SIGNAL DETECTION IN CLUTTER USING MAXIMUM ENTROPY PDF ESTIMATION BASED ON FRACTIONAL MOMENTS

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ABSTRACT

In this paper, we introduce a new method to improve the detection performance of weak unknown radar signals in the presence of unknown clutter. We use maximum entropy (MAXENT) probability density function (PDF) estimation with a new approach based on a few sampled fractional moments (FM). These moments; i.e, their fractional orders, are obtained from the observed sample variates. Using the fractional moments instead of the integer moments the estimated PDF is quite close to the true PDF. The test statistics is a fractional polynomial of very low order of the received samples.

1. INTRODUCTION

Over the last two decades, significant progress has been made towards the development of widely applicable radar clutter models. The K-distribution has proved to be an appropriate model for characterizing the amplitude of microwave sea clutter. The parameter-estimation task for K-distributed radar clutter is to estimate some moments of the K-distribution given N independent samples of the radar signal [1], one of the simplest choices for the two moments is to use the sample mean and variance of the data. However, there are situations that other models fit the clutter environment; i.e., Rayleigh, Lognormal, Weibull, ..., etc., under these circumstances the RADAR system must be intelligent enough to capture the appropriate model. This can only be possible if the underlying random behavior of the clutter, PDF, is known. RADAR system performance for target detection relies on false alarm probability (P_{fa}), the target detection probability (P_d), and the time for detection. These parameters heavily depend on the clutter PDF, hence, the statistical characterization of clutter is a key point in the performance analysis. Statistical properties of clutter may only be known under very limiting circumstances, e.g., by assuming a homogeneous background, a RADAR signal return from two homogeneous environment such as sky and sea clutter definitely does not follow each one of them and detectors devised for this scenario may suffer a performance degradation.

If a set of moments meet the Carleman condition [2] then a unique PDF can be determined out of them. Moments are attractive because their computation is algorithmically simple and uniquely defined for any signal; it can be carried out in parallel and therefore very fast, and, since moments are global quantities, all available information is used making moment-based methods less vulnerable to losses or changes of details than methods that use few particular features of the signal. However, moments become very noise-sensitive with increasing order. Hence, the lowest possible orders should be used in a moment-based procedure. The classical moment based methods involves very few integer moments. We describe how a very few fractional- and possibly negative-order moments can be used to increase the accuracy of PDF estimation in MAXENT sense in a K or Weibull distributed clutter environment, they are the widely accepted models for the compound Gaussian clutter models. Generally speaking, the integer moments of spiky clutters cannot be reliably estimated from the sample returns unless for very large data set causing a prolonged detection time. In order to capture the nature of clutter from possibly nonhomogeneous background we should rely on low order; i.e, fractional moments. However, all fractional moments may not equally be suitable for estimating PDF of the cluttered environment. In this paper, we estimate the PDF of a clutter using MAXENT method using the optimum FM. In our scheme we use MAXENT method which has been involved in the solution of many statistical problems and we use it to fit the PDF for clutter. The chief assertion of the MAXENT PDF estimation is that the most unbiased PDF is the maximum entropy distribution satisfying some constraints which are usually a number of known moments, fractional or integer. The best way to give a short introduction to MAXENT is to offer a quote from one of the pioneers of these techniques, Edwin Jaynes [3]: The notion of entropy defines a kind of measure on the space of probability distributions, such that those of high entropy are in some sense favored over others. The maximum entropy distributions are "in some sense favored" can be backed up by mathematically proving what has come to be called the concentration theorem [3]. The result of this is that for a given set of constraints such as moments or their functions, if there is a family densities that could give us our solution, most of the solutions are concentrated, or close to the maximum entropy PDF. Thus, it is our best guess to take MAXENT PDF as the distribution of the desired variate. This paper is organized as follows. In section 2, a discussion about MAXENT PDF estimation based on FM is provided, in section 3, the detection problem in presence of clutter is presented, and in section4, we discuss the new detector using MAXENT and FM. In section 5, we provide the simulations and results, and some concluding remarks at the end.

