# COMBINING SHORT-TERM AND LONG-TERM CHANNEL STATE INFORMATION OVER CORRELATED MIMO CHANNELS

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## ABSTRACT

A simple structure to exploit both long-term and partial short-term channel state information at the transmitter (CSIT) over a family of correlated multiple-antenna channels is proposed. Partial short-term CSIT in the form of a weighting matrix is obtained via a resolution-constrained feedback link, combined with a unitary transformation based on the long-term channel statistics. The feedback link is optimized to maximize the expected achievable rate under different power constraints, using vector quantization techniques. Simulations indicate the benefits of the proposed scheme in all scenarios considered.

## 1. INTRODUCTION

The use of multiple antennas at the transmitter and receiver is by now a well recognized technique to achieve high data rates. A multitude of different transmission techniques have been proposed in the literature, especially for the special cases of full channel state information at the transmitter (CSIT) and no CSIT, respectively. At least when using a small number of antennas, the throughput can be significantly improved if CSIT is available. However, in practice this either requires carefully calibrated radio chains and duplex times lower than the channel coherence time, if the channel reciprocity is exploited in time-division duplex systems, or that a significant bandwidth is allocated to feed back channel estimates from the receiver. This has led to a great deal of interest in low-rate feedback schemes, see for example [1–5], that can achieve a significant portion of the full-CSIT performance using only a few bits of feedback for each channel realization.

The fading in wireless communications is generally governed by two components: A slowly-varying components caused by, e.g., shadowing, and a short-time variation caused by, e.g., multipath-fading. Even if it is impossible to obtain accurate short-term CSIT, the long-term channel characteristics can often be estimated with good accuracy. For fixed long-term channel statistics, short-term feedback design to maximize the ergodic capacity of the forward channel using vector quantization techniques is studied in [2,3]. The present work, on the other hand, proposes a simple scheme that successfully combines both long-term and quantized short-term CSIT over a family of multiple-input multiple-output (MIMO) channels. The idea presented here is related to [5], which uses the inverse square root of the channel covariance matrix to transform the channel estimates to a vector of uncorrelated variables before quantization.

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Our proposed transmission scheme includes a unitary transformation, which is influenced by the available knowledge of the channel statistics. Such a transformation can be motivated by the Karhunen-Loève transformation in vector quantization [6], and also by its optimality in the absence of short-term feedback [7–9]. The short-term CSIT regarding the current channel realization is exploited in the form of a weighting matrix, which is designed using a modified version of the Lloyd algorithm. It is shown that a major step in the design procedure can be cast as a variation of the determinant maximization problem [10], which can be solved very efficiently. Simulation results confirm the benefits of the proposed scheme. The results also indicate that temporal power control yields little extra gain over a system that only allocates power over spatial modes.

## 2. SYSTEM MODEL

Consider the discrete-time complex-baseband equivalent model of a MIMO communication system with $N_\text{t}$ transmit antennas and $N_\text{r}$ receive antennas. The received signal at time instant $k$ of *block* $l$ can be written as

$$\mathbf{y}_l(k) = \mathbf{H}_l(k)\mathbf{s}_l(k) + \mathbf{n}_l(k) \tag{1}$$

where $\mathbf{H}_l(k)$ denotes the channel matrix. The components of the temporally and spatially white noise $\mathbf{n}_l(k)$ are complex Gaussian with zero mean and unit variance. A block consists of $N$ consecutive channel uses, during which the vector $\text{vec}[\mathbf{H}_l(k)]$ is assumed to be independent and identically distributed (i.i.d.) zero-mean complex Gaussian with covariance $\mathbf{R}_l$, i.e., $\text{vec}[\mathbf{H}_l(k)] \sim \mathcal{CN}(0, \mathbf{R}_l)$. The channel covariance matrix $\mathbf{R}_l$, however, changes independently from one block to the next according to some distribution. A transmitted codeword is assumed to span a *single* block. We study the system in the limit of a very large block length $N$. This models a communication system where a codeword is sufficiently long to capture the ergodicity of short-term changes, but still short so that it experiences a single $\mathbf{R}_l$. Note that the system can also be viewed as coding over a family of ergodic channels, where each member of the family is parameterized by $\mathbf{R}_l$.

