ROBUST LINEAR PRECODING FOR UNCERTAIN MISO BROADCAST CHANNELS

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ABSTRACT

We consider linear precoding for the downlink of a multiuser communication system in the presence of uncertain channel state information (CSI) at the base station. We consider systems in which the base station has multiple antennas and each user has a single antenna; i.e. multiple-input single-output (MISO) systems. For systems with uplink-downlink reciprocity we propose a statistical model for the channel uncertainty and provide a convex optimization formulation for the precoder that maximizes an average mean square performance measure. For systems in which the channel measurements are quantized and fed back to the base station we propose a deterministically bounded model for the channel uncertainty and a convex formulation for the precoder that maximizes the worst-case performance. Both formulations allow the incorporation of power constraints on individual antennas in addition to the overall power constraint. Our simulations indicate that the proposed approach can significantly reduce the sensitivity of the linearly precoded downlink to uncertainty in the CSI.

1. INTRODUCTION

The downlink of many multiuser communication schemes operates in an interference-limited mode, and hence techniques that enable the transmitter to mitigate the effects of multiuser interference offer the potential for increased throughput and/or improved performance. For systems in which channel state information is available at the transmitter, precoding is one family of such techniques. Within this family, the trade-off between performance and complexity offered by the class of linear precoding techniques is often desirable in practice. For scenarios in which there is perfect CSI at the transmitter, several linear precoding systems have been proposed, including zero forcing [1] and regularized channel inversion [2]. However, in practice the CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback, and the performance of downlink linear precoding systems is rather sensitive to these inaccuracies; e.g., [3]. The goal of the present paper is to propose two robust linear precoding schemes for the downlink that explicitly take into account the uncertainties in the channel model.

Uncertainties in the CSI at the base station can arise via several different mechanisms. In systems with reciprocity between the uplink and the downlink (e.g., time division duplex systems), the base station can estimate the channel. In that case, a stochastic model for the uncertainty in the channel model and designs based on average performance are appropriate. On the other hand, for systems in which the channel is estimated and quantized at the receiver and fed back to the transmitter, one has a bound on the (quantization) error and hence a design in which the worst- case performance over these errors is optimized is appropriate. In this paper we will provide convex optimization formulations for robust linear precoders for both of these models of uncertainty. (An eloquent discussion of the corresponding designs for single-user systems appears in [4].) For the case of the stochastic uncertainty model, there is an existing approach to the design of robust linear precoders for the downlink [5]. Our approach is distinct from that in [5] in that our performance metric includes the effect of noise and our simulation results indicate that our approach can provide better performance. In addition, our approach can easily incorporate power constraints on individual antennas at the transmitter, in addition to the total power constraint.

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2. SYSTEM MODEL

We consider the downlink of a multiuser cellular communication system with n_t antennas at the transmitter and K users, each with one receive antenna. The received signal at the k^{th} receiver is:

$$y_k = \mathbf{h}_k \mathbf{x} + n_k, \tag{1}$$

where $\mathbf{x}$ is the transmitted vector, n_k is the noise at the k^{th} receiver, which has variance σ_n^2, and $\mathbf{h}_k \in \mathbb{C}^{1\times n_t}$ is the vector of channel coefficients of the k^{th} user, which are modelled as independent proper complex Gaussian random variables with zero mean and unit variance. The transmitted vector $\mathbf{x}$ is constructed by linearly precoding the vector $\mathbf{s}$ of data symbols destined for each user; i.e., $\mathbf{x} = \mathbf{P}\mathbf{s}$. The role of the precoder $\mathbf{P}$ is to mitigate the multiuser interference at the receivers subject to a bound on the transmitted power, $\mathrm{E}\{\mathbf{x}^H\mathbf{x}\} \leq P_{\text{total}}$. Without loss of generality we will assume that $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, and hence the power constraint simplifies to $\text{trace}(\mathbf{P}^H\mathbf{P}) \leq P_{\text{total}}$. We will find it convenient to write the received signals in (1) in the following vector form,

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where the k^{th} row of $\mathbf{H}$ is $\mathbf{h}_k$ and the noise vector has a covariance matrix given by $\mathrm{E}\{\mathbf{n}\mathbf{n}^H\} = \sigma_n^2\mathbf{I}$.

