ALTERNATIVE CONSTRAINT STRATEGIES TO THE ESMI ALGORITHM IN RADAR SYSTEMS

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ABSTRACT

This paper considers interference cancellation in radar systems when the signal environment is non-stationary and focuses on the extended sample matrix inversion algorithm (ESMI) first proposed by Hayward [1]. An explicit expression of the signal to noise plus interference ratio (SINR) obtained by this ESMI algorithm is given and analyzed. Compared to the SMI algorithm, it is shown that the performance improves for mainlobe jammers but degrades for sidelobe jammers. To overcome this drawback, an alternative constraint strategy is proposed which attains the good performances of the standard ESMI algorithm whatever the position of the jammers. Finally, the explicit expressions of these SINR are compared to Monte Carlo simulations w.r.t. implementation conditions.

1. INTRODUCTION

The problem of interference cancellation in radar environments has been extensively studied with stationary signals. For non-stationary conditions, Hayward proposed in 1996 an extension of the stationary effective Sample Matrix Inversion Algorithm (SMI, [2]) that he called Extended SMI (ESMI, [1]). It consists in calculating a time-varying spatial filter and is based on its series decomposition. The first objective of the algorithm is to perform well in the context of a rotating antenna with mainlobe jammers whereas the standard SMI degrades [3]. Simulations have thus shown an important improve of performances over those of the SMI algorithm [1].

However, in the case of a rotating antenna with jammers distant from the target, the performance of this ESMI algorithm can degrade compared to the standard SMI algorithm. Thus, a practical rule could consist in using the ESMI algorithm only in the neighbourhood of the jammers. However, because the jammers positions are in fact often unknown, the previous choice is not so easy. It would be preferable to use a single algorithm for all situations. The objective of this paper is to propose an alternative constraint to the constraints introduced in [1] to be used whatever the jamming situation and to analyze the performances of this ESMI algorithm.

This paper is organized as follows. The problem statement is given in Section 2, with a special attention to the data radar used to compute the adaptive weights and to form the beamformer. In Section 3, explicit expressions of the SINR are given for the standard ESMI algorithm and for the proposed ESMI algorithm based on an alternative constraint suggested by the Generalized Sidelobe Canceller (GSC).

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2. PROBLEM STATEMENT

2.1. Radar environment and training data hypotheses

Let suppose that the environment be composed of jammers, thermal noise, and a moving target. The ground-based radar emits an M- pulse waveform at pulse repetition interval T. In each PRI (Pulse Repetition Interval), the data is divided into two sets called primary and secondary data. The primary data consists of the samples to be filtered and is composed by interference (jammers and thermal noise) and possibly signal. The secondary data is the training data, and is supposed to be only made of jamming and thermal noise components. Let denote N, the number of secondary samples in each PRI. To calculate the spatial filter to apply on the primary data of the whole CPI (Coherent Processing Interval), we dispose of NM samples. We suppose that the J jammer signals $\{\mathbf{j}_n^{(m,j)}\}_{n=1..N,m=1..M,j=1..J}$ are zero-mean, with power σ_J^2. They are spatially correlated, but temporally white, independant from each other and supposed to be static.The thermal noise $\{\mathbf{n}_n^{(m)}\}_{n=1..N,m=1..M}$ is modelled by a white complex process, with power σ_N^2 (that will be unitary in the following). The signal is considered as deterministic, with unknown power σ_S^2 but known direction. We note $\{\mathbf{x}_n^{(m)}\}_{n=1..N,m=1..M}$ the K- dimensional secondary data (K being the number of sensors) and have :

$$\mathbf{x}_n^{(m)} = \mathbf{j}_n^{(m)} + \mathbf{n}_n^{(m)}$$

where $\mathbf{j}_n^{(m)} = \sum_{j=1}^{J} \mathbf{j}_n^{(m,j)}$.

2.2. Context and performance analysis

We consider an arbitrary array antenna. We consider a situation where it is rapidly rotating to the CPI scale, but slowly compared to a PRI [3]. Based on secondary samples, several implementations schemes for the SMI algorithm are possible, depending on the frequency update of the spatial filter in the CPI. To simplify, we restrict ourselves to the calculution of one filter per PRI or one per CPI and compare their advantages and drawbacks in the following table :

Alg.	SMI/PRI	SMI/CPI
+	robust to rotation	degrades with rotation
-	lack of samples	exploits all samples

In such a situation, we see that none of the previous solutions is satisfactory and the use of the ESMI algorithm is then justified. The latter is calculated from all the secondary data of the CPI and the resulting time varying spatial filters are applied on each PRI.

