JOINT POWER ALLOCATION FOR NONREGENERATIVE MIMO-OFDM RELAY LINKS

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ABSTRACT

We consider a two-hop MIMO-OFDM communication scheme with a source, a relay, and a destination. The relay is assumed to be nonregenerative (or amplify-and-forward (AF)). We assume channel state information at each transmitter (CSIT), source and relay. We present a jointly optimized power allocation (PA) over the subchannels in space and frequency domain at source and relay with a joint transmit power constraint. The PA is based on a high SNR approximation of the SNR expression at the destination. In the low SNR regime, the performance is still very tight to the optimal rate.

The presented PA can be computed with low computational complexity and achieves a considerable performance gain compared to a uniform PA at source and relay. To further enhance the performance of the considered scheme, all subchannels, in space and frequency domain, of the source to relay and relay to destination channel are paired according to their actual magnitude.

1. INTRODUCTION

Cooperative relaying strategies have become a major topic in the wireless research community. First research results on relay channels were obtained in the seventies in [1–3]. The interest in this topic was re-initiated recently by the seminal papers [4–6] and triggered a large amount of work in this area.

Most of the literature available today considers frequency-flat fading. In [7, 8] and [9] optimal PAs between single antenna source and relay (regenerative and nonregenerative) are discussed for the case that both share a total amount of transmit power over the two time-slots required for relaying. In [10] the optimal gain matrix for a nonregenerative MIMO relay which optimizes the mutual information for a given uniform PA at the source is presented.

The case of cooperative relaying in frequency-selective fading channels is much less examined so far. In [11], the authors determine the PA for multiple orthogonal nonregenerative relays (which is the same as having one relay using OFDM) maximizing the average SNR of the maximum-ratio combiner at the destination node. For single antenna nonregenerative OFDM relaying the optimal PA at the relay (source) that maximizes the instantaneous rate for a given source (relay) PA is presented in [12]. Furthermore, it is shown that alternate, separate optimization of source and relay PA converges to the solution of the joint PA optimization at source and relay with separate power constraint. In [13] the throughput of OFDM and OFDMA networks consisting of one source/destination pair and multiple relays is examined. In the case of OFDM only one amplification gain is used for all subcarriers at the nonregenerative relay. Therefore, the throughput is not optimized with respect to the frequency-selective channel. In the case of OFDMA only one nonregenerative relay is assigned to one subcarrier, which results in an optimization problem that can be solved by integer programming.

In this paper we present a *joint optimization* of source and relay transmit power with a *joint power constraint* for a MIMO-OFDM relay link. By means of CSIT we decouple the equivalent two-hop MIMO channel between source and destination into orthogonal channels. We present a PA that is an approximation of the optimal PA. It can be computed with very low computational complexity and approaches the optimal rate in the high SNR regime. In the low SNR regime, the performance is still very tight to the optimum. To further enhance the performance of the considered scheme, all subchannels, in space and frequency domain, of the source to relay and relay to destination channel are paired according to their actual magnitude.

The remainder of the paper is organized as follows. In the next section the system model is introduced. In Section 3 we present our joint PA in space and frequency domain with joint power constraint. Performance results are presented in Section 4. Conclusions are given in the last section.

2. SYSTEM MODEL

We consider a two-hop relay link consisting of one source/destination pair and one nonregenerative relay. The nodes are equipped with multiple antennas. For reasons of simplicity we assume that the number of antennas is equal to N_{a} at all three nodes. For broadband communication OFDM is applied with a cyclic prefix that is at least as long as the channel impulse responses. Thus, the available bandwidth is divided into N_{fft} subcarriers. In each subcarrier the channel is assumed to be frequency-flat. The channel matrix from source to relay and from relay to destination within the k-th subcarrier is denoted by $\mathbf{H}_{1,k}$ and $\mathbf{H}_{2,k}$, respectively. The singular value decomposition of the channel matrices is given by

$$\mathbf{H}_{1,k} = \mathbf{U}_{1,k}\mathbf{S}_{1,k}\mathbf{V}_{1,k}^H, \tag{1}$$

$$\mathbf{H}_{2,k} = \mathbf{U}_{2,k}\mathbf{S}_{2,k}\mathbf{V}_{2,k}^H, \tag{2}$$

respectively. Further, we assume that the destination is not able to receive the signal from the source directly, which may result from high shadowing between both nodes.