2.1. The Performance Metric

One of the advantages of transmitter precoding is that the tasks of channel equalization and interference mitigation are transferred to the transmitter and hence the receiver can be very simple. In order to focus on simple receivers, we will consider schemes in which the receiver makes its decision by simply scaling its decision regions by a parameter c and then quantizing y_k to the nearest constellation point; c.f., [2] [5]. That is, the k^{th} receiver decides in favor of the symbol $\hat{s}_k$ for which $c\hat{s}_k$ is closest to y_k. In the scheme in [2], the parameter c depends on the channel realization and the data vector $\mathbf{s}$, and that requires the parameter c to be updated with each symbol vector $\mathbf{s}$. In our approach, c will depend only on the (transmitter's estimate of the) channel realization, and hence for a block fading channel the communication overhead is significantly reduced. Furthermore, in our scheme the scaling parameter c will be optimized jointly with the precoder $\mathbf{P}$.

The performance metric that we will use in our design is the mean squared error (MSE) between the vector of received signals $\mathbf{y}$ and scaled version of $\mathbf{s}$, i.e.,

$$\begin{aligned} f_0(\mathbf{s},\mathbf{E}) &= \mathrm{E}_{\mathbf{n}}\{\|c\mathbf{s}-\mathbf{y})\|^2\} \\ &= \mathbf{s}^H(c\mathbf{I}-\mathbf{HP})^H(c\mathbf{I}-\mathbf{HP})\mathbf{s}+K\sigma_n^2. \end{aligned} \tag{2}$$

The last term in (2) is constant and will not affect the optimization of $\mathbf{P}$ and c, so only first term will appear in the objectives below.

2.2. Channel Uncertainty Models

We will develop design formulations for robust precoders under two models of channel uncertainty. The first model is suitable for systems with reciprocity in which the transmitter performs channel estimation. In that case, the error $\mathbf{E}$ between the actual channel $\mathbf{H}$ and the estimated channel $\hat{\mathbf{H}}$ can often be assumed to be a Gaussian random variable with zero mean and $\mathrm{E}\{\mathbf{E}^H\mathbf{E}\} = \sigma_E^2\mathbf{I}$. In the second model, the error is assumed to be deterministically bounded, $\|E\| \leq \Delta$, where $\|\cdot\|$ denotes spectral (maximum eigen value) norm [6]. This model is suitable for certain systems that involve quantization of the channel information.

3. ROBUST PRECODING FOR STOCHASTIC UNCERTAINTY

In this section we jointly design the precoder $\mathbf{P}$ and the constant c so as to minimize the average value, over the channel estimation errors, of the performance metric in (2). This robust precoding problem can be formulated as:

$$\min_{\mathbf{P},c} \quad f(\mathbf{P},c) = \mathrm{E}_{\mathbf{s},\mathbf{E}}\{f_0(\mathbf{s},\mathbf{E})\} \tag{3a}$$

$$\text{subject to} \quad \mathrm{trace}(\mathbf{P}^H\mathbf{P}) \leq P_{\text{total}}, \tag{3b}$$

$$c \geq c_{\text{th}}, \tag{3c}$$

where the lower bound constraint on c excludes the trivial solution $c = 0$, $\mathbf{P} = \mathbf{0}$. Since c represents the magnitude of the received signal, then an appropriate choice for c_{th} is the estimated magnitude of received signal based on the transmitter's knowledge of $\hat{\mathbf{H}}$ and P_{total}; see Appendix A for the details.

After taking the expectation with respect to $\mathbf{s}$, the objective in (3a) can be written as:

$$f(\mathbf{P},c) = \mathrm{E}_{\mathbf{E}}\{\mathrm{trace}((c\mathbf{I}-(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})^H(c\mathbf{I}-(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P}))\}.$$

If we write $\mathrm{vec}(c\mathbf{I}-(\hat{\mathbf{H}}+\mathbf{E})\mathbf{P}) = c\mathbf{b}-\mathbf{Ap}$, where the vec operator stacks the columns of the input matrix into one vector, $\mathbf{b} = \mathrm{vec}(\mathbf{I})$, $\mathbf{p} = \mathrm{vec}(\mathbf{P})$ and $\mathbf{A} = \mathbf{I}\otimes(\hat{\mathbf{H}}+\mathbf{E}) = \hat{\mathbf{A}}+\mathbf{\Delta}_A$, then $f(\mathbf{P},c)$ can be written as:

