Efficient Wideband Spreading Function Estimation Using Arbitrary Shaped LFM Signals via Hermite Decompositions

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ABSTRACT

In active imaging, information such as range and velocity of a target can be obtained by transmitting a signal and processing the received signals. Signal processing of acoustic signals scattered from distributed targets has become an increasing attention to the researchers. The delay-scale wideband spreading function (WSF) is often used to characterize the distributed targets environment. This paper presents an efficient technique for estimating WSF using Hermite decomposition. In realistic sonar and radar, it may be necessary to detect more than one target simultaneously. Under this scenario, resolution is a major concern to be considered to separate multiple targets. This paper focuses on the applications of Hermite decomposition to efficiently compute WSF and the use of multiple transmissions to improve resolution.

1. INTRODUCTION

In active sonar and radar, range and velocity information of a target can be obtained by transmitting a signal and processing the received echo. For this active sensing case, a signal is transmitted to the medium, which is reflected from the object and received at the receiver. Then the received signal is processed by correlating it with the hypothesized versions of the transmitted signal. This correlation processing is referred to as matched filtering. The magnitude squared of the output of matched filter is called wideband cross-ambiguity function (WCAF). The WCAF is a surface in the delay-scale plane. In [1], the WCAF was used as a means of estimating point target parameters. However, the simplest case of the received signal was used in [1], where the target was considered as a point reflector.

This paper aims to characterize a distributed target (DT) rather than a point target with varying ranges and velocities. The WSF is usually used to characterize the DT environment. Some applications of DT occur in sonar, radar, medical imaging, oceanography, tomography, remote sensing, etc. A DT is one that spreads in delay-scale plane as compared to a point target, which is associated with only particular delay-scale. In a complicated environment such as the ocean, the target is composed of several objects, or a physical large object with continuum of reflectors and the reflectors are often very close in delay-scale plane. In most cases the propagation and scattering is not ideal and the signal becomes spread in delay and scale so that it no longer resembles a replica of the transmitted waveform. This phenomenon usually occurs in two ways. The signal can be reflected from an elongated scatterer that may have moving components, or propagation involves multipath and intermediate reflections from boundaries.

Gaussian windowed linear frequency modulated (LFM) signals are good candidates for the transmitted signal as they are known to possess very good resolution properties for target parameter estimation [1]. However, this paper considers a more general transmitted waveform where the amplitude is not strictly a Gaussian function. This is important as even a Gaussian transmission signal may suffer from amplitude distortions due to various transmitter hardware limitations. Furthermore, it would be necessary for the sonar operator to use non-Gaussian amplitudes due to other system constraints. Therefore, in this paper it is proposed to decompose the transmitted signal amplitude using the Hermite expansions for efficient computation of WSF. As an arbitrary windowed signal is used in the transmission and hence in the processing, an appropriate analytical model is necessary to scale the signal in time to compute the WSF. However, the major problem associated with this is that the signal is not known analytically. A multirate sampling method maybe used to process this kind of signals; however, it is a computationally cumbersome procedure. Since the signal is not known analytically, creating the scaled and delayed replicas requires sampling the signal and then applying a multirate conversion technique. As such, these techniques are not efficient because of the very high computational complexity [2]. In this paper, we have used the Hermite decomposition to develop an analytical model for the arbitrary shaped LFM signal. Therefore, closed form expression for the transmitted signal can be easily obtained. Using this closed form expression, we propose algorithms to efficiently compute WSF. We will also use the closed form expression in the processing of a multiple transmissions system in Section 5. The development of Hermite decomposition based efficient algorithms is the novelty reported in this paper. In particular, the accurate estimation of Hermite spread parameter, the derivation of closed form expression for Fourier transform of LFM Hermite signals and the use of these for efficient computation of WSF and for multiple transmissions are the main contributions.

