EQUALIZATION OF A TIME-VARYING CHANNEL IN THE PRESENCE OF DOPPLER RATE

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ABSTRACT

In this paper, we consider the problem of equalizing a time-varying channel in the presence of Doppler rate. Blind estimation of a time-varying channel in the presence of Doppler frequency alone based on a Complex-Exponential, Basis Expansion Model (CE-BEM) has been studied by several researchers in the field. Recently, we proposed a fractionally spaced channel model that does not require basis expansion and possesses a structure that models Doppler rate. We presented a data-aided, LS method for channel estimation. In this work we analyze the performance of the channel estimator using exact and perturbed Doppler estimates. We present a symbol-by-symbol based, decision-feedback equalizer for the time-varying channel and assess its performance in terms of BER.

1. INTRODUCTION

Several researchers have studied the problem of estimating a deterministic, time-varying channel based on a CE-BEM model [1]-[3]. Reference [1] proposed a blind method using subspace decomposition. Reference [2] proposed a blind method using linear prediction. The CE-BEM channel model assumes that channel coefficients are time-invariant and that time-variation is induced by a first order Doppler component. However, several waveforms, such as SATCOM and HF require robust communications in the presence of Doppler rate. In [4], we presented a channel model whose structure is capable of modeling Doppler rate. This channel utilized over-sampling to map the channel impulse response on a fractional time grid. We proposed a data-aided method based on least-squares for channel estimation. We motivated the need to model Doppler rate by numerical as well as asymptotic analysis of Doppler rate due to kinematics of motion in 2-D space. We proposed a preamble signal structure that provided a means to estimate Doppler and Doppler rate with a high degree of accuracy. In this paper, we analyze the performance of the channel estimator in the presence of exact as well as perturbed estimates of Doppler parameters. We present the development of a channel-estimator based, symbol-by-symbol decision-feedback equalizer (DFE). We demonstrate the performance of the equalizer in terms of bit error rate (BER) via Monte Carlo simulations.

2. CHANNEL ESTIMATION

The Complex-Exponential, Basis Expansion Model (CE-BEM) is a deterministic, time-varying channel model. The coefficients of a CE-BEM channel are assumed to be time-invariant or slowly time varying due to changes in the ionosphere [3]. The dominant source of time variation is induced by Doppler due to kinematics. CE-BEM models Doppler up to a first order component. It uses a Doppler basis to span all the first order components present in the channel. The CE-BEM model is given by

$$x(n)=\sum_{l=0}^{M}\sum_{q=1}^{Q}h_q^*(m)e^{j\omega_q n}s(n-m)+v(n) \tag{1}$$

where M is the channel order and Q is the cardinality of the Doppler basis. In [4], we formulated the rate of change of radial distance between a source and an observer in 2-D space and provided analysis that motivated the need to model Doppler rate. The channel output, using the proposed model, is expressed by

$$x(n)=\sum_{k=0}^{M-1}h^*(k)e^{j\left(\alpha_k n^2+\omega_k n\right)}s(n-k)+v(n) \tag{2}$$

where α is the normalized Doppler rate and ω is the normalized initial Doppler frequency.

Equation (2) can be expressed in vector form as

$$\mathbf{x}(n)=\mathbf{\Psi h}+\mathbf{v}(n) \tag{3}$$

where $\mathbf{x}(n)$ is a $N\times 1$ observation vector defined as

$$\mathbf{x}(n)=\left[x(n)\quad\cdots\quad x(n-N+1)\right]^T \tag{4}$$

and $\mathbf{h}$ is a $(M+1)\times 1$ channel estimates vector defined as

$$\mathbf{h}=\left[h^*(0)\quad h^*(1)\quad\cdots\quad h^*(M)\right]^T \tag{5}$$

The channel estimates can be obtained by minimizing (3) in a least-squares sense.

$$\mathbf{h}_{LS} = \left(\mathbf{\Psi}^H \mathbf{\Psi}\right)^{-1} \mathbf{\Psi}^H \mathbf{x} \tag{6}$$