$$\begin{aligned} f(\mathbf{P},c) &= \mathrm{E}_{\mathbf{E}}\{(c\mathbf{b}-\mathbf{Ap})^H(c\mathbf{b}-\mathbf{Ap})\} \\ &= \|\hat{\mathbf{A}}\mathbf{p}-c\mathbf{b}\|^2+\mathbf{p}^H\mathrm{E}_{\mathbf{E}}\{\mathbf{\Delta}_A{}^H\mathbf{\Delta}_A\}\mathbf{p} \\ &= \|\hat{\mathbf{A}}\mathbf{p}-c\mathbf{b}\|^2+\sigma_E^2\|\mathbf{p}\|^2, \end{aligned} \tag{4}$$

where we have used the fact that $\mathbf{\Delta}_A{}^H\mathbf{\Delta}_A = \mathbf{I}\otimes\mathbf{E}^H\mathbf{E}$. Equation (4) shows that the robust precoding problem is a form of constrained regularized least-squares, where the cost function represents a trade-off between the MSE of the estimated channel (first term) and the total precoder power (second term). In particular, (4) indicates that as the (Frobenius) norm of $\mathbf{P}$ increases, the received signal $(\hat{\mathbf{H}}+\mathbf{E})\mathbf{Ps}+\mathbf{n}$ becomes more sensitive to channel estimation errors. As a result, the optimum precoder does not necessarily use all the available transmission power; i.e., the constraint in (3b) is not necessarily active at optimality.

Given the above analysis, the robust precoder design problem can be rewritten as:

$$\min_{\mathbf{P},c} \quad \|\hat{\mathbf{A}}\mathbf{p}-c\mathbf{b}\|^2+\sigma_E^2\|\mathbf{p}\|^2 \tag{5a}$$

$$\text{subject to} \quad \|\mathbf{p}\|^2 \leq P_{\text{total}}, \tag{5b}$$

$$c \geq c_{\text{th}}. \tag{5c}$$

We can show, by contradiction [7], that the constraint (5c) will be active at optimality, thus the optimization problem (5) reduces to:

$$\min_{\mathbf{P},c} \quad \|\hat{\mathbf{A}}\mathbf{p}-c_{\text{th}}\mathbf{b}\|^2+\sigma_E^2\|\mathbf{p}\|^2 \tag{6a}$$

$$\text{subject to} \quad \|\mathbf{p}\|^2 \leq P_{\text{total}}. \tag{6b}$$

The problem in (6) is convex and can be efficiently solved. In particular, it can be rewritten as the following rotated second order cone program [8]:

$$\min_{\mathbf{P},c} \quad t_1+\sigma_E^2 t_2 \tag{7a}$$

$$\text{subject to} \quad \|\hat{\mathbf{A}}\mathbf{p}-c_{\text{th}}\mathbf{b}\|^2 \leq t_1, \tag{7b}$$

$$\|\mathbf{p}\|^2 \leq t_2, \tag{7c}$$

$$t_2 \leq P_{\text{total}}. \tag{7d}$$

In fact, the optimal solution of the above problem can be computed in a closed form by solving the Karush-Kuhn-Tucker (KKT) conditions. The optimal solution is given be:

$$\mathbf{P}_{\text{opt}} = \begin{cases} c_{\text{th}}(\hat{\mathbf{A}}^H\hat{\mathbf{A}}+(\lambda+\sigma_E^2)\mathbf{I})^{-1}\hat{\mathbf{A}}^H\mathbf{b} & \lambda > 0 \\ c_{\text{th}}(\hat{\mathbf{A}}^H\hat{\mathbf{A}}+\sigma_E^2\mathbf{I})^{-1}\hat{\mathbf{A}}^H\mathbf{b} & \text{otherwise.} \end{cases} \tag{8}$$

where λ is the Lagrange multiplier associated with the inequality constraint in (6b). Using the singular value decomposition (SVD) of the block diagonal matrix[1] $\hat{\mathbf{A}} = \mathbf{I}\otimes\hat{\mathbf{H}} = \mathbf{U\Sigma V}^H$ and defining $\tilde{\mathbf{b}} = \mathbf{U}^H\mathbf{b}$, it can be shown that λ is equal to the unique positive root of the following equation [7]:

$$f(x) = \sum_{i=1}^{r}\beta_i\frac{\sigma_i^2}{(\sigma_i^2+(x+\sigma_E^2))^2}-\frac{P_{\text{total}}}{c_{\text{th}}^2} = 0 \tag{9}$$

where r is the rank of $\hat{\mathbf{A}}$, β_i are the diagonal elements of $\tilde{\mathbf{b}}\tilde{\mathbf{b}}^H$ and σ_i are the diagonal elements of $\mathbf{\Sigma}$. It can be shown that if the root of (9) is positive then its unique [7, 9]. If no positive root exists, then the alternate expression of $\mathbf{P}_{\text{opt}}$ in (8) is applicable. Furthermore, the value of λ can be efficiently found by applying the bisection search algorithm to find the positive root of (9).