For readability, we will omit the block index $l$ and the time index $k$ whenever this is unambiguous. Since $\mathbf{R}$ is a slowly-varying parameter, we assume that $\mathbf{R}$ is perfectly known at both sides of the link. Such information may be obtained from collected uplink measurements or using a low-rate feedback channel [11]. Furthermore, assume that $\mathbf{H}$ is fully known at the receiver. For a system with fixed long-term channel statistics, a transmitter consisting of an "i.i.d. Gaussian" codebook and a *weighting matrix* which depends *only* on short-term feedback information is optimal in a capacity sense under certain assumptions [2, 3, 12]. However, over a family of channels, this is not very practical as it requires infinitely many

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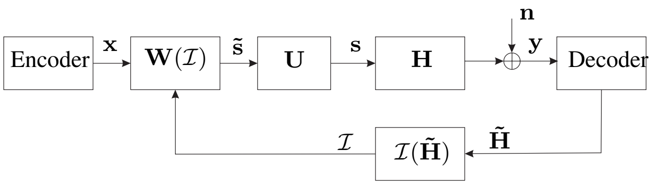


**Fig. 1**. System model.

quantization codebooks, one for each realization of $\mathbf{R}$. We therefore propose a simple alternative, illustrated in Fig. 1.

The transmitter first weights the symbols $\mathbf{x}$, taken from a Gaussian codebook, with $\mathrm{E}[\mathbf{x}\mathbf{x}^\dagger] = \mathbf{I}_{N_t}$, by $\mathcal{W}(\mathcal{I})$, producing $\tilde{\mathbf{s}}$. The notation $[\cdot]^\dagger$ denotes conjugate and transpose. Herein $\mathcal{W}$ is a mapping from a feedback index $\mathcal{I}$ to a finite set of weighting matrices. Such an index is obtained via a noiseless, zero-delay dedicated feedback link. The weighted signals $\tilde{\mathbf{s}}$ are then linearly transformed by a unitary matrix influenced by long-term channel statistics $\mathbf{U} \equiv \mathbf{U}(\mathbf{R})$. To produce the feedback index, the receiver employs an *index mapping* from the current *effective* channel realization $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{U}$ to integer $\mathcal{I} \equiv \mathcal{I}(\tilde{\mathbf{H}})$. Assume that $\mathcal{I}$ takes a value in the set $\{1, \ldots, K\}$ where $K$ is a constant positive integer. In other words, a resolution-constrained quantizer is considered. For convenience, let $\mathbf{W}_i \triangleq \mathcal{W}(i)$, and $\mathbf{Q}_i \triangleq \mathbf{W}_i\mathbf{W}_i^\dagger$, $i = 1, \ldots, K$.

Conditioned on a feedback index $\mathcal{I} = i$, the average transmit power is

$$\mathrm{E}\,\mathrm{tr}(\mathbf{s}\mathbf{s}^\dagger) = \mathrm{E}\,\mathrm{tr}(\tilde{\mathbf{s}}\tilde{\mathbf{s}}^\dagger) = \mathrm{E}\,\mathrm{tr}(\mathbf{W}_i\mathbf{x}\mathbf{x}^\dagger{\mathbf{W}_i}^\dagger) = \mathrm{tr}\,\mathbf{Q}_i,$$

where $\mathrm{tr}\,\mathbf{X}$ denotes the trace of matrix $\mathbf{X}$. We consider two different types of power constraints. A *short-term* power constraint requires that the transmit power does not exceed $P$ for any feedback index:

$$\mathrm{tr}\,\mathbf{Q}_{\mathcal{I}(\tilde{\mathbf{H}})} \le P, \quad \forall \tilde{\mathbf{H}}. \tag{2}$$