The source multiplies its transmit vector in the k-th subcarrier by $\mathbf{V}_{1,k}$, whereas the relay multiplies the received vector by $\mathbf{U}_{1,k}^H$. Thus, the equivalent received signal at relay if the vector $\mathbf{x}_k$ is transmitted can be expressed as

$$\mathbf{r}_k = \mathbf{S}_{1,k}\mathbf{x}_k + \mathbf{v}_k, \tag{3}$$

where $\mathbf{v}_k \sim \mathcal{CN}(0, \sigma_\mathrm{r}^2 \mathbf{I}_{N_\mathrm{a}})$ is the noise contribution at the relay. The transmit power of the source in the k-th subcarrier on the n-th spatial subchannel is $\mathrm{E}\left\{|x_m|^2\right\} = P_{\mathrm{s},m}$, where the subscript $m = (k-1)N_\mathrm{a} + n$ with $1 \le n \le N_\mathrm{a}$ is introduced to keep the total number of subscripts small.
Prior to the retransmission of the signal the relay multiplies the vector $\mathbf{r}_k$ with the diagonal matrix

$$\mathbf{G}_k = \mathrm{diag}\left[g_{k,1}, \dots, g_{k,N_\mathrm{a}}\right] \tag{4}$$

and the matrix $\mathbf{V}_{2,k}$, i.e., $\mathbf{s}_k = \mathbf{V}_{2,k}\mathbf{G}_k\mathbf{r}_k$. If the destination multiplies the incoming signals from the relay by $\mathbf{U}_{2,k}^H$ the equivalent channel between relay and destination is also diagonal. Therefore, the destination receives

$$\mathbf{y}_k = \mathbf{S}_{2,k}\mathbf{s}_k + \mathbf{w}_k, \tag{5}$$

$$= \mathbf{S}_{2,k}\mathbf{G}_k\mathbf{S}_{1,k}\mathbf{x}_k + \mathbf{S}_{2,k}\mathbf{G}_k\mathbf{v}_k + \mathbf{w}_k, \tag{6}$$

where $\mathbf{w}_k \sim \mathcal{CN}(0, \sigma_\mathrm{d}^2 \mathbf{I}_{N_\mathrm{a}})$ is the noise contribution at the destination. The relay chooses the amplification factor $g_{k,n} \equiv g_m$ as

$$g_m = \sqrt{\frac{P_{\mathrm{r},m}}{P_{\mathrm{s},m} \cdot \lambda_{1,m} + \sigma_\mathrm{r}^2}}, \tag{7}$$

where $\lambda_{1,m}$ denotes the squared singular value (i.e., eigenvalue) of the source to relay channel matrix in subchannel $m = (k-1)N_\mathrm{a} + n$. Therefore, by means of this amplification factor the relay transmit power on this subchannel is ensured to be $P_{\mathrm{r},m}$.

It can be seen that the two-hop MIMO channel between source and destination has been decoupled into N_a orthogonal SISO channels within the k-th subcarrier. Therefore, overall there are $M = N_\mathrm{fft}N_\mathrm{a}$ orthogonal subchannels between source and destination in which the transmit power values $P_{\mathrm{r},m}$ and $P_{\mathrm{s},m}$ can be chosen such that the rate of this subchannel is maximized.