The matrix $\mathbf{\Psi}$ is a $N \times (M+1)$ matrix defined as

$$\mathbf{\Psi} = \begin{bmatrix} e^{j\beta_0(n)} s(n) & \cdots & e^{j\beta_M(n)} s(n-M) \\ e^{j\beta_0(n-1)} s(n-1) & \cdots & e^{j\beta_M(n-1)} s(n-M-1) \\ \vdots & \vdots & \vdots \\ e^{j\beta_0(n-N+1)} s(n-N+1) & \cdots & e^{j\beta_M(n-N+1)} s(n-N-M+1) \end{bmatrix} \tag{7}$$

and the function $\beta_m(n)$ is defined as

$$\beta_m(n) = e^{j\left(\alpha_m n^2 + \omega_m n\right)} \tag{8}$$

2.1. Performance with exact Doppler Parameters

Given exact Doppler parameters, the matrix $\mathbf{\Psi}$ in (3) and (6) is identical. Substituting (3) in (6) results in

$$\begin{aligned} \mathbf{h}_{LS} &= \left(\mathbf{\Psi}^H \mathbf{\Psi}\right)^{-1} \mathbf{\Psi}^H \left[\mathbf{\Psi}\mathbf{h}_T + \mathbf{v}(n)\right] \\ &= \left(\mathbf{\Psi}^H \mathbf{\Psi}\right)^{-1} \left(\mathbf{\Psi}^H \mathbf{\Psi}\right)\mathbf{h}_T + \left(\mathbf{\Psi}^H \mathbf{\Psi}\right)^{-1} \mathbf{\Psi}^H \mathbf{v}(n) \Rightarrow \\ \mathbf{h}_{LS} &= \mathbf{h}_T + \left(\mathbf{\Psi}^H \mathbf{\Psi}\right)^{-1} \mathbf{\Psi}^H \mathbf{v}(n) \end{aligned} \tag{9}$$

Equation (9) shows that, in the presence of exact estimates of the Doppler parameters, the true channel is distorted by additive noise only. This indicates that channel estimates can be improved by increasing SNR. Blind techniques do not reap this benefit since the channel estimates are dominated by statistical errors at high SNR.

2.2. Performance with Perturbed Doppler Parameters

When the Doppler estimates are perturbed, the LS solution is expressed as

$$\begin{aligned} \mathbf{h}_{LS} &= \left(\hat{\mathbf{\Psi}}^H \hat{\mathbf{\Psi}}\right)^{-1} \hat{\mathbf{\Psi}}^H \left[\mathbf{\Psi}\mathbf{h}_T + \mathbf{v}(n)\right] \\ &= \left(\hat{\mathbf{\Psi}}^H \hat{\mathbf{\Psi}}\right)^{-1} \left(\hat{\mathbf{\Psi}}^H \mathbf{\Psi}\right)\mathbf{h}_T + \left(\hat{\mathbf{\Psi}}^H \hat{\mathbf{\Psi}}\right)^{-1} \mathbf{\Psi}^H \mathbf{v}(n) \end{aligned} \tag{10}$$

Equation (10) shows that the LS solution cannot recover the true channel estimates even in the absence of additive noise. We deduce that this result should exhibit a performance floor in terms of MSE as SNR is increased. Reducing (10) to a meaningful and mathematically tractable result in not practical. We consider qualitatively the impact of perturbed Doppler estimates on a one-tap channel and we will quantify the result for higher order channels via simulation. Assuming that the channel has one tap, the estimated channel coefficient is then given by

$$\hat{h} e^{j\hat{\beta}_0(n)n} s(n) = h_t e^{j\beta_0(n)n} s(n) \Rightarrow \hat{h} = h_t e^{j\left(\beta_0(n) - \hat{\beta}_0(n)\right)n} \tag{11}$$

This result indicates that the true channel is modulated by a phase term whose magnitude is proportional to the error in the Doppler estimates.

