MULTIPLE ANTENNA BROADCAST CHANNELS WITH LIMITED FEEDBACK

Peilu Ding

Motorola Labs
Schaumburg, IL 60169, USA
Email: pding@motorola.com

David J. Love, Michael D. Zoltowski*

School of ECE, Purdue University
West Lafayette, IN 47907, USA
Email: {djlove, mikedz}@ecn.purdue.edu

ABSTRACT

In this paper, we study the limited feedback model for partial CSI at the basestation (BS) for multiple antenna broadcast channels: the BS has the knowledge of quantized CSI of each user. We first give a practical limited feedback scheme designed for multiple antenna broadcast channels. Then, we study the sum rate performance of zero-forcing dirty paper coding under the proposed limited feedback scheme. An upper bound is also derived to get some insight about the impact of the use of limited feedback. Interestingly, we find that the systems experience a ceiling effect on the sum rate for a fixed feedback rate.

1. INTRODUCTION

Recently, multiple antenna broadcast channels have received significant research interest because of their spectral efficiency improvement and potential for commercial application in wireless systems. It was shown in [1, 2, 3] that multiple antennas at the basestation (BS) provide a sum rate capacity increase that grows linearly with the minimum of the number of transmit antennas and users. The resulting sum rate advantage can be achieved through dirty paper coding with perfect channel state information (CSI) available at BS. There also has been some work in the area of practical signaling for the multiple antenna broadcast channel, see [4] and reference there in.

On the other hand, the limited feedback model for partial CSI has received much interest for single user multiple antenna systems over the past few years. It describes some form of CSI as an index in a pre-determined codebook. The codebook is known at both the receiver and transmitter and is designed to capture the essential information of the CSI that is critical for channel capacity or error rate performance, see [5] and reference there in.

In this paper, we consider the limited feedback model for partial CSI at the BS of multiple antenna broadcast channels. This model was independently studied by Jindal[6] for channel inversion very recently. Here we focus on the zero forcing dirty paper coding (ZFDPC) signaling scheme and assume that the BS has N antennas while each mobile has a single antenna due to size and battery constraints. Either user selection or user ordering under limited feedback are beyond the scope of this paper. Therefore, the number of user K is assume to be equal to N, i.e., $K = N$. Each user sends the binary index of the best codevector from the codebook through a zero-error, zero-delay feedback channel to the BS. A limited feedback scheme designed for multiple antenna broadcast systems is proposed.

The key differences between our scheme and the limited feedback for single user MIMO channels are the following: i.) For multiple antenna broadcast channels, each receiver only knows its own channel instead of the full CSI and the users can not cooperate. Each user is unable to obtain the optimal precoding or beamforming structures which are usually computed from the full CSI. Therefore, in our scheme, the vector quantization is applied to the channel vector itself instead of to the beamforming vector or precoding matrices, which is usually the case for single user MIMO systems where the receiver has full CSI [7]; ii.) The codebook of each user should be different from others. Otherwise, there is a chance that two or more users quantize their channel vectors to the same codevector which will cause a rank loss in the channel matrix composed by those codevectors. To avoid this situation, we let every user rotate a general codebook by a random unitary matrix that is also known at the BS. So the channel matrix at the BS is full rank with probability one. Also under Rayleigh fading, the codebook used by different users is equivalent to each other in the sense of quantization error.

We analyze the ergodic sum rate of ZFDPC under limited feedback. An upper bound is derived for the ergodic sum rate and provide important insights into the impacts of the use of limited feedback. From both theoretical analysis and numerical results, we find that the mismatch between the quantized channel vectors and the perfect channel vectors results in additional cross user interferences and leads to a ceiling effect on the sum rate for a fixed codebook size as the SNR increases.