This models a system where temporal power control is not possible. Under the more relaxed *long-term* power constraint, the transmitter can vary the power over the transmission of infinitely many codewords so that

$$\mathrm{E}_{\tilde{\mathbf{H}}}\,\mathrm{tr}\,\mathbf{Q}_{\mathcal{I}(\tilde{\mathbf{H}})} \le P. \tag{3}$$

Note that the distribution of $\tilde{\mathbf{H}}$ depends on the distribution of $\mathbf{R}$.

Conditioned on a $\mathbf{R}$ and for a fixed feedback scheme, the mutual information between the transmitted and received signals has a deterministic value, $I(\mathbf{R})$. We are interested in the design of a feedback scheme that maximizes the *expected rate* over infinitely many blocks, i.e.,

$$\begin{aligned} \max_{\mathcal{I}(\tilde{\mathbf{H}}), \{\mathbf{Q}_i\}} \quad & \mathrm{E}_{\mathbf{R}}\, I(\mathbf{R}) \\ \text{s.t.} \quad & (2) \text{ or } (3). \end{aligned} \tag{4}$$

In practice, the distribution of $\mathbf{R}$ has to be determined a priori. However, as will be shown in Sec. 4, our proposed design does not require the exact distribution, but only an empirical distribution of $\mathbf{R}$. Methods to estimate this distribution is outside the scope of the paper.

## 3. DECORRELATING LINEAR TRANSFORMATION

The common knowledge of the channel covariance matrix at both the transmitter and receiver is used to decorrelate the channel coefficients before the quantization. We emphasize the simplicity of such a decorrelation, but do not claim its optimality, because unlike in [7–9], some form of short-term channel knowledge is available at the transmitter in our model.

For simplicity, we begin with the case of a single receive antenna. Let us use the notation $\mathbf{H} = \mathbf{h}^\dagger$, and $\mathbf{R} = \mathbf{R}^{\mathrm{Tx}}$. Thus the received signal can be written as

$$y = \mathbf{h}^\dagger\mathbf{s} + n$$

with $\mathbf{h} \sim \mathcal{CN}(0, \mathbf{R}^{\mathrm{Tx}})$. Introduce $\mathbf{U}^{\mathrm{Tx}}\mathbf{D}^{\mathrm{Tx}}(\mathbf{U}^{\mathrm{Tx}})^\dagger = \mathbf{R}^{\mathrm{Tx}}$ with unitary $\mathbf{U}^{\mathrm{Tx}}$ and diagonal $\mathbf{D}^{\mathrm{Tx}}$. Now if we choose $\mathbf{U} = \mathbf{U}^{\mathrm{Tx}}$, then

$$I(\mathbf{R}) = \mathrm{E}_{\tilde{\mathbf{h}}} \log(1 + \tilde{\mathbf{h}}^\dagger\mathbf{Q}\tilde{\mathbf{h}}),$$

where $\tilde{\mathbf{h}} = (\mathbf{U}^{\mathrm{Tx}})^\dagger\mathbf{h}$ is a vector of decorrelated variables, i.e., $\tilde{\mathbf{h}} \sim \mathcal{CN}(0, \mathbf{D}^{\mathrm{Tx}})$. This can be viewed as a property of the unitary-indepe-ndent-unitary model [13]. Such Karhunen-Loève transformations are commonly used in vector quantization [6]. In [5], it is proposed to use the transformation $\tilde{\mathbf{h}} = \mathbf{R}^{-1/2}\mathbf{h}$ to obtain a vector of i.i.d. variables. However, in case of highly correlated channels, such a transformation will amplify the channel vector along dimensions where it is known a priori that the coefficients are almost zero.