3. CHANNEL EQUALIZATION

In this section we develop an equalizer for the channel given by (2). The equalizer is a symbol-by-symbol based, decision-feedback equalizer similar to [1]. A DFE offers a significant advantage over a feed-forward equalizer (FFE) as it is able to remove both pre-cursor and post-cursor ISI [6]. Reference [6] proves that the MSE of a FFE reaches a floor that cannot be exceeded with increasing SNR since the MSE is dominated by residual ISI that the FFE cannot remove. To the contrary, the MSE of a DFE keeps improving as a function of increasing SNR.

We start by expanding (2) while ignoring the noise term without loss of generality. This expansion results in

$$\begin{aligned} x(n) = h^*(0) e^{j\beta_o(n)} s(n) + h^*(1) e^{j\beta_1(n)} s(n-1) + \\ \cdots h^*(M) e^{j\beta_M(n)} s(n-M) \end{aligned} \tag{12}$$

The desired signal at time n is $s(n)$. Therefore we solve for $s(n)$ in terms of the other parameters which results in

$$s(n) = \frac{x(n) - \sum_{k=1}^{M} h^*(k) e^{j\beta_k(n)} s(n-k)}{h^*(0) e^{j\beta_o(n)}} \tag{13}$$

Estimating the transmitted data, $\hat{s}(n)$, at time n, requires knowledge of the data estimates $\left\{\hat{s}(n-1), \cdots, \hat{s}(n-M)\right\}$. The complex exponentials $e^{j\beta_i(n)}$ $\forall i \in \{0 \cdots M\}$ are known by assumption. Reference [5] proposes a method that yields high quality estimates of these parameters. Channel estimates, $h(i)$ $\forall i \in \{0 \cdots M\}$ are available by (6). Hence, the transmitted data at time n can be estimated as

$$\hat{s}(n) = \frac{x(n) - \sum_{k=1}^{M} h^*(k) e^{j\beta_k(n)} \hat{s}(n-k)}{h^*(0) e^{j\beta_o(n)}} \tag{14}$$

The estimate in (14) could be significantly enhanced by using hard rather than soft decisions, provided that most of the hard decisions are not in error. Although decision-feedback equalizers are sensitive to feedback errors, they outperform the FFE with a significant margin in practice [6]. The hard decisions are obtained by mapping the soft estimates at the output of the equalizer onto the corresponding symbol in the signal space, $\mathcal{S}$, using the minimum Euclidean distance criterion

$$\tilde{s}_n = s_i \ : \underset{\forall s_i \in \mathcal{S}}{\arg\min} \left\{ \left\| \hat{s}_n - s_i \right\| \right\} \tag{15}$$

Hence, the estimated symbol at time n can be expressed as

$$\hat{s}(n)=\frac{x(n)-\sum_{k=1}^{M}h^{*}(k)e^{j\beta_k(n)}\tilde{s}(n-k)}{h^{*}(0)e^{j\beta_o(n)}} \tag{16}$$

The equalizer of (16) possesses two distinct advantages. The first is that it reaps the benefit of decision-feedback. The second is that is avoids the need for matrix inversion required by block-based LS equalizers, which is computationally inefficient and is prone to ill-conditioning.

Equation (16) reveals that the equalizer requires a memory of depth *M*. In practice, the equalizer computes the channel estimates during training and applies them to equalize the payload. Assuming that $\hat{s}(n)$ is the first payload symbol to be equalized, it is important to initialize the vector $\tilde{\mathbf{s}}_{n-1}=\mathbf{p}_{n-1}$ where $\mathbf{p}_{n-1}$ consists of the last *M* training symbols.

4. SIMULATIONS

In this section we assess the performance of the estimator in terms of MSE and its ability to reconstruct the channel response using exact and perturbed Doppler rate estimates. We also examine the performance to the channel estimator and the equalizer in terms of BER performance.

4.1. MSE Performance

The MSE is defined as

$$\varepsilon_{mse}=\frac{1}{M}\sum_{m=0}^{M}\left|h(m)-\hat{h}(m)\right|^{2} \tag{17}$$

The channel is a 3-tap channel whose coefficients were generated at random. The channel coefficients along with the simulated Doppler rates are listed in Table 1.