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2. BACKGROUND

2.1. Channel Model

Consider a broadcast channel consisting of an N-antenna BS and K single-antenna users. Assuming that the channel is flat-fading, it can be modeled as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{v} \tag{1}$$

where $\boldsymbol{H} = [\boldsymbol{h}_1, \ldots, \boldsymbol{h}_K]^T$ and $\boldsymbol{h}_k$ is the $N \times 1$ channel fading vector between the BS and the kth user. $\boldsymbol{x}$ is the $N \times 1$ transmitted signal, $\boldsymbol{v}$ is the $k \times 1$ zero mean complex white noise with variance one, and $\boldsymbol{y} = [y_1, y_2, \ldots, y_K]^T$ where y_k is the received signal at user k. Under an i.i.d. Rayleigh fading assumption, $\boldsymbol{h}_k \sim \mathcal{C}N(0, \boldsymbol{I}_N)$ and $E[\boldsymbol{h}_k \boldsymbol{h}_l^H] = \boldsymbol{0}$ if $k \neq l$. The following power constraint is applied to the transmitted signal

$$E[\|\boldsymbol{x}\|^2] = P \tag{2}$$

where P is the maximum total transmit power over one time slot. Since the noise power is normalized, P also represents the SNR.

The key difference between (1) and single user MIMO is that the receive antennas can not cooperate in the broadcast case. As a consequence, user i can only obtain the state information of its own channel which is $\boldsymbol{h}_i$ in this case. We assume that $K = N$ to avoid the discussion about user selection and user ordering both of which are very important but beyond the scope of the paper.

2.2. Zero-Forcing Dirty Paper Coding

ZFDPC is proposed in [1] and is shown to provide an optimal throughput for asymptotically large SNR. In the ZFDPC scheme, the BS collects the channel knowledge $\boldsymbol{H}$ from the users and then decomposes it into

$$\boldsymbol{H} = \boldsymbol{G}\boldsymbol{Q}$$

where $\boldsymbol{G}$ is a $K \times N$ lower triangular matrix and $\boldsymbol{Q}$ is an $N \times N$ unitary matrix. Applying $\boldsymbol{Q}^H$ to the original source signal $\boldsymbol{s} = [s_1, \ldots, s_N]^T$ as a precoding matrix gives $\boldsymbol{x} = \boldsymbol{Q}^H \boldsymbol{s}$, and the input-output relationship for the ith user

$$y_i = g_{ii} s_i + \sum_{j<i} g_{ij} s_j + v_i, \tag{3}$$

where g_{ij} is the (i, j) element in $\boldsymbol{G}$. By treating $\sum_{j<i} g_{ij} s_j$ as the known interference and judiciously generating s_i according to dirty paper coding, these N cross interfering subchannels have the same capacity as N parallel Gaussian channels with fading gains g_{ii}, $i = 1, \ldots, N$. The resulting sum rate is

$$R^{dpc} = \sum_{i=1}^{N} \log_2 \left(1 + |g_{ii}|^2 P_i\right) \tag{4}$$

where P_i is the power allocated to user i and satisfies $\sum_{i=1}^{N} P_i = P$. The maximum sum rate is achieved through waterfilling power allocation.

3. LIMITED FEEDBACK FOR MULTIPLE ANTENNA BROADCAST CHANNELS

At the receiver side the total CSI $\boldsymbol{H}$ is separated into $\{\boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_K\}$ and distributed among the K users. User i only knows $\boldsymbol{h}_i$ and no user can obtain the information about the optimal transmission scheme (for example, the precoding matrix $\boldsymbol{Q}^H$ in the ZFDPC scheme) which is based on the full knowledge of $\boldsymbol{H}$. Therefore, in our limited feedback scheme, the codebook is constructed for each user to directly quantize the channel vector itself.

Consider a codebook $\mathcal{W}$ which contains L codevectors $\mathcal{W} = \{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_L\}$. We use *minimum distance selection* and *mean square error* as the encoding function and distortion measure, respectively. User i encodes its channel vector $\boldsymbol{h}_i$ into

$$\mathcal{Q}_{\mathcal{W}}(\boldsymbol{h}_i) = \boldsymbol{w}_{l_i}$$

where $l_i = \arg\min_{1 \le j \le L} \|\boldsymbol{h}_i - \boldsymbol{w}_j\|$. Every user sends their index l_i back to the BS, so that the channel knowledge at the BS is

$$\boldsymbol{H}_w = [\boldsymbol{w}_{l_1}, \boldsymbol{w}_{l_2}, \ldots, \boldsymbol{w}_{l_K}]^T.$$

The average distortion introduce by quantization according to codebook $\mathcal{W}$ is defined as

$$D_{\mathcal{W}} = E_{\boldsymbol{h}} \left[\|\mathcal{Q}_{\mathcal{W}}(\boldsymbol{h}) - \boldsymbol{h}\|^2\right]. \tag{5}$$

An optimal codebook in the sense of (5) for a given size L can be construct by the generalized Lloyd algorithm [8].