The signal to noise ratio (SNR) at the destination in subchannel m is

$$\rho_m = \frac{P_{\mathrm{s},m}\lambda_{2,m}g_m^2\lambda_{1,m}}{\sigma_\mathrm{d}^2 + \sigma_\mathrm{r}^2 g_m^2 \lambda_{2,m}} = \frac{P_{\mathrm{s},m}a_m \cdot P_{\mathrm{r},m}b_m}{1 + P_{\mathrm{s},m}a_m + P_{\mathrm{r},m}b_m}, \tag{8}$$

where $a_m = \frac{\lambda_{1,m}}{\sigma_\mathrm{r}^2}$ and $b_m = \frac{\lambda_{2,m}}{\sigma_\mathrm{d}^2}$. $\lambda_{2,m}$ denotes the m-th eigenvalue of the relay to destination channel matrix. The instantaneous rate per complex dimension of the communication between source and destination with the nonregenerative half-duplex relay is given by

$$C_\mathrm{I} = \frac{1}{2N_\mathrm{fft}} \sum_{m=1}^{M} \log_2\left(1 + \rho_m\right), \tag{9}$$

where the factor $1/2$ is due to the two time slots (channel uses) which are needed in this traffic pattern, whereas the factor $1/N_\mathrm{fft}$ is due to the number of subcarriers of the OFDM system.

2.1. Pairing of Subchannels

A higher performance in terms of mutual information can be achieved if the subchannels of both channels, source to relay and relay to destination, are paired according to the actual magnitude of the eigenvalues. It can be seen that it is optimal to pair the subchannels over space and frequency domain. The subchannel SNR at the destination (8) is approximately given by the harmonic mean of the single hop SNRs of first and second hop [14]. Due to the fact that the harmonic mean is always limited to the value of the smaller single hop SNR it is favorable to couple a strong first hop subchannel with a strong second hop subchannel and not with a weak one.

3. JOINTLY OPTIMIZED POWER ALLOCATION

In the following we want to jointly optimize the transmit PA at the source and relay over the M subchannels with respect to a joint power constraint at both nodes, i.e.,

$$\sum_{m=1}^{M} P_{\mathrm{s},m} + \sum_{m=1}^{M} P_{\mathrm{r},m} = \mathbf{1}^T\mathbf{p}_\mathrm{S} + \mathbf{1}^T\mathbf{p}_\mathrm{R} = P_\Sigma. \tag{10}$$

The transmit power values over the subchannels is thereby stacked are the vectors $\mathbf{p}_\mathrm{S} = \left[P_{\mathrm{s},1}, P_{\mathrm{s},2}, \dots, P_{\mathrm{s},M}\right]^T$ and $\mathbf{p}_\mathrm{R} = \left[P_{\mathrm{r},1}, P_{\mathrm{r},2}, \dots, P_{\mathrm{r},M}\right]^T$, respectively. The joint optimization problem can be stated as

$$\begin{aligned} \underset{\mathbf{p}_\mathrm{S}, \mathbf{p}_\mathrm{R}}{\text{maximize}} \quad & \frac{1}{2N_\mathrm{fft}} \sum_{m=1}^{M} \log_2\left(1 + \rho_m\right) \\ \text{subject to} \quad & \mathbf{1}^T\mathbf{p}_\mathrm{S} + \mathbf{1}^T\mathbf{p}_\mathrm{R} = P_\Sigma \\ & \mathbf{p}_\mathrm{S} \succeq 0 \\ & \mathbf{p}_\mathrm{R} \succeq 0. \end{aligned} \tag{11}$$

By means of the joint power constraint this optimization is capable of responding more efficiently to the relative path losses between source and relay and between relay and destination. If, e.g., the attenuation between source and relay is much smaller than between relay and destination, this optimization would give a higher fraction of the overall transmit power P_{Σ} to the relay.