2. MAXENT DENSITY ESTIMATION VIA FRACTIONAL MOMENTS

It is a well known fact that a finite set of moments does not allow to calculate PDF of a random process. To get an unambiguous statistic, one has to approximate the unspecified moments in some sense. One way to do this is maximization of differential entropy. In this paper, we utilize the MAXENT principle as follows. Given the received samples of clutter, possibly in addition to noise, or signal plus clutter, and noise, we estimate PDF in MAXENT sense that matches the received data. We note that the traditional MAXENT [3] approach is based on a give set of moments or estimated sample moments to estimate PDF in MAXENT sense, but in this paper, we find the best set of moments, fractional or integer, that fit the received data set optimally in MAXENT sense. It is shown that MAXENT PDF estimation based on fractional moments has better performance than integer moments [4, 5]. We consider a positive random variable X with PDF $f(x)$. Our problem is to maximize the entropy functional $H[f] = -\int_0^\infty f(x)\ln f(x)dx$ subject to some FM $\mu_j = \{E(X^{\alpha_j})\}_{j=0}^{M}$ where the FM based MAXENT PDF is given as follows

$$f_M(x) = \exp\left(-\sum_{j=0}^{M}\lambda_j x^{\alpha_j}\right), \tag{1}$$

where $\lambda_0, \cdots, \lambda_M$ are the Lagrange multipliers corresponding to the following M FM constraints

$$\mu_{\alpha_j} = E(X^{\alpha_j}) = \int_0^\infty x^{\alpha_j} f_M(x)dx, \quad j = 0, \cdots, M, \tag{2}$$

where $\alpha_0 = 0$. Then the entropy is represented by

$$H[f_M] = -\int_0^\infty f_M(x)\ln f_M(x)dx = \sum_{j=0}^{M}\lambda_j\mu_{\alpha_j}. \tag{3}$$

If we assume that $F(x)$ and $F_M(x)$ are the cumulative distribution function for the exact and MAXENT solution, respectively, it has been shown [4, 5, 6] that we have the following bound for the difference between these two functions

$$\sup_{x\in[0,\infty)} |F_M(x) - F(x)| \leqslant 3\sqrt{-1 + \sqrt{1 + \frac{4}{9}(H[f_M] - H[f])}},$$

therefore, a convergence in entropy is translated into convergence in distribution. If we define the divergence measure of two PDF's as $\int_0^\infty f(x)\ \ln\ (f(x)/f_M(x))dx$, whenever the two PDF's have the same fractional moments we have

$$\int_0^\infty f(x)\ln\frac{f(x)}{f_M(x)}dx = H[f_M] - H[f] \tag{4}$$

Hence the two entropies converge to each other in the case of the FM's equivalence. Therefore we can always find an optimal choice for the fractional parameters α_j [4, 5, 6]. We assume $\{x_1, \cdots, x_N\}$ are the received samples, then, in order to determine the parameters of $f_M(x)$ in (1), we implement the following optimization for $j = 0, \cdots, M$

$$\min_{\alpha_j,\lambda_j} H[f_M] = \sum_{j=0}^{M}\lambda_j\hat{\mu}_{\alpha_j}, \quad \hat{\mu}_{\alpha_j} = \frac{1}{N}\sum_{n=1}^{N}x_n^{\alpha_j}. \tag{5}$$

Also, it has been proven that the convergence to the exact PDF holds as $M\to\infty$ [5, 6]. Our optimization results show that applying FM instead of the integer moments causes the MAXENT estimated PDF $f_M(x)$ to converge to $f(x)$ much faster. For example in Figures 1 and 2, we compare the resulting MAXENT density estimates by 5000 samples of a K–distributed random variable. Using the first four sample integer moments, and two sample FM, that the optimization (5) determines (1.1093,1.9989), we arrive at the following estimates for the PDF of K–distribution (7) with parameter $\nu = 0.5$ for $x \in [x_{\min} = 6.4\text{e}{-7}, x_{\max} = 12.2]$

$$f_M(x) = \exp\left(-0.22 - 0.58x - 0.186x^2 + 0.027x^3 - 0.001x^4\right),$$
$$f_M(x) = \exp\left(-0.0667 - 0.9558x^{1.1093} + 0.033x^{1.9989}\right)$$

Our comparative measure is the relative error defined as

$$\text{relative error} = \frac{|\text{True PDF} - \text{Approximated PDF}|}{\text{True PDF}}.$$

As it is shown in Figures 1 and 2, the PDF obtained via two optimized FM provide a better accuracy over the MAXENT PDF estimator based on four integer moments.