If a general codebook is used by all the users, $\boldsymbol{H}_w$ will become ill-conditioned when two or more users select the same codevector in the codebook. To avoid such degradation, we propose to use different codebooks at each user. Let $\mathcal{W}^{(i)} = \{\boldsymbol{w}_1^{(i)}, \ldots, \boldsymbol{w}_L^{(i)}\}$ be the codebook used by user i. We want

$$\boldsymbol{w}_l^{(i)} \neq \boldsymbol{w}_m^{(j)} \text{ for } i \neq j; l, m = 1, \ldots, L \tag{6}$$

We also require that the codebooks provide the same average distortion, i.e.

$$D_{\mathcal{W}_i} = D_{\mathcal{W}_j} \text{ for } i, j = 1, \ldots, L. \tag{7}$$

To achieve (6) and (7), a general codebook $\mathcal{W}$ is first generated. Then every user rotates the common codebook by a random unitary matrix $\boldsymbol{T}_i, \boldsymbol{T}_i^H \boldsymbol{T}_i = \boldsymbol{I}$. Thus, the codebook used at user i is

$$\mathcal{W}_i = \boldsymbol{T}_i \mathcal{W} = \{\boldsymbol{T}_i \boldsymbol{w}_1, \ldots, \boldsymbol{T}_i \boldsymbol{w}_L\}.$$

Since the distribution of the channel vectors is invariant to unitary rotation, these rotated codebooks have the same mean square quantization errors. The CSI at the BS is then

$$\boldsymbol{H}_w = [\mathcal{Q}_{\mathcal{W}_1}(\boldsymbol{h}_1), \ldots, \mathcal{Q}_{\mathcal{W}_K}(\boldsymbol{h}_K)]^T,$$

and $\mathrm{rank}(\boldsymbol{H}_w) = N$ with probability one.

Furthermore, we will model the codebook's conditional behavior as

$$\mathbb{E}\left\{\|\boldsymbol{h}_i - \mathcal{Q}_{\mathcal{W}_i}(\boldsymbol{h}_i)\|^2 \mid \mathcal{Q}_{\mathcal{W}_i}(\boldsymbol{h}_i)\right\} = D \tag{8}$$

and

$$\mathbb{E}\left\{\boldsymbol{h}_i - \mathcal{Q}_{\mathcal{W}_i}(\boldsymbol{h}_i) \mid \mathcal{Q}_{\mathcal{W}_i}(\boldsymbol{h}_i)\right\} = \boldsymbol{0}. \tag{9}$$

4. SUM RATE UNDER LIMITED FEEDBACK

4.1. Sum Rate Performance

When limited feedback is used with ZFDPC, the BS assumes $\boldsymbol{H}_w$ to be the perfect CSI and applies the QR-type decomposition $\boldsymbol{H}_w = \boldsymbol{G}_w \boldsymbol{Q}_w$ to get the precoding matrix $\boldsymbol{Q}_w^H$. The resulting input-output relation is

$$\begin{aligned} \boldsymbol{y} &= (\boldsymbol{H}_w + \boldsymbol{\Delta})\boldsymbol{Q}_w^H \boldsymbol{s} + \boldsymbol{v} \\ &= \boldsymbol{G}_w \boldsymbol{s} + \boldsymbol{\Delta}\boldsymbol{Q}_w^H \boldsymbol{s} + \boldsymbol{v} \end{aligned}$$

where $\boldsymbol{\Delta} = \boldsymbol{H} - \boldsymbol{H}_w$. For the ith user, we have

$$y_i = g_{ii}^w s_i + \sum_{j<i} g_{ij}^w s_j + (\boldsymbol{\Delta})_i^T \boldsymbol{Q}_w^H \boldsymbol{s} + v_i \qquad (10)$$

where $(\boldsymbol{\Delta})_i^T$ is the ith row vector of $\boldsymbol{\Delta}$ and g_{ij}^w is the (i, j) element in $\boldsymbol{G}_w$. The gain g_{ii}^w is revealed to user i. We will assume that $\boldsymbol{s}$ is generated using successive dirty paper coding with Gaussian codebooks such that $\mathbb{E}\left\{\boldsymbol{s}\boldsymbol{s}^H\right\} = \frac{P}{N}\boldsymbol{I}$. The BS encoder is assumed to have perfect knowledge of the noise variance and the quantizer distortion. The receiver i uses minimum distance decoding to recover the transmitted codeword assuming a multiplicative channel value of g_{ii}^w. A discussion of decoding for dirty paper coding can be found in [9].

