GEAR SIGNAL SEPARATION BY EXPLOITING THE SPECTRAL DIVERSITY AND CYCLOSTAIONARITY

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ABSTRACT

This paper deals with the problem concerning the framework of rotating machines diagnostics by using signal processing advanced tools and more precisely Blind Source Separation (BSS) methods. An application on gear box is given, the objective is to separate gear mesh signals corresponding to each reducer's wheel. It enables us to diagnose and separate each defect in the event of degradation. The proposed method exploits the information redundancy around the meshing frequency and its harmonics resulting from cyclostationarity properties. This redundancy allows us to separate the contribution of each wheel from only one sensor, by tacking advantage of the non-uniformity of the Mechanical Structure Frequency Response (MSFR) connecting the exciting source to the sensor.

Key words: Mechanical Structure Frequency Response, Blind Source Separation, Cyclostationarity, Diagnostics.

1. INTRODUCTION

The aim of the vibratory analysis is to detect mechanical systems' defects early in order to monitor machines. Three phases can mainly be distinguished : acquisition, signal processing, decision-making. Our work concerns the signal processing phase and more particularly the source separation techniques, to separate different contributions in the signal in order to extract information about faulted elements of the machine. Moreover, this can be combined with the exploitation of the cyclostationarity property of the vibratory signal of rotating machines [1]. It's worth emphasizing that in the literature, vibratory sources separation received less attention owing to the complexity of the problem (Non stationarity of the signals and convolutive nature of the vibratory mixtures). One of the possible solutions to this last constraint is to reduce the convolutive temporal mixture to a set of instantaneous complex mixture for each frequency bin inside the frequency domain ([2], [3] and [4]). In this communication, we tackled the gear signal separation problem in the temporal domain by working with bandpass filtered signals centered on the meshing frequency harmonics. Thus we assume that the mixture of the reducer's two wheels signals in each band is instantaneous. Contrary to other BSS methods that use spatial diversity, we use only one physical sensor but we take advantage of frequential diversity. Many artificial sensors are generated by the demodulation of a bandpass filtered versions of the physical sensor signal. The bandpass filter is centered around one of the meshing frequency harmonics. Each artificial sensor is a different observation due to the different local weighting in each band by MSFR. Spectral redundancy resulted from cyclostationarity property [5] guarantee a certain coherence between each artificial sensor. This paper proceeds as follows. Sections 2, 3 and 4 present a survey and two algorithms of instantaneous BSS. Section 5 is devoted to describing the details of the proposed method. In section 6 we explain the separation result. Ultimately, in section 7 we conclude our study and also we introduce some perspectives.

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2. REMINDER ON THE SEPARATION OF INSTANTANEOUS SOURCES

The problem of instantaneous blind source separation in a general context can be summarized in the estimation of the useful information $s(t)$ (that have weighed sums in the transmission channel) in noise-contaminated observation $x(t)$. BSS is based only on the records of sensor signals (spatial diversity) whose number m is greater than or equal to the number of source signals n and the strong assumption of the statistical independence of the sources. Finally, we consider that we have at most one Gaussian source, in order to be able to restore the primary signals. The instantaneous BSS model that bind the j^{th} observation $x_j(t)$ and the i^{th} source $s_i(t)$ can be written as :

$$x_j(t) = \sum_{i=1}^{n} a_{ji} s_i(t) + b(t), \qquad (j = 1, 2, \cdots, m) \quad (1)$$

Or in the matrix notation :

$$x(t) = As(t) + b(t) \quad (2)$$

where

- $s(t) = [s_1(t), s_2(t), \ldots, s_n(t)]^T$ contains the n unknown sources.

- $x(t) = [x_1(t), x_2(t), \ldots, x_m(t)]^T$ represents the m observations or sensors .
- $a_j = [a_{j1}, a_{j2}, \ldots, a_{jn}]^T$ is the directional vector (or signature) associated with the source j.
- $A = [a_1, a_2, \ldots, a_n]$ is an unknown full rank $m \times n$ mixing matrix.
- $b(t)$ is the noise vector, assumed to be zero-mean, stationary, Gaussian and spatially white.