In order to evaluate the performance of the proposed processing, we need to take into account the filtered signal on the whole CPI. We choose the spatio-temporal SINR corresponding to a space-time processing [4] with a non adaptive temporal filter. The expression of the SINR is given by : $SINR = \frac{\sigma_S^2 \left|\mathbf{W}^H \mathbf{\Phi}\right|^2}{\mathbf{W}^H \mathbf{R} \mathbf{W}}$, where $\mathbf{R}$ is the total noise spatio-temporal correlation matrix. In the latter expression, we have introduced the spatio-temporal filter : $\mathbf{W} = \begin{pmatrix} \mathbf{w}_S^1 \\ \vdots \\ \mathbf{w}_S^M \end{pmatrix}$ where $\{\mathbf{w}_S^m\}_{m=1..M}$ are spatial filters and where $\mathbf{\Phi}$ is the spatio-temporal steering vector. Because the angle variation due to rotation is weak, we neglect the loss due to the spatial steering vector $\boldsymbol{\phi}$ (with $\|\boldsymbol{\phi}\|^2 = K$) variation. Thus, we will write $\mathbf{\Phi} = \begin{pmatrix} \phi_T(1)\boldsymbol{\phi} \\ \vdots \\ \phi_T(M)\boldsymbol{\phi} \end{pmatrix}$ whether the antenna is rotating or not and where $\phi_T(m)$ represents dephasing due to Doppler effect. Moreover, we will withdraw the signal power, because it disappears in both normalizations.

3. ESMI ALGORITHMS

3.1. Standard ESMI algorithm

In our context, the spatial noise covariance is time-varying, that is $\mathbf{R}_S = \mathbf{R}_S(t) = E\left\{\mathbf{x}_t \mathbf{x}_t^H\right\}$. Hayward's idea [1] has been to write the spatial filter $\mathbf{w}_S(t) = \mathbf{w}_0 + t\mathbf{\Delta w}$. In that case, the Minimum Variance Distortionless Response (MVDR) problem (see e.g., [5]) becomes :

$$min_{(\mathbf{w_0},\mathbf{\Delta w})/\mathbf{w}(t)^H \mathbf{c}_t = 1} \; E\left\{\left|(\mathbf{w}_0 + t\mathbf{\Delta w})^H \mathbf{x}_t\right|^2\right\} \quad (1)$$

where different constraints $\mathbf{c}_t$ are proposed in [1].

With $\tilde{\mathbf{x}}_t \overset{def}{=} \begin{pmatrix} \mathbf{x}_t \\ t\mathbf{x}_t \end{pmatrix}$, $\tilde{\mathbf{w}} \overset{def}{=} \begin{pmatrix} \mathbf{w}_0 \\ \mathbf{\Delta w} \end{pmatrix}$, and $\tilde{\mathbf{c}}$ a constraint associated with $\mathbf{c}_t$, (1) becomes :

$$min_{\tilde{\mathbf{w}}/\tilde{\mathbf{w}}^H \tilde{\mathbf{c}} = 1} \; E\left\{\left|\tilde{\mathbf{w}}^H \tilde{\mathbf{x}}_t\right|^2\right\}$$

The implementation of this standard minimization is then realized by a SMI algorithm which in that case is called the ESMI algorithm :

$$\hat{\tilde{\mathbf{w}}} = \hat{\mathbf{R}}^{-1} \tilde{\mathbf{c}} \quad (2)$$

where $\hat{\mathbf{R}} = \begin{pmatrix} \hat{\mathbf{R}}_{(0)} & \hat{\mathbf{R}}_{(1)} \\ \hat{\mathbf{R}}_{(1)} & \hat{\mathbf{R}}_{(2)} \end{pmatrix}$ with $\hat{\mathbf{R}}_{(j)} = \frac{\sum_{m,n=1}^{M,N} t_{m,n}^j \mathbf{x}_n^{(m)} \mathbf{x}_n^{(m)H}}{NM}$.