Comparing (10) with (3), we see there is an additional interference $u_i = (\boldsymbol{\Delta})_i^T \boldsymbol{Q}_w^H \boldsymbol{s}$ which is unknown at both the BS and the receiver. Since $\boldsymbol{Q}_w^H \boldsymbol{s} \sim \mathcal{C}N(\boldsymbol{0}, P\boldsymbol{I}/N)$ and the source signal $\boldsymbol{s}$ is independent of the quantization error $(\Delta)_i^T$, the variance of the interference is

$$E[u_i u_i^*] = \frac{P}{N} E\left[\|\mathcal{Q}_{\mathcal{W}_i}(\boldsymbol{h}_i) - \boldsymbol{h}_i\|^2\right] = \frac{DP}{N} \qquad (11)$$

where D is the average distorting of the codebook defined in (5). Here we take average over the quantization error by assuming each codeword experiences the channel state ergodically.

Thus, under limited feedback, the ZFDPC precoding gives us a lower triangular channel $\boldsymbol{G}_w$ with interference-plus-noise power of each subchannel is equal to $1 + DP/N$. By applying successive dirty paper coding to the lower triangular channel $\boldsymbol{G}_w$, the supremum of all achievable sum rates using limited feedback and Gaussian ZFDPC encoding, $R_{\lim}^{dpc}$, is bounded as

$$R_{\lim}^{dpc} \le \sum_{i=1}^{N} \mathbb{E}\left\{\log_2\left(1 + \frac{|g_{ii}^w|^2 P/N}{1 + DP/N}\right)\right\} \qquad (12)$$

where equal power allocation is assumed and the expectation is over $\boldsymbol{H}_w$. This follows from the generalized mutual information work in [10, 11] using i) the assumption of Gaussian codebooks, ii) (8), and (9), and (11) and iii) the fact that D and $E[|v|_i^2] = 1$ are known at the encoder. Note that the bound in (12) is a necessary, rather than sufficient, condition for achievability.

Since it is hard to quantify g_{ii}^w, we derive the following upper bound.

Theorem 1 *$R_{\lim}^{dpc}$ in (12) is upper bounded by*

$$R_{\lim}^{dpc} \le N \log_2\left(1 + \frac{P\bar{\eta}}{N + PD}\right) \qquad (13)$$

where $\bar{\eta} = \mathbb{E}\{\|\boldsymbol{w}_{l_i}\|^2\} = N - D$.

Proof : Using Jensen's inequality on (12), we have

$$R_{\lim}^{dpc} \le N\mathbb{E}\left\{\log_2\left(1 + \frac{P}{N + DP}\frac{1}{N}\sum_{i=1}^{N}|g_{ii}^w|^2\right)\right\}. \qquad (14)$$

For a QR-type decomposition, we have

$$|g_{ii}^w|^2 \le \sum_{j=1}^{i} |g_{ij}^w|^2 = \|(\boldsymbol{H}_w)_i^T\|^2 = \|\boldsymbol{w}_{l_i}\|^2 \qquad (15)$$

where the equality $\sum_{j=1}^{i}|g_{ij}^w|^2 = \|(\boldsymbol{H}_w)_i^T\|^2$ comes from the orthogonality of the columns of the $\boldsymbol{Q}$ in the decomposition. By substituting (15) into (14), we have

$$\begin{aligned} R_{\lim}^{dpc} &\le N\mathbb{E}\log_2\left(1 + \frac{P\sum_{i=1}^{N}\|\boldsymbol{w}_{l_i}\|^2}{N(N + DP)}\right) & (16) \\ &\le N\log_2\left(1 + \frac{P\sum_{i=1}^{N}\mathbb{E}\{\|\boldsymbol{w}_{l_i}\|^2\}}{N(N + DP)}\right) & (17) \end{aligned}$$

where the second inequality is from Jensen's inequality. Since the channel vectors are i.i.d. distributed, we have $\bar{\eta} = \mathbb{E}\{\|\boldsymbol{w}_{l_i}\|^2\} = \mathbb{E}\{\|\boldsymbol{w}_{l_j}\|^2\} = (N - D)$ according to assumption (8) for $i, j = 1, \ldots, K$ and get the bound. *Q.E.D.*

For the case $L \to \infty$, i.e., the perfect feedback case, we have $D \to 0$ and $\bar{\eta} \to \mathbb{E}\|\boldsymbol{h}_i\|^2 = N$. The bound changes into

$$R^{dpc} \le N\log_2(1 + P)$$

which can be seen as the result of applying the Jensen's inequality to the throughput of N non-interfering MISO channels with i.i.d. Rayleigh fading and transmit power P/N.