2. WIDEBAND SPREADING FUNCTION

When narrowband signals have been transmitted, narrowband scattering functions can be estimated by computing the outputs of narrowband cross-correlation receivers. For narrowband signal model, the bandwidth of the transmitted signal is narrow as compared to main frequency component. The narrowband technique operates on the assumption that the narrowband condition holds [2]. Mathematically, the narrowband condition is written as $2\upsilon / c << 1/TB$, where υ is the scatterer's radial velocity, c is the propagation velocity in the medium under observation, and TB is the time bandwidth product of the signal $g(t)$. However, in many cases like ocean acoustics the narrowband condition is violated. Moreover, the wide sense stationary and uncorrelated scattering conditions assume wide-sense stationarity in both time and frequency, which are not strictly true for the ocean medium. In such scenario the wideband signal model is more appropriate. In this paper, we will use wideband signal model to compute WSF. In environments with multiple scatterers, the delay, τ, and scale, α, are assumed to be distributed according to the WSF so that the received signal, $r(t)$, is written as [2]-[5],

$$r(t) = \iint S(\alpha,\tau) \frac{1}{\sqrt{|\alpha|}} g\left(\frac{t-\tau}{\alpha}\right) \frac{d\alpha d\tau}{\alpha^2} \tag{1}$$

where $S(\alpha,\tau)$ is the WSF associated with each time-delayed and time-scaled version of the transmitted signal and $\alpha = (c-\upsilon)/(c+\upsilon)$. Equation (1) is called the wideband model of the received signal.

By correlating the received signal, $r(t)$, with the hypothesized replicas of the transmitted signal, $g(t)$, the wideband cross-correlation (WC) function (matched filter) is defined as [2],

$$WC_{rg}(\alpha',\tau') = \frac{1}{\sqrt{|\alpha'|}} \int_{-\infty}^{\infty} r(t) g^*\left(\frac{t-\tau'}{\alpha'}\right) dt\,. \tag{2}$$

Equation (2) is identical to cross-wavelet transform, $W_g r(\alpha',\tau')$, of the signal $r(t)$ with respect to the signal $g(t)$, which is assumed to be a mother wavelet. For this interpretation, the transmitted signal $g(t)$ should satisfy the admissibility condition of being a mother wavelet, i.e.,

$$c_g = \int_{-\infty}^{\infty} \frac{|G(\omega)|^2}{|\omega|} d\omega < \infty \tag{3}$$

where c_g is the admissibility constant and $G(\omega)$ is the Fourier transform of $g(t)$.

When the wideband signal model in (1) is inserted into (2), the resulting output is expressed as,

$$\begin{aligned} WC_{rg}(\alpha',\tau') &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\alpha,\tau) WA_{gg}\left(\frac{\alpha'}{\alpha}, \frac{\tau'-\tau}{\alpha}\right) \frac{d\alpha d\tau}{\alpha^2} \\ &= S(\alpha',\tau') *_{wb} WA_{gg}(\alpha',\tau') \end{aligned} \tag{4}$$

where $*_{wb}$ denotes wideband convolution and $WA_{gg}(\alpha',\tau')$ is the auto-wavelet transform of the transmitted signal $g(t)$.

In [3], Naparst has proposed a method to reconstruct $S(\alpha',\tau')$ for the affine group. However, Naparst's method failed to work in some cases. Naparst did not consider noise effects and the sensitivity to changing environments. The method also depends on the choice of the signal. There are also various iterative techniques such as gradient search algorithms [4], least-squares method [5], and regularization techniques such as the method of Tikhonov [5]. However, the Tikhonov method has also some drawbacks such as computationally intensiveness and the lack of guaranty of convergence [5]. To overcome the above drawbacks, wideband deconvolution of (4) has been used to estimate $S(\alpha',\tau')$ [2]

$$\begin{aligned} S(\alpha,\tau) &= \frac{1}{c_g} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} WC_{rg}(\alpha',\tau') WA_{gg}^*\left(\frac{\alpha'}{\alpha}, \frac{\tau'-\tau}{\alpha}\right) \frac{d\alpha' d\tau'}{\alpha'^2} \\ &= WC_{rg}(\alpha,\tau) *_{wb} WA_{gg}(\alpha,\tau)\,. \end{aligned} \tag{5}$$