3. SIGNAL DETECTION IN CLUTTER ENVIRONMENTS

In the past two decades there have been lots of works in the literature to claim the validity of non-Gaussian clutter models. Among these models the compound-Gaussian model is of great interest. In this model, which its properties are close to the Gaussian vectors, a vector of size N of clutter returns mathematically corresponds to a spherically invariant random vector (SIRV). If we consider the complex clutter envelope vector as $\boldsymbol{c}$; (the boldfaced characters are to denote vectors), we have

$$\boldsymbol{c} = \boldsymbol{x}s, \tag{6}$$

where s is a non-negative random variable and $\boldsymbol{x} \sim \mathcal{N}(0, \mathcal{M})$ where $\mathcal{N}(\cdot)$ is to denote the Gaussian PDF, and $\mathcal{M}$ is the covariance matrix. Based on the PDF of s, $f(s)$, the clutter variable $\boldsymbol{c}$ may have different densities. In the case of K–distributed clutter, $f(s)$, is a generalized Chi-PDF and it has been also shown that for the Weibull clutter $f(s)$ can be expressed in terms of Meijers G functions[9]. The densities

$$f_k(c) = \frac{b^{\nu+1}}{\Gamma(\nu)2^{\nu-1}}c^{\nu}K_{\nu-1}(bc), b = \sqrt{2\nu}, \quad c \geqslant 0 \tag{7}$$
$$f_w(c) = abc^{b-1}\exp(-ac^b), \quad c \geqslant 0 \tag{8}$$

are respectively K and Weibull clutter densities. According to the SIRV properties, the problem of detecting the signal in a correlated clutter is equivalent to that of detecting its filtered version [7]. Therefore we can assume the uncorrelated disturbance without loss of generality. The hypothesis tests are established as follows

$$H_0 \ : \ \boldsymbol{y} = \boldsymbol{c} \tag{9}$$
$$H_1 \ : \ \boldsymbol{y} = \boldsymbol{\alpha p} + \boldsymbol{c} \tag{10}$$

where $\boldsymbol{y}, \boldsymbol{p}$ and $\boldsymbol{c}$ are N dimensional complex vectors of the received signal, the transmitted signal, and the uncorrelated clutter, respectively. $\boldsymbol{\alpha}$ is a complex vector in the form of $A\exp(j\theta)$; A is a random variate, specified according to the target fluctuating model and θ is a uniform random variable. Now we assume that we have no prior information about the target parameter $\boldsymbol{\alpha} = A\exp(j\theta)$ in N pulses, then, in this case, the generalized likelihood ratio test (GLRT) is a suitable detection approach [8]. GLRT is a Neyman-Pearson likelihood ratio test in which the unknown parameter is substituted by its maximum likelihood solution. Since GLRT optimum test usually has a complex structure, we can consider the asymptotic forms of the GLRT instead of its optimum tests [7, 10] as a comparison to the new technique presented in this paper.

4. THE NEW DETECTION SCHEME USING FM MAXENT

Our detection scheme is based on the well known Neyman-Pearson likelihood ratio test and the MAXENT PDF estimation of the hypothesis test sample variates with the aid of FM. The Neyman-Pearson log-likelihood test statistics for the N received pulses is

$$\begin{aligned} T(\boldsymbol{y}) &= \sum_{i=1}^{N} a_i(y_i), \\ a_i(y_i) &= \ln\left(\frac{f_i(y_i|H_1)}{f_i(y_i|H_0)}\right). \end{aligned} \tag{11}$$

Since the samples returned from different transmitted pulses are IID, by solving the optimization problem in (5), for a single pulse returned sample variates, we have

$$f\left(y_i|H_1\right) = \exp\left(-\sum_{j=0}^{M} \lambda_{1j} y_i^{\alpha_{1j}}\right) \tag{12}$$

$$f\left(y_i|H_0\right) = \exp\left(-\sum_{j=0}^{M} \lambda_{0j} y_i^{\alpha_{0j}}\right). \tag{13}$$