The same ideas can be applied to a MIMO channel, but one problem is that it is in general difficult to derive the unitary transformation matrix $\mathbf{U}$ from a decorrelating transformation of the form $\mathrm{vec}[\tilde{\mathbf{H}}] = \mathbf{V}\,\mathrm{vec}[\mathbf{H}]$. Therefore, we assume that the second-order statistics of the channel follows the so-called Kronecker model [14], i.e. $\mathrm{vec}[\mathbf{H}] \sim \mathcal{CN}(0, \mathbf{R})$, where $\mathbf{R} = (\mathbf{R}^{\mathrm{Tx}})^{\mathrm{T}} \otimes \mathbf{R}^{\mathrm{Rx}}$. Herein $[\cdot]^{\mathrm{T}}$ denotes transpose. Introduce the eigenvalue decompositions

$$\mathbf{U}^{\mathrm{Tx}}\mathbf{D}^{\mathrm{Tx}}(\mathbf{U}^{\mathrm{Tx}})^\dagger = \mathbf{R}^{\mathrm{Tx}}$$

and let us choose $\mathbf{U} = \mathbf{U}^{\mathrm{Tx}}$. Recall that $\tilde{\mathbf{H}} = \mathbf{H}\mathbf{U}$, we then have

$$I(\mathbf{R}) = \mathrm{E}_{\tilde{\mathbf{H}}} \log\det\left(\mathbf{I}_{N_r} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^\dagger\right),$$

with $\mathrm{vec}[\tilde{\mathbf{H}}] \sim \mathcal{CN}(0, \mathbf{D}^{\mathrm{Tx}} \otimes \mathbf{R}^{\mathrm{Rx}})$. We can further decorrelate $\tilde{\mathbf{H}}$ by an additional linear transformation at the *receiver*. To see that, consider the eigenvalue decomposition

$$\mathbf{U}^{\mathrm{Rx}}\mathbf{D}^{\mathrm{Rx}}(\mathbf{U}^{\mathrm{Rx}})^\dagger = \mathbf{R}^{\mathrm{Rx}}$$

and let $\hat{\mathbf{H}} = (\mathbf{U}^{\mathrm{Rx}})^\dagger\tilde{\mathbf{H}}$. Then, $\mathrm{vec}[\hat{\mathbf{H}}] \sim \mathcal{CN}(0, \mathbf{D}^{\mathrm{Tx}} \otimes \mathbf{D}^{\mathrm{Rx}})$, i.e., we obtain an equivalent fully uncorrelated channel. However, given any fixed codebook $\{\mathbf{Q}_i\}_{i=1}^K$, a unitary transformation at the receiver does not change the value of $I(\mathbf{R})$ for any $\mathbf{R}$. Thus, omitting this step at the receiver does not affect the solution to (4).

## 4. FEEDBACK LINK DESIGN

### 4.1. Short-Term Power Constraint

The feedback link is designed using a modified version of the Lloyd algorithm [2, 3, 6]. However, instead of using an ad-hoc approximation that does not necessarily guarantee convergence as in [3], we herein exploit some results in determinant maximization [10].

We first discretize the problem (4) and consider

$$\max_{\mathcal{I}(\tilde{\mathbf{H}}), \{\mathbf{Q}_i\}} \frac{1}{|\mathcal{S}|} \sum_{\tilde{\mathbf{H}} \in \mathcal{S}} \log\det(\mathbf{I}_{N_r} + \tilde{\mathbf{H}}\mathbf{Q}_{\mathcal{I}(\tilde{\mathbf{H}})}\tilde{\mathbf{H}}^\dagger) \text{ s.t. } (2)$$

where $\mathcal{S}$, with finite cardinality denoted by $|\mathcal{S}|$, is a database of the equivalent channels $\tilde{\mathbf{H}}$. Note that $\mathcal{S}$ is actually a sample distribution, used to approximate the true continuous distribution of $\tilde{\mathbf{H}}$ [6].