Note that if $g(t)$ satisfies (3), then the right hand side of (5) is equal to $WC_{rg}(\alpha,\tau)$ (a property of cross wavelet transform). Therefore, if $g(t)$ is available in analytic form $S(\alpha,\tau)$ can be easily computed using (2) rather than (5). In this paper, however, we will consider semi-active sonar/radar case where the transmitted signal is available only as a noise corrupted version [6]. Under this scenario, the transmitted signal is not known analytically. In this case the receiver, situated away from the transmitter, receives two signals; one directly from the transmitter; other reflected from the targets. That is, for single target

$$r_1(t) = g(t) + n_1(t) \quad \text{and} \quad r_2(t) = \frac{1}{\sqrt{\alpha_0}} g\left(\frac{t-\tau_0}{\alpha_0}\right) + n_2(t) \tag{6}$$

where $g(t)$ is the transmitted signal, $n_1(t)$ and $n_2(t)$ are assumed to be zero-mean additive white Gaussian noise (AWGN) process. We will use $\hat{g}(t)$, an approximation to $g(t)$ obtained from $r_1(t)$, as the mother wavelet to compute (5). Using analytical expression for $\hat{g}(t)$ and using (5), we propose algorithms to efficiently compute WSF. This is achieved via Hermite decomposition of $r_1(t)$.

3. HERMITE DECOMPOSITIONS

Hermite series expansion is a well known decomposition technique useful in many signal processing applications. The Hermite expansion is useful because of the following properties and advantages: (i) the Hermite basis functions are orthogonal, (ii) Hermite polynomials can easily be computed via a recurrence relation, (iii) any type of signal can be represented to a high degree of accuracy by using a sufficient number of terms in the expansion, (iv) Hermite expansion is well suited for signals with finite time support [7], (v) Hermite expansion helps to scale the signal easily, which can be used for efficient evaluation of the WSF. The *n*th degree Hermite polynomial is given by,

$$H_n(t/\lambda) = (-1)^n \lambda^n e^{t^2/\lambda^2} \frac{d^n}{dt^n}\left(e^{-t^2/\lambda^2}\right). \tag{7}$$

Using $H_n(t)$, the *n*th order associated Hermite (AH) function $z_{n,\lambda,t_c}(t)$ is defined by,

$$z_{n,\lambda,t_c}(t) = \frac{1}{\sqrt{\lambda 2^n n! \sqrt{\pi}}} H_n\left(\frac{t-t_c}{\lambda}\right) \exp\left\{-\frac{(t-t_c)^2}{2\lambda^2}\right\} \tag{8}$$

where λ and t_c are the spread and shift parameters. Any signal $w(t)$ can be represented using an AH series as,

$$w(t) = \sum_{n=0}^{N} a_{n,\lambda,t_c} z_{n,\lambda,t_c}(t) \text{ with } a_{n,\lambda,t_c} = \int_{-\infty}^{\infty} w(t) z_{n,\lambda,t_c}(t) dt\,. \tag{9}$$

The proper selection of λ and t_c are useful to obtain a good approximation of the signal with a minimum number of Hermite expansion terms. To expand a signal using the Hermite basis functions, it is necessary to center the expansion around a suitable $t = t_c$. This is due to the fact that the Hermite basis functions provide equal support on either side of the center of expansion [7]. Therefore, centering the expansion about $t = t_c$ would require lesser terms for the Hermite expansion. The proper selection of λ is also important because λ decides the amount of support in the Hermite functions. In this paper we have used the centroid of square of $w(t)$ to estimate t_c. An appropriate λ is then evaluated around $t = t_c$. Consider the following minimization that optimally decomposes the signal in a mean square sense. The optimization is accomplished by estimating the values of λ, t_c, C_e, and C_o that minimize a cost function,