Different types of constraints can be used for the extended filter. One and several independant constraints have been proposed in [1]. However, the most classical one consists in imposing a constraint on $\mathbf{w}_0$, while leaving $\mathbf{\Delta w}$ unconstrained (see e.g., [7] where $\tilde{\mathbf{c}} = \begin{pmatrix} \boldsymbol{\phi} \\ \mathbf{0} \end{pmatrix}$). The performance analysis in SINR of this ESMI algorithm is analysed in the following.

3.2. Performance analysis of the standard ESMI

First, let us show that the performance analysis done on thermal noise alone is also valid in presence of a single sidelobe jammer. Consider a stationary scenario for which $\mathbf{R}_S = \sigma_J^2 \boldsymbol{\phi}_J \boldsymbol{\phi}_J^H + \sigma_N^2 \mathbf{I}$ where $\boldsymbol{\phi}_J$ denotes the jammer spatial steering vector. After straightforward algebra manipulations, the following SINR is obtained :

$$SINR = \boldsymbol{\phi}^H \mathbf{R}_S^{-1} \boldsymbol{\phi} = \frac{K}{\sigma_N^2} \left(1 - \frac{\left|\boldsymbol{\phi}^H \boldsymbol{\phi}_J\right|^2}{K\frac{\sigma_N^2}{\sigma_J^2} + K^2}\right)$$

This implies that $SINR \approx \frac{K}{\sigma_N^2}$ if $\frac{\left|\boldsymbol{\phi}^H \boldsymbol{\phi}_J\right|^2}{K^2} \ll 1$, i.e., if the jammer is in antenna sidelobes. When several jammers are present in antenna sidelobes, an analytic proof of the equivalence is more delicate. However, it can be shown through simulations that the previous result remains valid.

The analysis is performed in exact statistics where the estimated matrices $\hat{\mathbf{R}}_{(j)}$ are replaced by their expectation. Thus, we have $\hat{\mathbf{R}}_{(j)} \approx \sigma_N^2 s_j \mathbf{I}$ with $s_j = \frac{1}{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} t_{m,n}^j$. Then suppose that the secondary data corresponds to the last samples of each PRI which contains the total number L of samples. After withdrawing the sampling period which does not intervene in the calculutions, we have : $t_{m,n} = mL - N - 1 + n$. The following normalized (w.r.t. the optimal SINR obtained with a motionless array) spatio-temporal SINR at the filtered range gate k is proved in Appendix 6.1.

$$\rho = \frac{\left(s_2 - ks_1 - s_1\left(\frac{M-1}{2}\right)L\right)^2}{\left((s_2 - ks_1)^2 - s_1(s_2 - ks_1)(M-1)L + \frac{s_1^2 (M-1)(2M-1)L^2}{6}\right)} \quad (3)$$

To illustrate this involved formula, we first note that it is observed by simulation, this SINR approximately depends on L and N through their ratio $\frac{L}{N}$. Then we set default values to $k = 0$, $M = 10$, $N = 100$ and $L = 2000$ (first column of the following tabular) and test the influence of each of the parameters (other columns) while keeping the others unchanged.

		$k{=}1899$	$M{=}1$	$M{=}100$	$\frac{L}{N}{=}2$	$\frac{L}{N}{=}200$
ρ	-3.65	-6.54	0	-5.71	-4.02	-3.65

We notice that k and M have a greater influence on the SINR than L and N.

In order to remedy this loss in performance, we propose to derive its (GSC) form [6].

3.3. GSC form of ESMI

The algorithm ESMI corresponds to the implementation of the direct solution of a MVDR problem. But the problem is equivalent to an unconstrained one written in its GSC form [6] represented on the following figure :

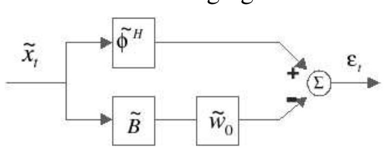

Figure 1 : GSC form corresponding to ESMI

We denote $\tilde{\mathbf{B}}$ the blocking matrix such that $\tilde{\mathbf{B}}\tilde{\boldsymbol{\phi}} = \mathbf{0}$. The relation between the spatial filter $\tilde{\mathbf{w}}$ and the GSC filter $\tilde{\mathbf{w}}_0$ is the following :

$$\tilde{\mathbf{w}} = \tilde{\boldsymbol{\phi}} - \tilde{\mathbf{B}}^H \tilde{\mathbf{w}}_0$$

where $\tilde{\mathbf{w}}_0 = \left(\tilde{\mathbf{B}}\tilde{\mathbf{R}}\tilde{\mathbf{B}}^H\right)^{-1}\left(\tilde{\mathbf{B}}\tilde{\mathbf{R}}\tilde{\boldsymbol{\phi}}\right)$.