The design procedure iteratively optimizes the index mapping and the weight mapping to obtain at least a local optimum. This is summarized as follows, where $n$ indicates the iteration index.

Given a set $\{\mathbf{Q}_i^{(n)}\}_{i=1}^K$ satisfying $\operatorname{tr}\mathbf{Q}_i^{(n)} \le P, \forall i$, the optimal index mapping is given by

$$\mathcal{I}^{(n)}(\tilde{\mathbf{H}}) = \arg\max_i \log\det\left(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}_i^{(n)}\tilde{\mathbf{H}}^\dagger\right).$$

Let us now fix $\mathcal{I}^{(n)}(\tilde{\mathbf{H}})$ and define the quantization regions

$$\mathcal{R}_i^{(n)} \triangleq \{\tilde{\mathbf{H}} \in \mathcal{S} : \mathcal{I}^{(n)}(\tilde{\mathbf{H}}) = i\}.$$

The elements of the weight codebook can be optimized individually:

$$\mathbf{Q}_i^{(n+1)} = \arg\max_{\mathbf{Q}} \frac{1}{|\mathcal{R}_i^{(n)}|} \sum_{\tilde{\mathbf{H}}\in\mathcal{R}_i^{(n)}} \log\det\left(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}\tilde{\mathbf{H}}^\dagger\right)$$
$$\text{s.t. } \operatorname{tr}\mathbf{Q} \le P, \mathbf{Q} \succeq 0.$$

This convex problem is a slightly modified version of the determinant maximization problem [10]. In the design we have been able to solve this maximization with relative ease using a barrier method with Newton steps.

### 4.2. Long-Term Power Constraint

The technique outlined above can also be applied to the long-term power constraint case. The design however becomes more involved as we have to optimize the elements of the codebook $\{\mathbf{Q}_i\}_{i=1}^K$ jointly.

Given a fixed $\{\mathbf{Q}_i^{(n)}\}_{i=1}^K$ and the Lagrange multiplier associated with the power constraint $\lambda^{(n)}$, we assume that a constraint qualification holds so that the optimal index mapping solves

$$\max_{\mathcal{I}(\tilde{\mathbf{H}})} \quad \frac{1}{|\mathcal{S}|}\sum_{i=1}^K \sum_{\tilde{\mathbf{H}}\in\mathcal{R}_i} \log\det(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}_i^{(n)}\tilde{\mathbf{H}}^\dagger) - \lambda^{(n)} \sum_{i=1}^K \frac{|\mathcal{R}_i|}{|\mathcal{S}|} \operatorname{tr}\mathbf{Q}_i^{(n)}.$$

This can be rewritten as

$$\max_{\mathcal{I}(\tilde{\mathbf{H}})} \quad \frac{1}{|\mathcal{S}|}\sum_{i=1}^K \sum_{\tilde{\mathbf{H}}\in\mathcal{R}_i} \left(\log\det(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}_i^{(n)}\tilde{\mathbf{H}}^\dagger) - \lambda^{(n)} \operatorname{tr}\mathbf{Q}_i^{(n)}\right).$$

Note that the maximization can also be seen as one performed over all possible ways of partitioning $\mathcal{S}$ into $K$ subsets $\mathcal{R}_1, \dots, \mathcal{R}_K$. Thus the solution is readily given by

$$\mathcal{I}^{(n)}(\tilde{\mathbf{H}}) = \arg\max_i \left\{\log\det\left(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}_i^{(n)}\tilde{\mathbf{H}}^\dagger\right) - \lambda^{(n)} \operatorname{tr}\mathbf{Q}_i^{(n)}\right\}.$$

In the initial step, we can select $\mathbf{Q}_i^{(0)}$ so that $\operatorname{tr}\mathbf{Q}_1^{(0)} = \cdots = \operatorname{tr}\mathbf{Q}_K^{(0)}$ to remove the dependence of $\mathcal{I}^{(0)}(\tilde{\mathbf{H}})$ on $\lambda^{(0)}$.