The expression of the SNR in (8) is intractable to find an analytical solution for the optimization problem (11). Therefore, we use a high SNR approximation of (8) given by

$$\rho_m = \frac{P_{\mathrm{s},m} a_m \cdot P_{\mathrm{r},m} b_m}{P_{\mathrm{s},m} a_m + P_{\mathrm{r},m} b_m}. \quad (12)$$

Using the Karush-Kuhn-Tucker (KKT) conditions [15] we get the solution of the optimization problem (11) as

$$P_{\mathrm{s},m} = \frac{1}{1+\sqrt{\frac{a_m}{b_m}}} \left[\frac{1}{\nu} - \frac{\left(\sqrt{a_m} + \sqrt{b_m}\right)^2}{a_m b_m} \right]^+ \quad (13)$$

$$P_{\mathrm{r},m} = \frac{1}{1+\sqrt{\frac{b_m}{a_m}}} \left[\frac{1}{\nu} - \frac{\left(\sqrt{a_m} + \sqrt{b_m}\right)^2}{a_m b_m} \right]^+, \quad (14)$$

where $[x]^+ = \max\{0, x\}$. The Lagrange multiplier ν has to be chosen such that the sum power constraint $\mathbf{1}^T \mathbf{p}_{\mathrm{S}} + \mathbf{1}^T \mathbf{p}_{\mathrm{R}} = P_{\Sigma}$ is fulfilled. It can be computed very efficiently. By adding $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$ it turns out that the sum power of source and relay in subchannel m is

$$P_m = P_{\mathrm{s},m} + P_{\mathrm{r},m} = \left[\frac{1}{\nu} - \frac{\left(\sqrt{a_m} + \sqrt{b_m}\right)^2}{a_m b_m} \right]^+. \quad (15)$$

The calculation of ν such that $\sum_{m=1}^{M} P_m = P_{\Sigma}$ is done by the standard parallel gaussian waterfilling [16] procedure. Therefore ν can be easily computed in at most M steps. From the sum power P_m in subchannel m source and relay transmit power $P_{\mathrm{s},m}$ and $P_{\mathrm{r},m}$ are calculated. Note that the computational complexity is very low compared to waterfilling solutions presented in [10–12]. These PAs have the drawback that the Lagrange multiplier ν cannot be calculated easily in at most M steps. In [10, 12] ν has to be rather found iteratively with no reasonable bound on the number of iterations.

In a practical low mobility time division duplex (TDD) system this joint optimization could be computed by the relay. It is more meaningful to think of source and destination as two nodes communicating via the relay. Thus, the relay estimates both channels, first and second hop, by means of pilots from both nodes. The nodes only estimate the corresponding node to relay channel by means of pilots that are transmitted by the relay. After calculating the PA, the relay transmits the values to the corresponding node. Therefore, the signaling overhead which is necessary for this joint power optimization only consists of the dissemination of the PA values to the nodes, whereas it can be assumed that the pilots have to be transmitted for channel estimation anyway.

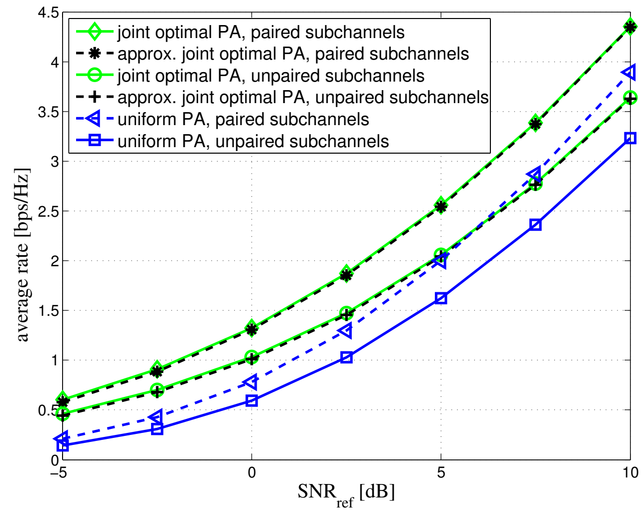

Fig. 1. Normalized average rate vs. $\mathrm{SNR}_{\mathrm{ref}}$; $N_{\mathrm{a}} = 4$; $N_{\mathrm{fft}} = 16$;