Coefficient	0.3706 + 0.7166i	0.4295 + 0.3953i	0.8235 + 0.5747i
α	0	0.1571	-0.1571

TABLE 1: CHANNEL PARAMETERS

The Doppler rate values in Table 1 are used to synthesize output of the channel. We perturb these values and use them to construct $\hat{\mathbf{\Psi}}$ in (10) for the case of perturbed Doppler estimates. The MSE performance for the case of exact Doppler estimates is shown in Figure 1. As predicted by (9), the true estimates are perturbed only by additive noise and the MSE continues to improve linearly as SNR is increased. We also investigate the sensitivity of the MSE to an increasing number of samples. We notice an average improvement of 4 dB as *N* is increased from 20 to 100 samples. The result for the perturbed Doppler rate is shown in Figure 2. We observe that, at lower SNR, additive noise is dominant and the MSE tracks the unperturbed case. However, at higher SNR, the perturbation errors become more dominant and the MSE reaches a performance floor with a significant loss as compared to the unperturbed case.

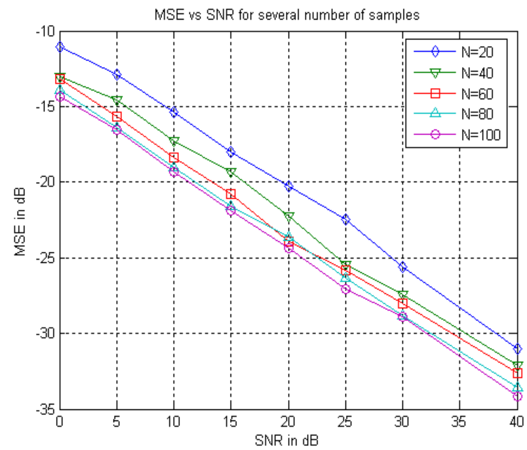

Figure 1: MSE vs. SNR with Exact Doppler Estimates.

A direct comparison between this estimator and those of [1]-[3] is not plausible since the channel models are different. However, some qualitative conclusions can be drawn. The blind estimators exhibit a MSE floor even in the presence of exact Doppler frequency estimates. This is due to the fact that at higher SNR, the error in the channel estimates is dominated by errors in estimating channel statistics. Additionally, blind estimators require a larger sample set, 1000 in the case of [2], and can resolve the channel up to a scalar ambiguity. Our estimator requires 20 to 100 samples depending on multipath severity and SNR [4], and can resolve the channel unambiguously.

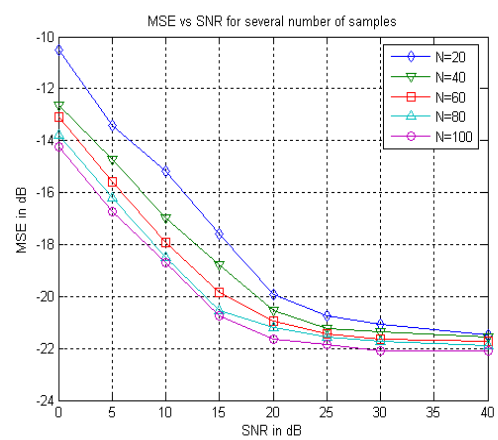

Figure 2: MSE vs. SNR with Perturbed Doppler Estimates.

4.2. Channel Response

In this section we demonstrate the estimator's ability to reconstruct the channel response using exact and perturbed Doppler rate estimates. Results are shown in Figure 3. This figure demonstrates the sensitivity of the channel estimator to the accuracy of the Doppler rate estimates and illustrates the need for a high quality Doppler parameter estimator. Reference [5] presented an estimator that achieves a CR bound and is capable of yielding estimates with the desired quality.