¿From the implementation point of view it is often desirable to consider transmitter designs with power constraints on individual antennas in addition to (or instead of) a constraint on the total

[1] If $\hat{\mathbf{H}} = \hat{\mathbf{U}}\hat{\mathbf{\Sigma}}\hat{\mathbf{V}}^H$ denotes the SVD of $\hat{\mathbf{H}}$, then $\mathbf{U} = \mathbf{I}\otimes\hat{\mathbf{U}}$, $\mathbf{\Sigma} = \mathbf{I}\otimes\hat{\mathbf{\Sigma}}$ and $\mathbf{V} = \mathbf{I}\otimes\hat{\mathbf{V}}$

power transmitted by antennas. A transmitter design for a broadcast channel with such constraints was considered in [10] under perfect CSI assumptions. The formulation of our robust precoding problem in (6) can be easily modified to include such constraints. Since $\mathrm{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}$, the power transmitted by the i^{th} antenna is $\mathbf{P}_i\mathbf{P}_i^H$, where $\mathbf{P}_i$ is the i^{th} row of $\mathbf{P}$, and hence the constraint on the power transmitted by the i^{th} antenna can be written as the rotated second order cone constraint, $\|\mathbf{P}_i\|^2 \leq P_i$.

4. ROBUST MINIMAX PRECODING

In this section we will design a precoder that provides robust performance in a minimax sense over a deterministically bounded set of uncertainties. We will assume that each symbol in $\mathbf{s}$ is chosen from a constellation that satisfies $|s_k|^2 \leq p_{\text{peak}}$ and hence $\|\mathbf{s}\|^2 \leq Kp_{\text{peak}}$ for all possible $\mathbf{s}$. The robust worst-case precoding can then be stated as:

$$\min_{\mathbf{P},c} \max_{\substack{\|\mathbf{E}\|\leq\Delta \\ \|\mathbf{s}\|^2\leq Kp_{\text{peak}}}} f_0(\mathbf{s},\mathbf{E}) \tag{10}$$
$$= \min_{\mathbf{P},c} \max_{\substack{\|\mathbf{E}\|\leq\Delta \\ \|\mathbf{s}\|^2\leq Kp_{\text{peak}}}} \mathbf{s}^H(c\mathbf{I}-\mathbf{HP})^H(c\mathbf{I}-\mathbf{HP})\mathbf{s}$$

subject to the constraints on $\mathbf{P}$ and c given by (3c) and (3b). Considering the inner maximization and carrying out the maximization with respect to $\mathbf{s}$ first, we can rewrite the objective as:

$$\min_{\mathbf{P},c} \max_{\|\mathbf{E}\|\leq\Delta} \lambda_{\max}(\mathbf{M}), \tag{11}$$

where $\mathbf{M} = (c\mathbf{I} - (\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})^H(c\mathbf{I} - (\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})$ and $\lambda_{\max}$ denotes the maximum eigen value. By upper bounding the maximum eigen value of the matrix $\mathbf{M}$ for every $\|\mathbf{E}\| \leq \Delta$, the objective of worst-case precoding problem can be written as single minimization problem, namely

$$\min_{\mathbf{P},c,t} t \tag{12a}$$
$$\text{s.t. } (c\mathbf{I} - (\hat{\mathbf{H}}+\mathbf{E})\mathbf{P})^H(c\mathbf{I} - (\hat{\mathbf{H}}+\mathbf{E})\mathbf{P}) \leq t\mathbf{I} \qquad \forall \|\mathbf{E}\| \leq \Delta, \tag{12b}$$
$$\operatorname{trace}(\mathbf{P}^H\mathbf{P}) \leq P_{\text{total}}, \tag{12c}$$
$$c \geq c_{\text{th}}. \tag{12d}$$