The objective of the BSS is to estimate the source signals even if they are *not statistically independent*, while the goal of the Independent Component Analysis (ICA) is to make transformations on the sensors so that the output signals are as independent as possible. The BSS methods have a tendency to use only Second order statistics (SOS) and work with Gaussian signals. The ICA methods use often Higher-order statistics (HOS) and can not be applied to Gaussian signals. Other methods, known as semi-blind, exploit a priori knowledge of the sources, even if very weak. This information can lead to simpler and very powerful algorithms. One can exploit a priori information on the probability distribution of the sources. Other forms of diversity are exploitable [6] like the frequential diversity which allows us to write as many different equations as there are frequency bands. Here we exploit the frequential diversity and a priori knowledge. In the present paper we have chosen to test two source separation algorithms, one based on SOS and the other on HOS. Both algorithms that we will present hereafter articulate on two distinct phases : whitening [7] and joint diagonalization. [7][8][9].

3. ALGORITHM SOBI

If the source signals have temporal structure, it is possible to separate the different signals by using a set of covariance matrices. As its name indicates SOBI (Second Order Blind Identification) [7] is a blind identification algorithm based on SOS. It uses a set of covariance matrices of whitened processes at different lags. The aim is to find a unitary matrix where the column vectors constitute an orthonormal basis. This latter gathers the set of the eigenvectors that simultaneously and jointly diagonalizes the set of covariance matrices. Thus it has the property to form a vectorial subspace, in spite of the degeneration of certain eigenvalues associated with some matrices from the set of covariance matrices.

4. ALGORITHM JADE

The independence condition imposes constraints on the possible choice of separation algorithms. The decorrelation (Principal Component Analysis) is far from ensuring this independence except for the Gaussian signals case. For a better description of independence, many criteria are based on fourth-order cummulants. Of course, the cancellation of the higher-order cross cummulants is a consequence of sources independence. When the signals sources do not present temporal structure, or when they have identical standardized spectra, it is not possible to use only SOS to carry out the separation of sources. One has recourse to statistics of an order higher than two. In these cases, it's necessary to assume that sources must have non-Gaussian distributions. JADE (Joint Approximate Diagonalization of Eigen-matrices) [8] belongs to the orthogonal techniques, i.e. it combines the information of second order and higher-order. The SOS are used for the whitening, and the Joint Approximate Diagonalization of the eigen-matrices to find unitary matrix. The statistical effectiveness is envisaged to increase by using at the same time the SOS and HOS, see [10], [11] and [12] for more details.

5. THE PROBLEM STATEMENT

We are interested on the separation of gear mesh reducer signals. The gears are composed of two toothed wheels R_1 and R_2, (N_1 and N_2 teeth) and rotate at the speed n_1 and n_2 rpm, (i.e. F_1 and F_2 Hz). The gear mesh is run at the rhythm of gear tooth engagement at the meshing frequency : $F_E = N_1 \times F_1 = N_2 \times F_2$. In the case of a healthy operation (correct teeth, normal condition of use), vibratory signal $s_e(t)$ consists of lines spectrum spaced by the meshing frequency "Fig. 1". The gear mesh signal $s_c(t)$ is modulated by each wheel $s_1(t)$ and $s_2(t)$ ([1] and [13]):

$$s_e(t) = s_c(t)(1 + s_1(t) + s_2(t))$$

"Fig. (1 and 2)" below shows the nonuniform frequency distribution : the shape of the gear mesh spectrum displays the influence of the mechanical structure (whose the impulse response is $h(t)$) which connects the physical sensor to the exiting source. "Fig. 2", is a zoom to represent better the influence of the MSIR on the spectrum, one can see a resonance in the vicinity of $2F_E$:

$$y(t) = s_e(t) * h(t) + b(t) \tag{3}$$

while $y(t)$ is the accelerometer vibratory signal.
When a tooth's defect appears on one of the two wheels, the associated sidebands amplitudes will be increased.In our experimentation (reducer $N_1 = 20, N_2 = 21$ teeth), the frequencies brought into play are very close $F_1 = 16.66Hz$ ($1000rpm$) and $F_2 = 15.86Hz$ ($952rpm$). Therefore, the modulating signals $s_1(t)$ and $s_2(t)$ appear superimposed in the spectrum, a bandpass filtering does not manage to isolate them. We propose to use the well-known source separation algorithms [14] evoked previously. The system inputs come only from one sensor. In fact, in classical BSS the spatial diversity is of primary importance to make a success of separation. Admittedly. In our problem we have only one sensor, but the collected signal $y(t)$ is obviously rich in a priori information (Cyclostationarity, MSFR), that will enable

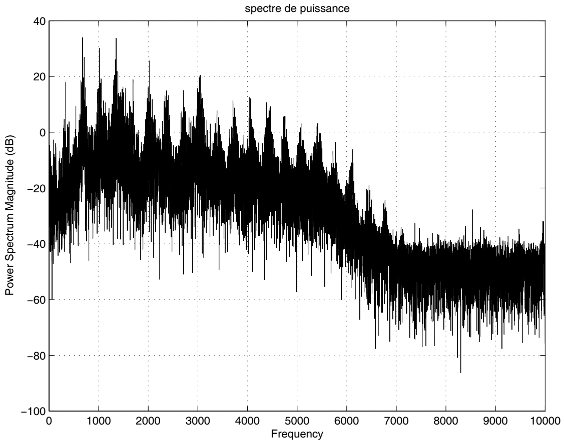


Fig. 1. The gears real spectrum

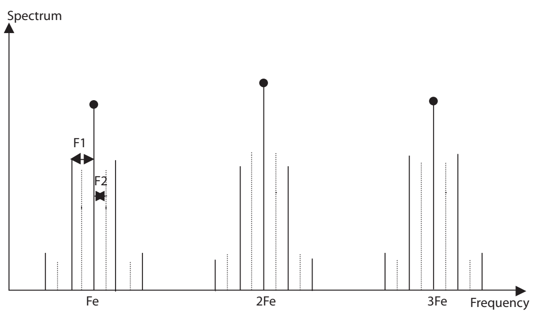


Fig. 2. Approximate description of the gears spectrum

us to compensate the spatial diversity. Thus, to make observations that contain only frequential components corresponding to the two wheels, we begin by isolating the frequency bands at the meshing frequency F_E and its multiples by the means of bandpass filters. Then, we demodulate around the meshing frequencies "Fig. 3", in order to generate several observations (sensors) $x_1(t), x_2(t), \dots x_j(t), \dots, x_m(t)$ while $x_j(t) = TF^{-1}(X_j(\nu))$. Thereafter, one consider each observation $\{x_j(t), j = 1, 2, \dots, m\}$ as being a sensor. The operation of demodulation (after a series of bandpass filtering around the gear mesh harmonics) makes possible the simplification of "(3)", in order to take into account only the signals corresponding to the two wheels. Mechanical structure impulse reponse $h(t)$, generates a 'spatial' diversity between the different harmonics of the gear meshing frequency "Fig. 3". This last remark allows us to solve the problem :

$$x(t) = s(t) * h(t) + b(t) \tag{4}$$

while $s(t) = [s_1(t), s_2(t)]^T$ comprises source signals of the wheels R_1 and R_2. A local examination of the amplitude gain associated with each wheel frequency at different meshing frequency harmonics reveals a local stationarity, therefore one can assume that the linear mixture of the two sources (the

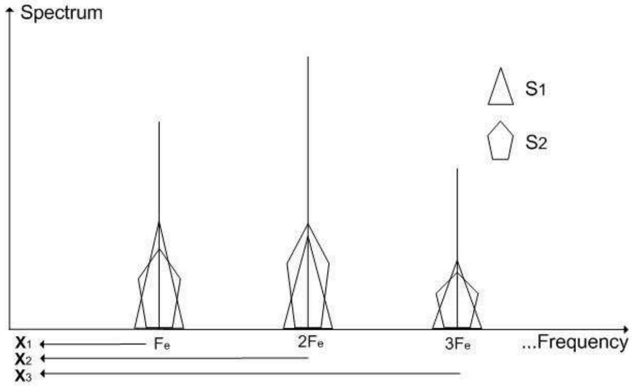


Fig. 3. Illustration of the gear power spectrum as well as the amplitude and frequency modulation.

