymptotic outage probability of 1 (for any R), which means that the transmission rate per user goes to zero asymptotically. Direct transmission has a diversity order of 1, so its outage probability decreases only linearly with the increase of channel SNR. Partial-repetition-cooperation outperforms direct transmission, but attains a diversity order of no more than 2 despite the existence of infinite terminals. On the other hand, space-time-cooperation and adaptive-coded-cooperation both achieve an asymptotically unbounded diversity order. Their outage probabilities drop extremely fast and exhibit an interesting threshold phenomenon. For the case shown in Figure 2, as long as the average channel SNR γ exceeds 3.010dB and 3.998 dB (when θ_2 and θ_3 equal 0), STCC and ANCC respectively achieves asymptotically zero outage. Further, STCC and ANCC outperform repetition-cooperation by more than 27dB at an outage of 10^{-6}. We should note that ANCC achieves this remarkable performance without stringent symbol-level inter-user synchronization which is required for STCC.

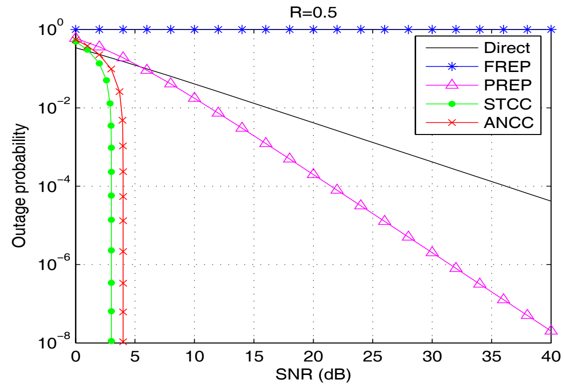

Fig. 2. Asymptotic outage probabilities.

VI. Conclusions

User cooperation in large wireless networks is a potentially fruitful research area, which can bring remarkable performance gain. However, immigrating the existing cooperation schemes developed for the basic three-terminal scenario to the many-terminal scenario is either inefficient or impractical as the network size increases. Adaptive-network-coded-cooperation opens a new direction to perform large-scale wireless user cooperation in an adaptive, de-centralized and efficient manner.

The simplicity, practicality and efficiency of ANCC were demonstrated by simulations in [3]. This paper provides a theoretical support for its excellent performance by means of outage analysis. A closed-form outage expression is derived for ANCC as well as other existing schemes including space-time-coded-cooperation and full/partial-repetition-cooperation. A threshold phenomenon for ANCC (and STCC) is also revealed, which demonstrates the possibility to completely eliminate the system outage. Considering that ANCC performs only marginally worse than STCC but obviates the need for stringent inter-user synchronization, it is therefore a very attractive protocol in practice.

References

[1] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," accepted by *IEEE Trans. Inform. Theory*.

[2] J. N. Laneman, and G. W. Wornell, "Distributed Space-Time-Coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans Inform. Theory,* vol. 49, NO. 10, Oct 2003, pp. 2415-2425.

[3] X. Bao, and J. Li, "Matching Code-on-Graph with Network-on-Graph: Adaptive Network Coding for Wireless Relay Networks," *Proc. Allerton Conf. on Commun., Control and Computing*, Urbana Champaign, IL, Sept. 2005. http://www.ece.lehigh.edu/∼jingli/publication.html

[4] R. Ahlswede, N. Cai, S.-Y.R. Li and R.W. Yeung, "Network information flow", *IEEE Trans. Inform. Theory*, vol. 46, pp. 1204-1216, 2000.

[5] R. Koetter, and M. Medard, "An algebraic approach to network coding," *IEEE Trans. Networking*, Oct. 2003.

[6] R. G. Gallager, *Low Density Parity-Check Codes*, *MIT Press*, Cambridge, MA, 1963.

[7] T. J. Richardson, R. L. Urbanke, "Efficient encoding of low-density parity-check codes," *IEEE Trans. Inform. Theory*, Feb 2001, pp. 638-656.