Similar to Section 3, we can show by contradiction that the constraint in (12d) is always active at optimality [7]. Hence, (12d) can be removed and the value $c = c_{\text{th}}$ substituted in (12b). We can formulate the constraint in (12b) as a simpler Linear Matrix Inequality (LMI) constraint using the Schur Complement Theorem [6], which states that if $\mathbf{C} > 0$ then

$$\mathbf{A} \geq \mathbf{B}^H\mathbf{C}^{-1}\mathbf{B} \quad \text{if and only if} \quad \begin{bmatrix} \mathbf{A} & \mathbf{B}^H \\ \mathbf{B} & \mathbf{C} \end{bmatrix} \geq \mathbf{0}.$$

Using the Schur Complement stated above and by moving terms containing $\mathbf{E}$ to the other side of the inequality, we can rewrite constraint (12b) in the following form:

$$\begin{bmatrix} t\mathbf{I} & (c_{\text{th}}\mathbf{I} - \hat{\mathbf{H}}\mathbf{P})^H \\ (c_{\text{th}}\mathbf{I} - \hat{\mathbf{H}}\mathbf{P}) & \mathbf{I} \end{bmatrix} \geq \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix} \mathbf{E} \begin{bmatrix} \mathbf{0} & \mathbf{P} \end{bmatrix} + \begin{bmatrix} \mathbf{P}^H \\ \mathbf{0} \end{bmatrix} \mathbf{E}^H \begin{bmatrix} \mathbf{0} & \mathbf{I} \end{bmatrix} \quad \forall \quad \|\mathbf{E}\| \leq \Delta. \tag{13}$$

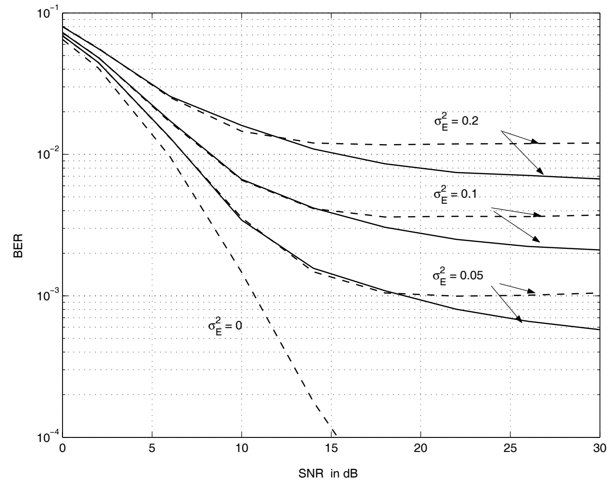

Fig. 1. Comparison between the performance of the proposed robust statistical precoding (solid) and regularized channel inversion [2] (dashed) for different values of channel uncertainty $\sigma_E^2 = 0.05, 0.1, 0.2$ for a system with $n_t = 5$ and $K = 4$.

The constraint (13) represents an infinite number of LMIs since it needs to hold for every $\|\mathbf{E}\| \leq \Delta$. However, the following lemma [11] will be useful in casting it as single LMI.

Lemma 1 [Eldar, Ben-Tal and Nemirovski]: Let $\mathbf{A}$ be a Hermitian matrix. Then $\mathbf{A} \geq \mathbf{C}^H\mathbf{X}^H\mathbf{B} + \mathbf{B}^H\mathbf{X}\mathbf{C}$ for all $\|\mathbf{X}\| \leq \Delta$ if and only if there exists a $\lambda \geq 0$ such that

$$\begin{bmatrix} \mathbf{A} - \lambda\mathbf{C}^H\mathbf{C} & -\Delta\mathbf{B}^H \\ -\Delta\mathbf{B} & \lambda\mathbf{I} \end{bmatrix} \geq \mathbf{0}.$$

Using Lemma 1, the worst case precoding problem can be formulated as the following convex conic optimization problem:

$$\min_{\mathbf{P},c,t,\lambda} \quad t$$
$$\text{subject to} \quad \lambda \geq 0,$$
$$\begin{bmatrix} t\mathbf{I} & (c_{\text{th}}\mathbf{I} - \mathbf{HP})^H & -\Delta P^H \\ (c_{\text{th}}\mathbf{I} - \mathbf{HP}) & (1-\lambda)\mathbf{I} & \mathbf{0} \\ -\Delta\mathbf{P} & \mathbf{0} & \lambda\mathbf{I} \end{bmatrix} \geq 0,$$
$$\|\operatorname{vec}(\mathbf{P})\|^2 \leq P_{\text{total}}. \tag{14a}$$

This problem can be efficiently solved using general purpose implementations of interior point methods, such as SeDuMi [12].