4. PERFORMANCE

In this section we present the performance of our presented approximation of the optimal joint PA for MIMO-OFDM nonregenerative relay links by means of Monte-Carlo simulations. We model our MIMO channels, first and second hop, as uncorrelated Rayleigh fading channels. The channel impulse response between receive antenna i and transmit antenna j is given by

$$h^{(i,j)}(t) = \sum_{l=0}^{L-1} h_l^{(i,j)} \cdot \delta(t - lT), \quad (16)$$

where $h_l^{(i,j)}$ is the complex amplitude of path l and L the number of channel taps. We assume $L = 4$ channel taps with the power delay profile such that attenuation of the tap $i = 1, .., 3$ compared to tap $i = 0$ is 3 dB, 6 dB, and 9 dB higher, respectively. The number of antennas at each node is $N_{\mathrm{a}} = 4$. The frequency domain channel is given by Fourier Transformation with $N_{\mathrm{fft}} = 16$ subcarriers. We assume that the distances between source and relay and relay and destination are equal, which results in the same average path loss in both hops.

We define a reference $\mathrm{SNR}_{\mathrm{ref}}$ as the SNR at the relay that would be achieved by a uniform PA at the source with half of the total sum power P_{Σ}. Further, we set $\sigma_{\mathrm{d}}^2 = \sigma_{\mathrm{r}}^2$.

In Fig. 1 the average rate of the joint optimization of source and relay transmit PA with joint power constraint versus the $\mathrm{SNR}_{\mathrm{ref}}$ is depicted, with and without subchannel pairing[1]. As reference also the average rate for a uniform PA over space and frequency dimension are depicted. In this case each

[1]The average rate curves for the optimal PA have been computed by means of numerical convex optimization tools.

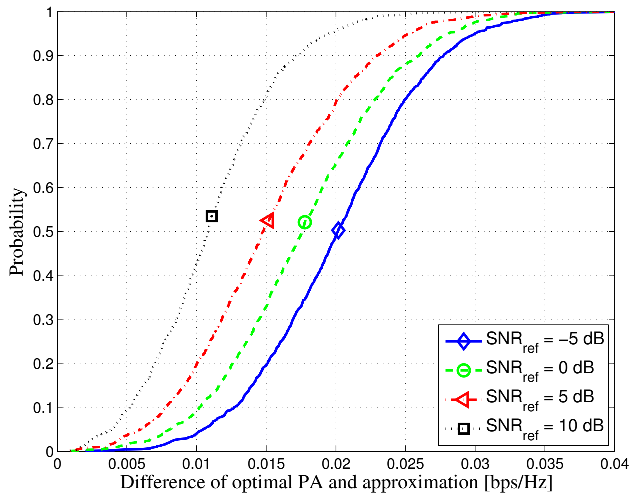

Fig. 2. CDF of difference between rate of joint optimal PA and approximated joint optimal PA for different $\mathrm{SNR}_{\mathrm{ref}}$; $N_{\mathrm{a}} = 4$; $N_{\mathrm{fft}} = 16$;

node, source and relay, spread the transmit power of $P_{\Sigma}/2$ uniformly over the antennas and subcarriers. Note, that in the case of the uniform PA without pairing of subchannels the knowledge of the relay to destination channel is not required at the relay, whereas with pairing of subchannels the relay has to know both channels.

It can be seen that our proposed joint PA with sum power constraint achieves a average rate that is very tight to the optimal PA over the whole SNR range. Furthermore, compared to the uniform PA without subchannel pairing it achieves a SNR gain of 5 dB and 3 dB at $\mathrm{SNR}_{\mathrm{ref}} = -5$ dB and 5 dB, respectively. This gain would surely increase for nonsymmetric channels (different pathloss of first and second hop), because in this case the fixed allocation of $P_{\Sigma}/2$ to source and relay is not a good choice.
The performance improvement due to pairing of subchannels is clearly visible. In the high SNR regime, a uniform PA with pairing achieves an average rate that is even superior to joint optimal PA without pairing.