$$\varepsilon(\lambda,t_c,C_e,C_o)=E_e\int_{-\infty}^{\infty}\left(w_e(t)-\frac{C_e}{\sqrt{\lambda\sqrt{\pi}}}e^{-(t-t_c)^2/2\lambda^2}\right)^2 dt + E_o\int_{-\infty}^{\infty}\left(w_o(t)-\frac{C_o(t-t_c)}{\sqrt{\lambda^5\sqrt{\pi}}}e^{-(t-t_c)^2/2\lambda^2}\right)^2 dt \quad (10)$$

where $w_e(t)$ and $w_o(t)$ are the even and odd parts of the signal $w(t)$, evaluated at $t=t_c$, while E_e and E_o are the energy of the even and odd parts. Note that if the signal $w(t)$ has either odd or even symmetry then the optimization in (10) becomes rather simplified. For even symmetry signals (i.e., $E_o=0$), the simplified form of (10) can be written as,

$$\varepsilon(\lambda,t_c,C)=\int_{-\infty}^{\infty}\left(w(t)-\frac{C}{\sqrt{\lambda\sqrt{\pi}}}e^{-(t-t_c)^2/2\lambda^2}\right)^2 dt \quad (11)$$

and for odd symmetry signals (i.e., $E_e=0$), the simplified form of (10) can be written as,

$$\varepsilon(\lambda,t_c,C)=\int_{-\infty}^{\infty}\left(w(t)-\frac{C}{\sqrt{\lambda^5\sqrt{\pi}}}(t-t_c)e^{-(t-t_c)^2/2\lambda^2}\right)^2 dt\,. \quad (12)$$

The simplified approach in (11) or (12) has been used in all reported literature. However, for neither even nor odd symmetry signals ($E_e, E_o \neq 0$) it can be shown that (10), the cost function proposed here, would be a better choice. The optimization in (10) can be done via gradient based methods such as the steepest-descent or the Newton's method. In the following simulations, a signal containing both even and odd components and having an arbitrary amplitude expressed as in (13) has been used.

$$w(t)=\sum_{n=0}^{3}a_n\, z_{n,\lambda,t_c}(t) \quad (13)$$

where $\lambda=0.126$, $t_c=-0.1$, and $a=[0.47\ \ 0.411\ \ 0.101\ \ -0.474]$.

Fig. 1 shows the true and the Hermite synthesized signal's envelopes for the three methods. Simulations show that the use of (10) performs better than the use of (11) and (12). In the decomposition, truncation at $N=10$ resulted in a residual mean squared error (MSE) of less than 10^{-4}. Based on the result of Fig. 1, all the simulations in subsequent sections use (10) for the estimation of λ for Hermite decomposition.

4. EFFICIENT COMPUTATION OF WSF

Here we consider the approximate sonar transmitter signal $\hat{g}(t)$ as an LFM signal, given by,

$$\hat{g}(t)=w(t)\exp(j[2\pi\kappa t+\pi\beta t^2]) \quad (14)$$

where $w(t)$ is an arbitrary window function. The instantaneous frequency of signal $\hat{g}(t)$ at $t=0$ is given by κ, $\beta=B/T$ is the frequency sweep rate, B is the bandwidth, and T is the duration.

Applying Hermite expansion, the Fourier transform of $\hat{g}(t)$ in (14) can be expressed as,

$$G(f)=\int_{-\infty}^{\infty}\hat{g}(t)e^{-j2\pi ft}dt=\sum_{n=0}^{N}G_nZ_n(f) \quad (15)$$

where the coefficients G_n are the weights of the Hermite expansion of the transmitted signals envelope that can be computed from (9), and $Z_n(f)$ is the Fourier transform of the LFM AH function, $z_n^{LFM}(t)$, given by

$$F\{z_n^{LFM}(t)\}=Z_n(f)=\int_{-\infty}^{\infty}z_n(t)e^{j[2\pi\kappa t+\pi\beta t^2]}e^{-j2\pi ft}\,dt\,. \quad (16)$$