Given the quantization regions $\mathcal{R}_i^{(n)} \triangleq \{\tilde{\mathbf{H}} \in \mathcal{S} : \mathcal{I}^{(n)}(\tilde{\mathbf{H}}) = i\}$, the optimal weight codebook $\{\mathbf{Q}^{(n+1)}\}$ is the solution to

$$\max_{\{\mathbf{Q}_i\}} \frac{1}{|\mathcal{S}|}\sum_{i=1}^K \sum_{\tilde{\mathbf{H}}\in\mathcal{R}_i^{(n)}} \log\det\left(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}_i\tilde{\mathbf{H}}^\dagger\right)$$
$$\text{s.t.} \quad \sum_{i=1}^K \frac{|\mathcal{R}_i^{(n)}|}{|\mathcal{S}|} \operatorname{tr}\mathbf{Q}_i \le P, \mathbf{Q}_i \succeq 0.$$

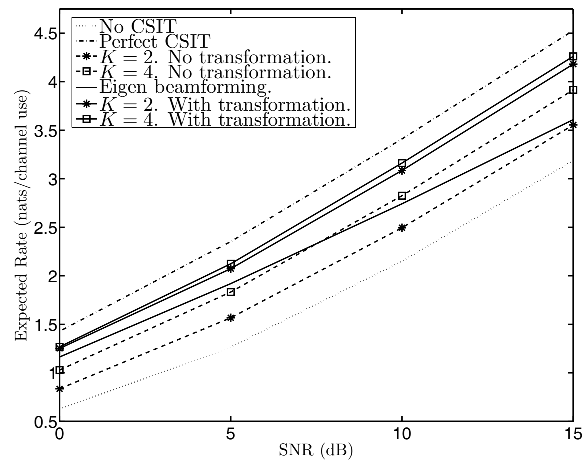


**Fig. 2**. Performance of the proposed scheme over a family of $4 \times 1$ channels. A short-term power constraint is assumed.

We also solve this convex optimization using a barrier method with Newton steps. The optimal Lagrange multiplier can be shown to be

$$\lambda^{(n+1)} = \frac{\operatorname{tr}\sum_{\tilde{\mathbf{H}}\in\mathcal{R}_1}\mathbf{X}_1\mathbf{Q}_1}{|\mathcal{R}_1|\operatorname{tr}\mathbf{Q}_1} = \cdots = \frac{\operatorname{tr}\sum_{\tilde{\mathbf{H}}\in\mathcal{R}_K}\mathbf{X}_K\mathbf{Q}_K}{|\mathcal{R}_K|\operatorname{tr}\mathbf{Q}_K},$$

where $\mathbf{X}_i \equiv \mathbf{X}_i(\tilde{\mathbf{H}}) \triangleq \tilde{\mathbf{H}}^\dagger\left(\mathbf{I}_{N_\mathrm{r}} + \tilde{\mathbf{H}}\mathbf{Q}_i\tilde{\mathbf{H}}^\dagger\right)^{-1}\tilde{\mathbf{H}}$.

## 5. SIMULATION RESULTS

In this section we present simulation results over a specific class of correlated channels. The family of channel covariance matrices is taken as Toeplitz matrices with $[1, \rho, \rho^2, \dots, \rho^{N_\mathrm{t}-1}]$ as the top row, where $\rho$ is a complex-valued random variable with phase uniformly distributed in $[0, 2\pi]$ and modulus distributed as $f(|\rho|) = C\exp(-\lambda(1-|\rho|))$ if $|\rho| < 1$ and $f(|\rho|) = 0$ otherwise, where $\lambda > 0$ and $C$ is a normalization factor. All the simulations are obtained with $\lambda = -\log(0.01)$. Thus, the channels varied from highly correlated to fully uncorrelated. Since the noise variance is normalized to unity, we define the signal-to-noise ratio as $\mathrm{SNR} \triangleq P$. The feedback link is trained with $10^5$ channel realizations, using 20 random starting points and 15 iterations in the Lloyd algorithm.