5. SIMULATION RESULTS

In order to compare the performance of the proposed design approach with the existing approaches, we have simulated these methods for the case of QPSK transmission over an independent Rayleigh block fading channel. For each system we will plot the average uncoded bit error rate (BER) of all users against the signal-to-noise-ratio (SNR), which is defined as the ratio of the total transmitted power to the total noise power; i.e., SNR $= P_{\text{total}}/(K\sigma_n^2)$. The elements of $\mathbf{E}$, the error of the transmitter's model of the channel, are generated independently from a zero-mean Gaussian distribution of variance σ_E^2. In the Fig. 1 we compare the performance of the proposed robust precoder for the statistical uncertainty model (see

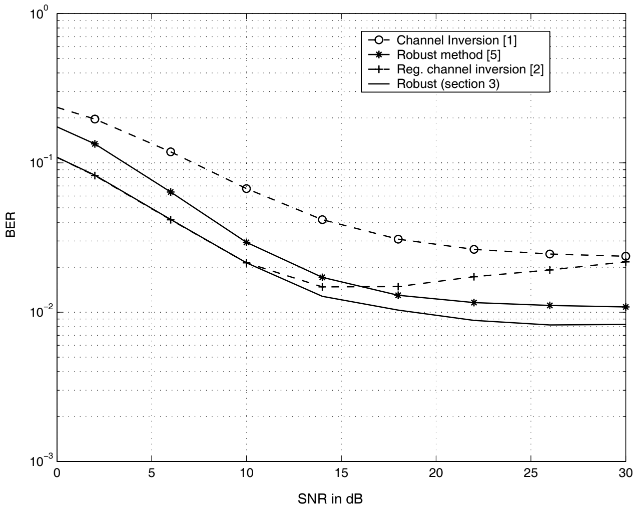

Fig. 2. Comparison of performance of robust and non-robust methods for a system with $n_t = 4$, $K = 4$ and $\sigma_E^2 = 0.05$

Section 3) with that of the regularized channel inversion (RCI) approach introduced in [2], for a system with 5 transmit antennas and 4 users. The performance of each method is plotted for different values of $\sigma_E^2 = 0.05, 0.1, 0.2$ along with the performance of RCI method with perfect channel knowledge. It can be seen that the effect of noise is dominant at low SNR, while channel uncertainty dominates at high SNR where the robust precoding approach performs significantly better than the RCI approach.

In Fig. 2 we consider a scenario with 4 transmit antennas and 4 users. We compare our approach with that of an existing robust method for a stochastic model of uncertainty in [5]. We will also compare with RCI and zero forcing channel inversion (CI) [1]. It can be seen from Fig. 2 that the proposed robust method can outperform the CI and RCI methods and the robust method in [5]. Fig. 2 also shows that in the presence of channel uncertainty, both RCI and CI have the same limit at high SNR. This is due to the fact that the RCI method involves the addition of a regularization term whose value is inversely proportional to $P_{\text{total}}/\sigma_n^2$; see [2, 13].

6. CONCLUSION

We have proposed two different robust precoding schemes for the broadcast channel. The first is based on a statistical model of channel uncertainty at the transmitter. The problem formulation showed similarities to least-squares problems with regularization. The second robust precoding scheme assumes no model for channel uncertainty apart from its being bounded and minimizes the performance metric for the worst-case uncertainty. The resulting conic programming formulation for each robust precoding has advantage that power constraints on individual transmitter antennas can be easily incorporated.

APPENDIX A

In this appendix we provide an appropriate choice for c_{th} based on the available channel knowledge $\hat{\mathbf{H}}$ at the transmitter. We choose c_{th} to be the norm, averaged over $\mathbf{s}$, of the received signal that would be achieved if the precoder of [2] and [13] was used. In absence of noise, the received signal vector for this precoder is:

$$\tilde{\mathbf{y}} = a\hat{\mathbf{H}}\hat{\mathbf{H}}^H(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + b\mathbf{I})^{-1}\mathbf{s},$$

where a is a scaling factor chosen so that the power constraint is met with equality and $b = K/(P_{\text{total}}/\sigma_n^2)$. We suggest that c_{th} be chosen to be $\mathrm{E}_{\mathbf{s}}\{\tilde{\mathbf{y}}\}$.

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