signal of the entry wheel and the signal of the wheel at exit) is an instantaneous mixture, i.e. the model described by the following equation :

$$x_j(t) = \sum_{i=1}^{2} h_{ji} s_i(t) + b(t), \qquad (j = 1, 2, \cdots, m) \tag{5}$$

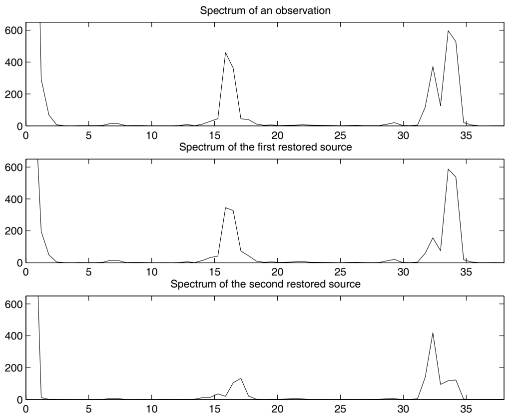
while h_{ji} is the local effect of the MSIR on the source s_i.
In the vectorial form :

$$x(t) = Hs(t) + b(t) \tag{6}$$

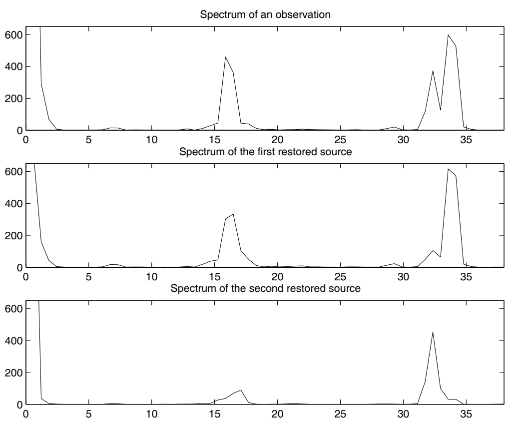
while $H = [h_{11}, h_{12}; h_{21}, h_{22}]$ is the mixing matrix.
Furthermore, it should be noted that we assume that the two sources are statistically independent and jointly uncorrelated to the noise.

6. SEPARATION PART

At this stage, we can dealt with the signals. In our experimentation, we are being restricted to $m = 3$ or even 5 sensors. To evaluate the separation quality of SOBI and JADE, one can compute the ratio of frequency values corresponding to the peak of each estimate and see if it's equal to the ratio of the two wheels' teeth number i.e. $20/21 = 0,95$. "Fig. 4" represents the power spectral densities of a specimen of the three sensors and both of the two wheels signals estimates. On the first harmonic the separation is not possible due to the weak frequential resolution. On the second harmonic we can see that we have separated the source at $2F_1$ from the source at $2F_2$ (different peak position) on the source spectrum with JADE and SOBI. The ratio of the frequencies values taken on "Fig. 4" corresponding to each peak estimate give a value about 0.96, which is within 0.01 to the true ratio. Furthermore, in our case the separation quality at SOBI output is better, because of the small number of average compared with JADE.



(a) JADE separation result



(b) SOBI separation result

Fig. 4. Separation result.

7. CONCLUSION AND OUTLINES

In this article, we saw the implication of the MSFR and the cyclostationarity in the source separation process starting from only one sensor. Of course, the frequential diversity and the different local signatures of the MSFR gave rise to spatial diversity, which authorizes the use of BSS to separate the signals of the two wheels. In the future, we consider the exploitation of these tools, for the separation of the bearings and gear mesh contributions, In order to locate the defects. In addition, we project the exploitation of the cyclostationarity properties to design criteria of separability and also to extend our study to the convolutive mixtures.

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