The performance of the proposed scheme under a short-term power constraint over a family of $4 \times 1$ channels and $2 \times 2$ channels is plotted in Fig. 2 and Fig. 3, respectively. In the no-CSIT case, we assume that $\mathrm{E}[\mathbf{s}\mathbf{s}^\dagger] = \frac{P}{N_\mathrm{t}}\mathbf{I}_{N_\mathrm{t}}$. A consistent gain over a system that only uses short-term CSIT can be seen in all scenarios. A particularly large gain is observed in the $4 \times 1$ case, where one bit of short-term CSIT already provides 2 dB gain at an expected rate of 2 nats per channel use. Combining with long-term channel statistics yields more than 2 dB extra gain. In the same figure, we also plot the performance of the so-called "eigen-beamforming" technique [7–9, 15, 16], where the transmitter allocates power in a water-filling manner based on the eigenvalues of $\mathbf{R}$. Interestingly, this simple approach outperforms a system with a few bits of short-term CSIT in the low-SNR region, but becomes more suboptimal as the SNR increases.

In the $2 \times 2$ case, the effect of CSIT diminishes as the SNR increases, as expected. Nevertheless, at an SNR of 10 dB, for instance, using only one bit of short-term CSIT in combination with long-term

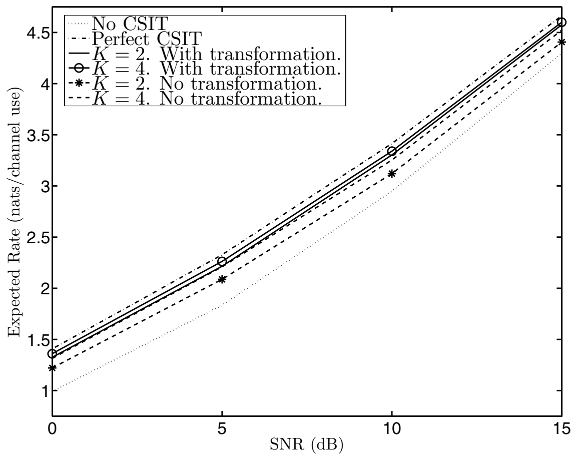


**Fig. 3**. Performance of the proposed scheme over a family of $2 \times 2$ channels. A short-term power constraint is assumed.

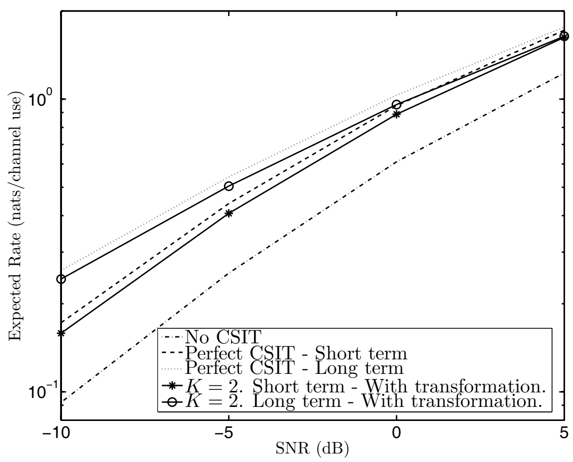


**Fig. 4**. Performance of the proposed scheme over a family of $2 \times 1$ channels under different power constraints.

statistics provides a gain of 0.35 nats per channel use (more than 10 percent) over a no-CSIT system.

Fig. 4 compares the performance of the proposed scheme under different power constraints over a family of $2 \times 1$ channels. The results indicate that temporal power control provides negligible gain for moderate and high SNRs. This is consistent with some results under the assumption of perfect CSIT [17]. At relatively low SNRs, however, a long-term power-constrained system outperforms a short-term one by a wide margin. For instance, at $-5$ dB, a long-term power system using one bit of feedback information even outperforms a perfect-CSIT system without temporal power control. However, the validity of the assumption about perfect channel knowledge at the receiver at so low SNR-values is a matter of debate.