A problem occuring with the use of the ESMI algorithm comes from the fact that is impossible to choose a blocking matrix $\tilde{\mathbf{B}}$ such that no signal component is present in the auxiliary data of the GSC algorithm. Indeed, the vector $\begin{pmatrix} \mathbf{0} \\ \boldsymbol{\phi} \end{pmatrix}$ which is orthogonal to $\tilde{\boldsymbol{\phi}}$ belongs to $\mathrm{span}(\tilde{\mathbf{B}})$. This statement suggests us to use another constraint, which is expressed in the following.

3.4. An optimal constraint strategy

In order to avoid suppressing signal part after filtering, we propose to choose a constraint vector $\tilde{\boldsymbol{\phi}} = \begin{pmatrix} \boldsymbol{\phi} \\ \alpha\boldsymbol{\phi} \end{pmatrix}$ and look for an optimal (in a certain sense) value for α.

After using the same method as in the previous subsection, the following normalized SINR is obtained when this alternative constraint is used :

$$\rho = 1 - s_4^2 \left(\frac{\frac{(M-1)(M+1)L^2}{12}}{s_3^2 - s_3 s_4 (M-1)L + \frac{s_4^2 (M-1)(2M-1)L^2}{6}} \right) \quad (4)$$

where $s_3 = s_2 - k s_1 - \alpha' s_1$ and $s_4 = s_1 - \alpha'$ with $\alpha' \stackrel{def}{=} \frac{\alpha}{T_e}$.

We know that the normalized SINR is upper bounded by one and see in (4) that this upper bound is achieved when $s_4 = 0$, that is for $\alpha_{opt} = s_1 T_e = [\frac{(M+1)L-(N+1)}{2}]T_e$.

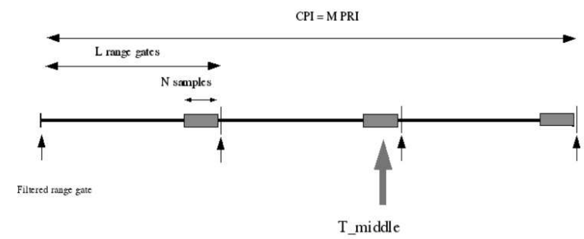

Figure 2 : Position of α in the CPI

As shown on Figure 2, the optimal constraint obtained by our approach is intuitive because it consists in imposing that the weight vector has a unit gain in the steering direction at a time corresponding approximately to the "middle" of the CPI that we can note $T_{middle} = \alpha$. The constraint writes : $\mathbf{w}_S(T_{middle})^H \boldsymbol{\phi} = 1$.

It is worth noticing that this optimal value $\rho = 1$ does not depend on the tested range gate k contrary to the standard ESMI algorithm (see (3)). However, the next figure shows that the SINR is very sensitive to the value of α at the different distant range gates.

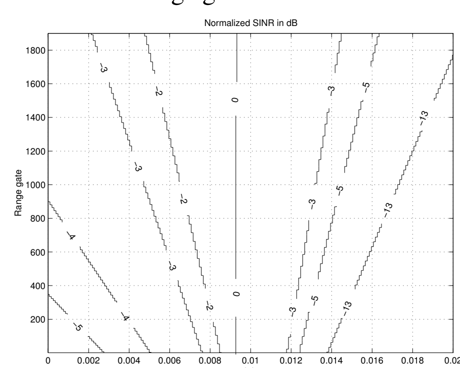

Figure 3 : Normalized SINR as a function of α

4. SIMULATIONS

We now present simulations to compare the performance of the two previous constraints used with the ESMI algorithm with a uniform linear array antenna. For the simulation, we use the following typical radar parameters:

K	M	N	L	θ_J(deg)
8	10	100	2000	35
σ_N^2(dB)	σ_J^2(dB)	N_J	α	T(s)
0	50	1	0.0109	2e-4

We compare performances when the antenna is rotating to the realistic speed of $1/2$ turn/sec, which corresponds to a rotation of 0.06 radians during the CPI or 0.25 beamwidths. Figure 4 represents the normalized SINRs (w.r.t. the optimum value) for three algorithms (SMI, ESMI with the standard constraint and ESMI with the proposed constraint). The plots are obtained from Monte Carlo simulation of 100 runs. We compare them to the optimal SINR curve obtained with a full STAP processing when the covariance $\mathbf{R}(t)$ is known.