The bound implies that a ceiling effect occurs on the sum rate under limited feedback for asymptotical high SNR, because

$$\lim_{P\to\infty} R_{\lim}^{dpc} \le N\log_2\left(1 + \frac{\bar{\eta}}{D}\right). \qquad (18)$$

This ceiling effect is also found in [6] for in the setting of zero-forcing beamforming (channel inversion).

Intuitively speaking, the ceiling effect is because the power of the cross user interference is related to the signal power P. To avoid the ceiling effect, we should at least let the interference power keep constant as P increases. This also enables us to roughly compute the feedback rate required for an applicable limited feedback system. For the interference power $\frac{DP}{N}$ to be at least constant as P increases, we should have D of order $O(N/P)$. From the rate-distortion theorem [8], we know that the number of bits b necessary for each user to represent its $N \times 1$ channel vector $\boldsymbol{h}_i$ with average distortion D is

$$b = N \log_2(N/D). \quad (19)$$

By replacing D with $O(N/P)$ in the rate-distortion function (19), we have

$$b = O(N \log_2 P)$$

which is the approximate number of bits necessary for the system to avoid the sum rate ceiling. We see that b has to increase logarithmically with P and should scale linearly as the number of transmit antenna N grows. For example, when $P = 10\,\text{dB}$ and $N = K = 4$, we have $b \approx 13$ bits for each user.

4.2. Simulation Results

In this section, we give numerical result on the ergodic sum rate performance of limited feedback. Since the noise power is normalized, the plotted SNR in the figures is $\text{SNR} = 10\log_{10} P$.

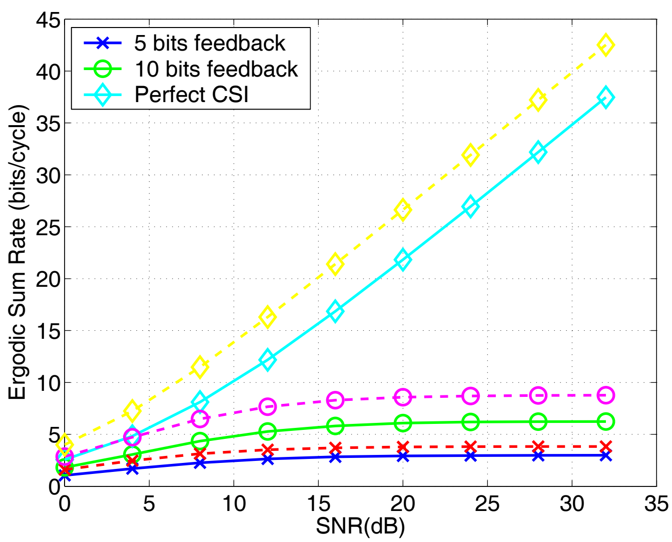

Fig. 1. Ergodic sum rate (solid lines) and upper bound (dashed lines) of limited feedback with ZFDPC for $N = K = 4$.

Fig. 1 shows the ergodic performance of $R_{\lim}^{dpc}$ and the respective upper bound for $N = K = 4$ when the feedback rate of each user is 5bits and 10bits. The ceiling effect can be clearly observed. The sum rate curves increase linearly with SNR for low SNR situation and becomes flat for high SNR regime. The result also indicates that these overhead rate is not enough for a system operating at $\text{SNR} \geq 20\,\text{dB}$.

5. CONCLUSION

We proposed a limited feedback scheme for multiple antenna broadcast channels which avoids the ill-conditioning of BS CSI by randomly rotating the general codebook at each receiver. We derive an upper bound for sum rate of limited feedback under ZFDPC. The bound gives critical insight about the sum rate performance of limited feedback. It shows that the systems experience a ceiling effect on the sum rate for a fixed feedback rate.

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