Using some properties of the Hermite function [8], after some manipulation, it can be shown that

$$Z_n(f)=(-j)^n\sqrt{\frac{2\lambda\sqrt{\pi}}{2^n n!}}\frac{\exp(j\phi_1+Ae^{j\phi_2})}{(1+4\pi^2\beta^2\lambda^4)^{1/4}} \times H_n\left\{\frac{2\pi\lambda(f-\kappa-\beta t_c)}{(1+4\pi^2\beta^2\lambda^4)^{1/2}}\right\}\exp\left\{-\frac{2\pi^2\lambda^2(f-\kappa)^2e^{j\phi_3}}{(1+4\pi^2\beta^2\lambda^4)^{1/2}}\right\} \quad (17)$$

where $A=\dfrac{\pi t_c(2\kappa-2f+\beta t_c)}{\sqrt{1+4\pi^2\beta^2\lambda^4}}$, $\phi_1=\dfrac{1}{2}(2n+1)\tan^{-1}(2\pi\beta\lambda^2)$, $\phi_2=\pi/2+\tan^{-1}(2\pi\beta\lambda^2)$, and $\phi_3=\tan^{-1}(2\pi\beta\lambda^2)$.

Using the semi-active case in (6) and using (17), (5) can be expressed using the Hermite expansion as,

$$S(\alpha,\tau)=\left\{\sqrt{\alpha}\int_{-\infty}^{\infty}\sum_{n=0}^{N}R_2(f)G_nZ_n^*(\alpha f)e^{j2\pi f\tau}\,df\right\}*_{wb}\left\{\sqrt{\alpha}\int_{-\infty}^{\infty}\sum_{m=0}^{N}R_1(f)G_mZ_m^*(\alpha f)e^{j2\pi f\tau}\,df\right\} \quad (18)$$

where $R_1(f)$ and $R_2(f)$ are the Fourier transforms of the two received signals $r_1(t)$ and $r_2(t)$, respectively.

Because $Z_n(f)$ has been obtained in closed form, it is easier to evaluate (18). The procedure of computing the WSF in (18) efficiently using the Hermite expansion of the signal can be then outlined as follows: (i) obtain the amplitude envelope, $w(t)$, and estimate the optimal values of spread and the shift parameters using (10), (ii) compute the Hermite decomposition coefficients, G_n, of the envelope, $w(t)$, using (9), (iii) evaluate the Fourier transform of the LFM AH function, $Z_n(f)$, using (17), (iv) using the Fourier transform of received signals, $R_1(f)$, $R_2(f)$, evaluate WSF in (18). Fig. 2 shows the estimation of WSF obtained using (18) for arbitrary shaped LFM signals with AWGN in the presence of three targets. The signal-to-noise ratio (SNR) of the received signals $r_1(t)$ and $r_2(t)$ were considered 40 dB and 30 dB, respectively. The three targets can be distinguished clearly in Fig. 2.

5. MULTIPLE TRANSMISSIONS FOR DETECTION

In realistic sonar and radar, it may be necessary to detect more than one target simultaneously. The targets may also be close to each other. Under these conditions, the WCAF using single transmission often fails to separate close targets due to the noise as well as resolution effects. Multiple targets detection and tracking have been an active research area for over few decades [9]-[11]. The recent methods are reported to perform well under high SNR environments. However, the methods may exhibit poor performance when the noise level is high [9].

In this part, we investigate the use of multiple transmissions by sending arbitrary windowed LFM signals at known interval of time to track and detect multiple targets in noisy environment. The target velocity can be computed for known values of scale. Then the target velocity can be used to correct for the delay for every scale and subsequently align all the WCAFs. This method could be implemented efficiently if the transmitted signal is known in analytic form. Therefore, we will use Hermite decomposition technique to have an analytical model of the transmitted signal.