Using the above PDF models and the Neyman-Pearson test statistics in (11), we are led to the following new test statistics for N pulses

$$T(\boldsymbol{y}) = \sum_{i=1}^{N} a_i(y_i) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{14}$$

$$a_i(y_i) = \left(\sum_{j=0}^{M} \lambda_{0j} y_i^{\alpha_{0j}} - \sum_{j=0}^{M} \lambda_{1j} y_i^{\alpha_{1j}}\right),\ i = 1, \cdots, N. \tag{15}$$

Since, by means of optimized FM, the MAXENT PDF of (5) converges to the true PDF, the above test in (14) is a suitable statistics consisting of only a very few terms and easily applicable and implementable into a digital system. On the other hand, the method does not require any prior information about the clutter density, therefore, we could apply the new detection scheme in various types of clutter environments. Moreover, the estimated PDF via FM-MAXENT definitely converge to the true, but (un)known (non)homogeneous clutter PDF.

5. SIMULATIONS AND RESULTS

We consider the N IID uncorrelated clutter samples and compare the performance of the new detection scheme with the GLRT detection technique. We assume that in the target parameter $\boldsymbol{\alpha} = A\exp(j\theta)$ both the amplitude and phase are unknown and fluctuating according to the SwerlingII target model. In Figure 3, P_d is illustrated versus the signal to clutter ratio (SCR) for a single pulse. P_{fa} is set to 10^{-3}, the K–distributed parameter is $\nu = 0.5$ and the number of pulses are $N = 10$. Figure 4 shows the ROC curves for two detectors in the presence of the K–distributed clutter. The parameters are $\nu = 1, 0.5$, $N = 10$ and SCR $= -5$dB. We have summarized some of the simulation results in the Weibull clutter environment in tables 1 and 2, we also note that the GLRT detector in literature for Weibull clutter is too complex to be implemented [10]. In this case, because of the complexity of the optimum GLRT test, we have used the asymptotic GLRT for our comparison. All these performances are obtained via the Monte Carlo simulation method. We see the improvement in the performance of the new detector over all other detectors currently available.

6. CONCLUSION

In this paper, we introduced a new test statistics for detection of unknown RADAR signals in the presence of the cluttered environment without any prior knowledge about the clutter density function. Our detection test is based on the MAXENT PDF estimation by means of the optimum FM. We compared our simulations with the GLRT strategy. The results show a promising performance using the new detection scheme.

7. REFERENCES

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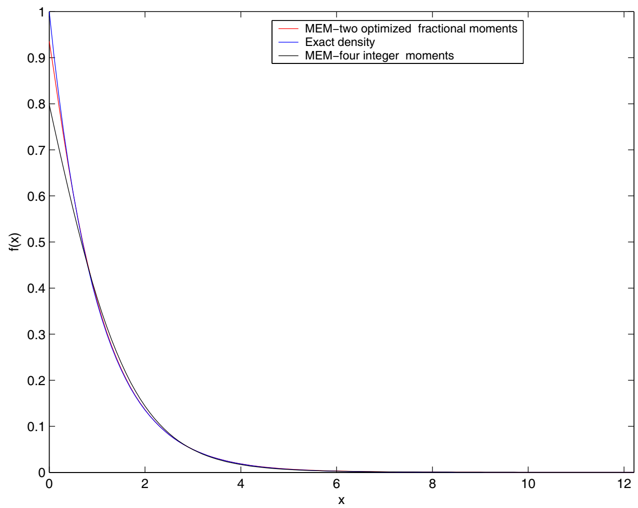

Fig. 1. comparison of MAXENT density estimates for a $K-$ distributed random variable($\nu = 0.5$), using four integer moments, and two optimum FM.