## 6. REFERENCES


[1] A. Narula, M. J. Lopez, M. D. Trott, and G. W. Wornell, “Efficient use of side information in multiple-antenna data transmission over fading channels,” *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1423–1436, Oct. 1998.

[2] V. K. N. Lau, Y. Liu, and T.-A. Chen, “Capacity of memoryless channels and block-fading channels with designable cardinality-constrained channel state feedback,” *IEEE Trans. Inform. Theory*, vol. 50, pp. 2038–2049, Sept. 2004.

[3] V. Lau, Y. Liu, and T.-A. Chen, “On the design of MIMO block-fading channels with feedback-link capacity constraint,” *IEEE Trans. Commun.*, vol. 52, pp. 62–70, Jan. 2004.

[4] G. Jöngren and M. Skoglund, “Quantized feedback information in orthogonal space-time block coding,” *IEEE Trans. Inform. Theory*, vol. 50, pp. 2473–2486, Oct. 2004.

[5] D. J. Love and R. W. Heath, Jr., “Limited feedback diversity techniques for correlated channels,” *IEEE Trans. Veh. Technol.*, 2005, to appear.

[6] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Norwell, MA: Kluwer Academic, 1992.

[7] E. Visotsky and U. Madhow, “Space-time transmit precoding with imperfect feedback,” *IEEE Trans. Inform. Theory*, vol. 47, pp. 2632–2639, Sept. 2001.

[8] S. A. Jafar and A. Goldsmith, “Transmitter optimization and optimality of beamforming for multiple antenna systems,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 4, pp. 1165–1175, July 2004.

[9] E. Jorswieck and H. Boche, “Channel capacity and capacity-range of beamforming in MIMO wireless systems under correlated fading with covariance feedback,” *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1543–1553, Sept. 2004.

[10] L. Vandenberghe, S. Boyd, and S.-P. Wu, “Determinant maximization with linear matrix inequality constraints,” *SIAM J. Matrix Anal. Appl.*, vol. 19, no. 2, pp. 499–533, 1998.

[11] M. Bengtsson and B. Ottersten, “Optimal and suboptimal transmit beamforming,” in *Handbook of Antennas in Wireless Communications*, L. C. Godara, Ed. CRC Press, Aug. 2001, ch. 18.

[12] M. Skoglund and G. Jöngren, “On the capacity of a multiple-antenna communication link with channel side information,” *IEEE J. Select. Areas Commun.*, vol. 21, pp. 395–405, Apr. 2003.

[13] A. M. Tulino, A. Lozano, and S. Verdú, “Impact of antenna correlation on the capacity of multiantenna channels,” *IEEE Trans. Inform. Theory*, vol. 51, no. 7, pp. 2491–2509, July 2005.

[14] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, “A stochastic MIMO radio channel model with experimental validation,” *IEEE J. Select. Areas Commun.*, vol. 20, no. 6, pp. 1211–1226, Aug. 2002.

[15] C. Brunner, W. Utschick, and J. A. Nossek, “Exploiting the short-term and long-term channel properties in space and time: Eigenbeamforming concepts for the BS in WCDMA,” *Eur. Trans. Telecommun.*, vol. 12, no. 5, pp. 365–378, 2001. [Online]. Available: http://www.chrisbrunner.org/publ.htm

[16] S. Zhou and G. B. Giannakis, “Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback,” *IEEE Trans. Signal Processing*, vol. 50, no. 10, pp. 2599–2613, Oct. 2002.

[17] A. J. Goldsmith and P. P. Varaiya, “Capacity of fading channels with channel side information,” *IEEE Trans. Inform. Theory*, vol. 43, pp. 1986–1992, Nov. 1997.