For known value of scale parameter, α, the velocity of the target, υ, can be computed as $\upsilon = c(1-\alpha)/(1+\alpha)$. Then for every α, a delay correction term, $e^{-j4\pi f(\upsilon p T_P / c)}$, is introduced. Here p is the number of sonar ping, T_p is the interval of transmission, and the transmitter is sending multiple sonar pings in a regular interval. Then the WCAF is computed for all the received signals and added together to get the final WCAF. Figs. 3 and 4 show the simulation results for a single transmission and multiple transmissions, respectively, with SNR = 10 dB (same targets as in Fig. 2). There are ten sonar pings used to generate Fig. 4. Comparing Fig. 4 with Figs. 2 and 3 it is seen that the multiple transmissions provide much better resolution and posses higher noise immunity than the use of a single transmission.

6. CONCLUSIONS

In this paper we have presented an efficient technique for estimating WSF using Hermite decomposition. The main advantage of the proposed algorithms is that the signals are analyzed in the frequency domain; hence it is faster than the conventional time domain approach. We have also proposed the use of multiple transmissions to resolve close targets using the WCAF. It appears that the multiple transmissions can distinctively separate multiple targets with increased robustness to noise.

7. REFERENCES

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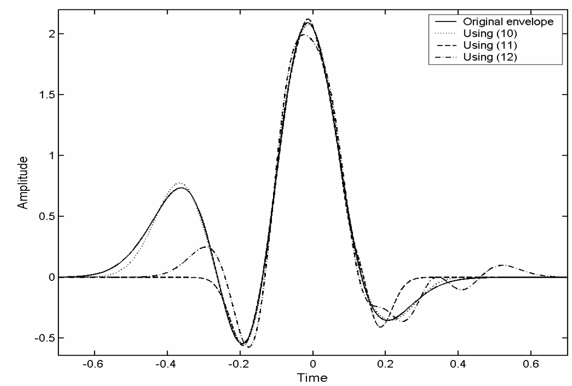

Fig. 1. Original envelope and its Hermite synthesized forms.

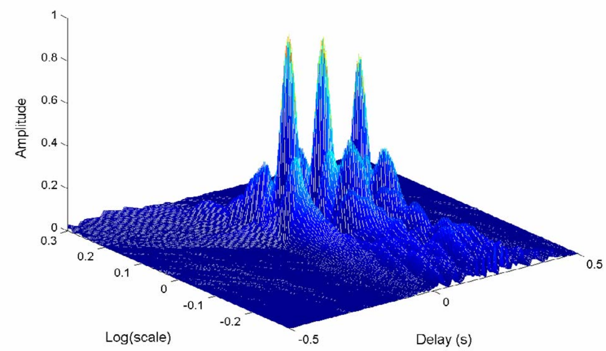

Fig. 2. 3-D plot of WSF.

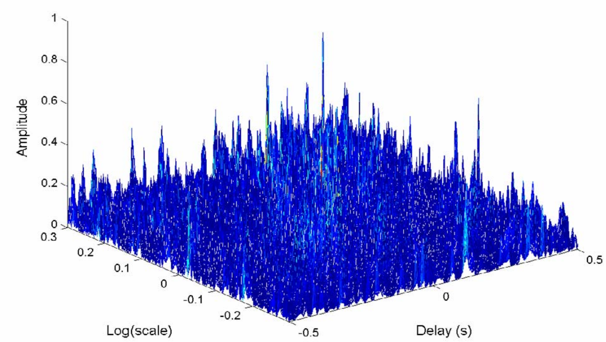

Fig. 3. 3-D plot of WCAF for a single transmission.

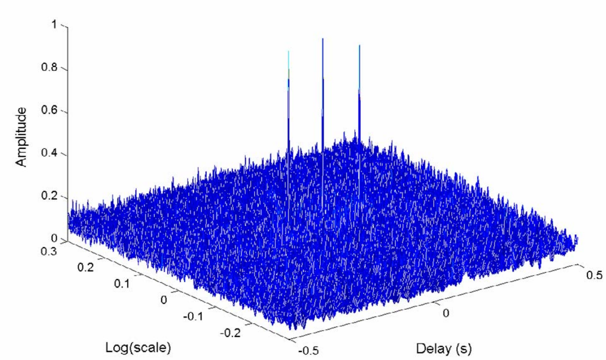

Fig. 4. 3-D plot of WCAF for multiple transmissions.