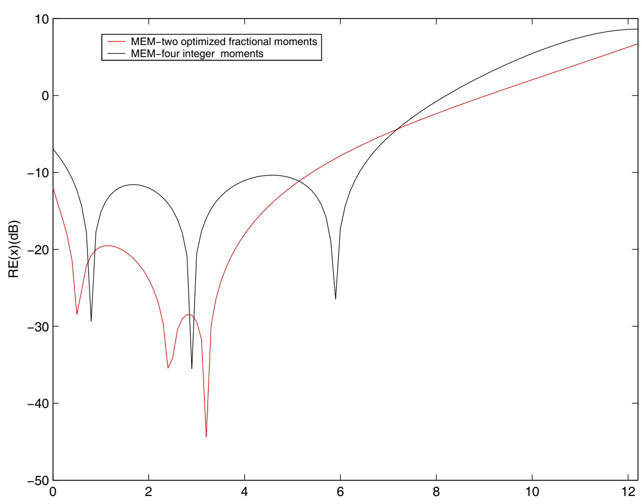

Fig. 2. Relative errors of MAXENT density estimates for a $K-$ distributed random variable($\nu = 0.5$), using four integer moments, and two optimum FM.

P_{fa}	GLRT(P$_d$)	New(P$_d$)
10^{-4}	0.38	0.5
10^{-3}	0.5	0.7
10^{-2}	0.6	0.8
10^{-1}	0.75	0.95

Table 1. Performance comparisons between the GLRT and the new detection scheme in the presence of the Weibull distributed clutter and the SwerlingII target fluctuating model, $a = 1$, $b = 0.5$ and SCR= -5dB.

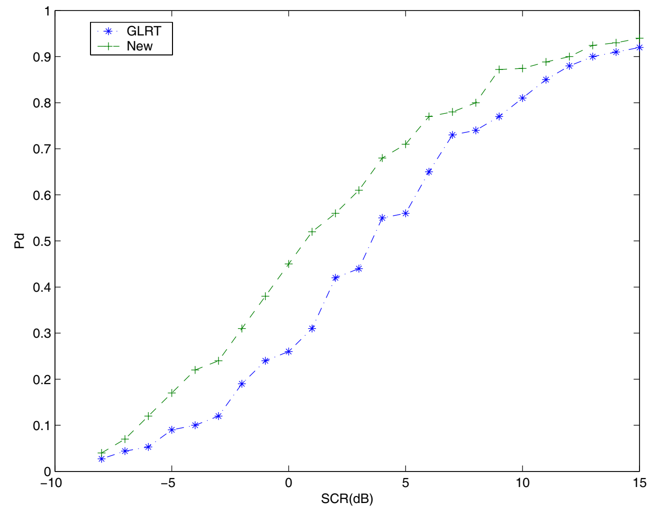

Fig. 3. Performance comparisons between the GLRT and the new detection scheme in the presence of the K–distributed clutter and the SwerlingII target fluctuating model, $\nu = 0.5$ and $P_{fa} = 10^{-3}$

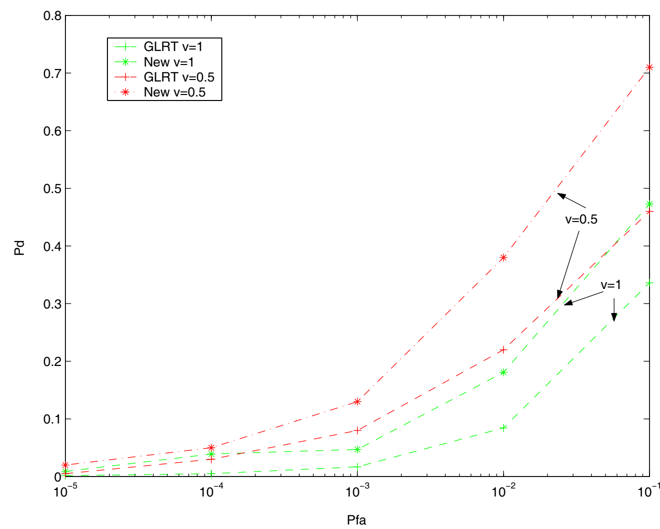

Fig. 4. The ROC curves for the GLRT and the new detection scheme in the presence of the K–distributed clutter and the SwerlingII target fluctuating model, $\nu = 1, 0.5$ and SCR= -5dB

SCR(dB)	GLRT(P$_d$)	New(P$_d$)
-5	0.23	0.32
-3	0.35	0.41
-1	0.3	0.5
1	0.5	0.7

Table 2. Performance comparisons between the GLRT and the new detection scheme in the presence of the Weibull distributed clutter and the SwerlingII target fluctuating model, $a = 1$, $b = 0.8$ and $P_{fa} = 10^{-3}$.