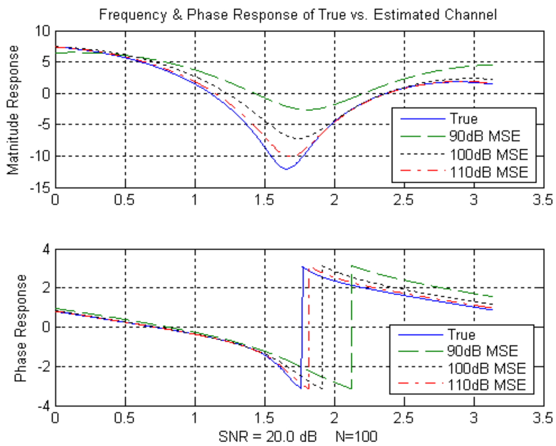

Figure 3: Frequency Response with Imperfect Doppler Estimates.

4.3 BER Performance

Figure 4 demonstrates the BER results of the equalizer given exact Doppler parameter estimates. This simulation shows that with N=100 samples, the channel estimator is able to yield results identical to using exact channel coefficients. The figure illustrates the ability of the DFE to restore the exponential nature of the BER curve and achieve results that are within 2.5 dB from that of an ideal channel. The BER results show that the data-aided channel estimator with DFE does not exhibit a noise floor as in the case of the blind estimators in [1] and [2]. However, our estimator requires training. Training could be made available from existing preambles that are typically used for detection and synchronization purposes.

5. CONCLUSIONS

In this paper, we considered the problem of estimating and equalizing a channel in the presence of Doppler rate. We presented a data-aided LS estimator for the channel and demonstrated the performance of the estimator in the presence of exact and perturbed Doppler estimates. We developed a symbol-by-symbol based decision-feedback equalizer and demonstrated its performance in terms of BER. The proposed method of estimating and equalizing the channel has several advantages over a blind estimator. However, it requires training. Training could be made available from existing preambles that are used for detection and synchronization. An existing method that delivers high quality Doppler estimates was identified.

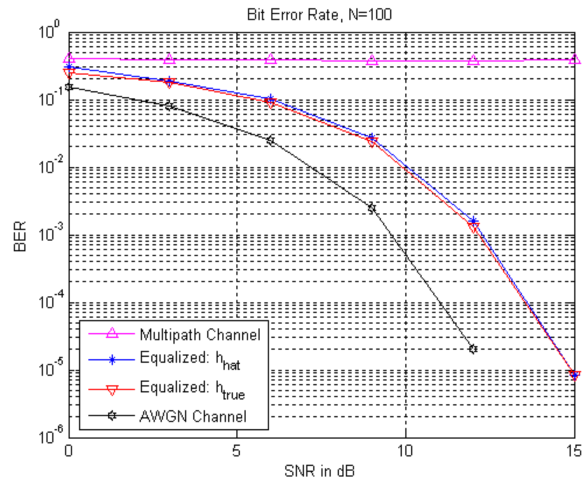

Figure 4: BER Plots.

5. REFERENCES

[1] M. K. Tsatsanis and G. B. Giannakis, “Subspace Methods for the Blind Estimation of Time-Varying FIR Channels,” *IEEE Transactions on Signal Processing*, Vol. 45, No. 12, pp 3084-3093, December 1997.

[2] J.K. Tugnait, “Linear Prediction Error Method for Blind Identification of Time-Varying Channels Theoretical Results,” *Proc. IEEE 2001, International Conf. on Acoustics, Speech & Signal Processing*, Salt Lake City, UT, pp. 2125 – 2128, May 2001.

[3] G.B. Giannakis and C. Tepedelenlioglu, “Basis Expansion Models and Diversity Techniques for Blind Identification and Equalization of Time-Varying Channels,” *Proceedings of the IEEE*, Vol. 86, pp. 1969-1986, Oct. 1998.

[4] G. Maalouli and A. Spanias, “Estimation of a Time-Varying Channel in the Presence of Second Order Dynamics,” *IEEE Signal Processing Letters*, Submitted for Publication, Sept. 2005

[5] Y. Lin, Y. Peng, X. Wang, “Maximum Likelihood Parameter Estimation of Multiple Chirp Signals by a New Markov Chain Monte Carlo Approach,” *Proc. of the IEEE, Radar Conf. 2004*, pp. 559-562, April 26-29, 2004

[6] John Proakis, “*Digital Communications*”, 2nd Edition, McGraw Hill, pp. 621-623, 1994