In Fig. 2 CDFs of the difference between the rate of the joint optimal PA and the approximated joint optimal PA for different $\mathrm{SNR}_{\mathrm{ref}}$ are shown for the case of no pairing of subchannels. It can be seen that with increasing reference SNR the difference between the optimum and our proposed PA decreases.

5. CONCLUSIONS

We proposed a PA which approximates the joint optimal PA of source and relay with a joint transmit power constraint over the subchannels in space and frequency domain. It can be computed with low computational complexity and requires only a small signaling overhead. We showed that the performance is very tight to the optimum and achieves considerable performance gains compared to a uniform PA at source and relay. Furthermore, we showed that pairing the orthogonal subchannels in frequency and space domain improves the performance significantly.

6. REFERENCES

[1] Eric C. van der Meulen, “Three-terminal communication channels,” *Adv. Appl. Prob.*, vol. 3, pp. 120–154, 1971.

[2] H. Sato, “Information transmission through a channel with relay,” Tech. Rep. B76-7, The Aloha System, University of Hawai, Honolulu, Mar. 1976.

[3] Thomas M. Cover and Abbas A. El Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, Sept. 1979.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, “Increasing uplink capacity via user cooperation diversity,” in *Proc. IEEE Int. Symposium on Inf. Theory*, Cambridge, MA, Aug. 16–21, 1998, p. 156.

[5] Piyush Gupta and Piyush R. Kumar, “The capacity of wireless networks,” *IEEE Trans. Inform. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[6] J. Nicholas Laneman, David N.C. Tse, and Gregory W. Wornell, “An efficient protocol for realizing cooperative diversity in wireless networks,” in *Proc. IEEE Int. Symposium on Inf. Theory*, Washington DC, June 24–29, 2000, p. 294.

[7] Jingmei Zhang, Qi Zhang, Chunju Shao, Ying Wang, Ping Zhang, and Zhang Zhang, “Adaptive optimal transmit power allocation for two-hop non-regenerative wireless relaying system,” in *Proc. 60th IEEE Veh. Tech. Conf.*, Los Angeles, CA, Sep. 26–29, 2004.

[8] Qi Zhang, Jingmei Zhang, Chunju Shao, Ying Wang, Ping Zhang, and Rong Hu, “Power allocation for regenerative relay channels with rayleigh fading,” in *Proc. 59th IEEE Veh. Tech. Conf.*, Los Angeles, CA, Sep. 26–29, 2004.

[9] Mazen O. Hasna and Mohamed-Slim Alouini, “Optimal power allocation for relayed transmissions over rayleigh-fading channels,” *IEEE Trans. Wirel. Comm.*, vol. 3, no. 6, pp. 1999–2004, Nov. 2004.

[10] Olga Muñoz, Josep Vidal, and Adrian Agustín, “Non-regenerative MIMO relaying with channel state information,” in *Proc. ICASSP*, Philadelphia, PA, Mar. 18–23, 2005.

[11] Ivana Maric and Roy Yates, “Bandwidth and power allocation for cooperative strategies in gaussian relay networks,” in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov. 2004.

[12] Ingmar Hammerstroem and Armin Wittneben, “On the optimal power allocation for nonregenerative OFDM relay links,” in *Proc. ICC*, Istanbul, Turkey, Jun. 11–15, 2006, to appear.

[13] Guoqing Li and Hui Liu, “On the capacity of the broadband relay networks,” in *Proc. Asilomar Conf. Signals, Syst., Comput.*, Pacific Grove, CA, Nov. 7–10, 2004.

[14] Mazen O. Hasna and Mohamed-Slim Alouini, “Harmonic mean and end-to-end performance of transmission systems with relays,” *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130–135, Jan. 2004.

[15] Stephen Boyd and Lieven Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[16] Thomas M. Cover and Joy A. Thomas, *Elements of Information Theory*, John Wiley & Sons, 1991.