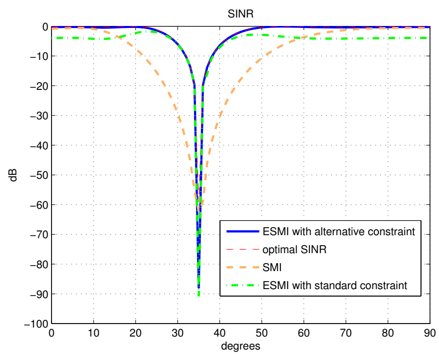

Figure 4 : Performance of algorithms

On this figure, we first verify that the improve on performance due to the use of the ESMI algorithm is significant in the jammer zone (a gain of about 20 dBs). Then, we notice that the choice of the constraint has not much influence when the target is close to the jammer. However, when the jammer is seen in a distant sidelobe, that is when the situation becomes similar to a situation without jamming, an important loss (-4 dB in accordance with (3) appears when the standard constraint is used. On the contrary, the use of our alternative constraint leads to performances close to the optimal SINR.

5. CONCLUSION

Though the choice of the constraint used with the standard ESMI algorithm does not seem to have an influence on its performance when the jammer is in the antenna mainlobe, we have shown in this paper that it leads to significant degradations in a situation without jammers or with sidelobe ones. However, we have proved that it was possible to choose a constraint which allows to attain optimal steady-states performances in terms of SINR, whatever the jamming situation. We have then proposed a method to derive the corresponding value of the constraint and validated the results by simulations.

6. APPENDIX

6.1. Derivation of the normalized SINR with the standard constraint

By using the Frobenius formula on the partitioned matrix inverse $\hat{\hat{\mathbf{R}}}$, we obtain from (2) with $\tilde{\boldsymbol{\phi}} = \begin{pmatrix} \boldsymbol{\phi} \\ \mathbf{0} \end{pmatrix}$:

$$\hat{\mathbf{w}}_0 = (\hat{\mathbf{R}}_{(0)} - \hat{\mathbf{R}}_{(1)}\hat{\mathbf{R}}_{(2)}^{-1}\hat{\mathbf{R}}_{(1)})^{-1}\boldsymbol{\phi} \quad (5)$$

$$\widehat{\boldsymbol{\Delta}\omega} = (\hat{\mathbf{R}}_{(1)}\hat{\mathbf{R}}_{(0)}^{-1}\hat{\mathbf{R}}_{(1)} - \hat{\mathbf{R}}_{(2)})^{-1}\hat{\mathbf{R}}_{(1)}\hat{\mathbf{R}}_{(0)}^{-1}\boldsymbol{\phi} \quad (6)$$

Then, after simple calculations, we derive the expressions for the quantities $(s_j)_{j=0..2}$:

$$\begin{aligned} s_0 &= 1 \\ s_1 &= \tfrac{(M+1)L}{2} - \tfrac{(N+1)}{2} \\ s_2 &= \tfrac{L^2(M+1)(2M+1)}{6} + \tfrac{(N+1)(2N+1)}{6} - \tfrac{(N+1)L(M+1)}{2} \end{aligned}$$

By replacing the estimated covariance by their expectation and using the previous notations in (5) and (6), we obtain the expression of the spatial filter at time t:

$$\hat{\mathbf{w}}_S(t) = \hat{\mathbf{w}}_0 + t\widehat{\boldsymbol{\Delta}\omega} = \frac{(s_2 - ts_1)}{(s_2 - s_1^2)}\boldsymbol{\phi} \quad (7)$$

Then, choosing to study the performance at the range gate number k, we get:

$$\widehat{\mathbf{W}} = \begin{pmatrix} \hat{\mathbf{w}}_S(k) \\ \hat{\mathbf{w}}_S(L+k) \\ \vdots \\ \hat{\mathbf{w}}_S((M-1)L+k) \end{pmatrix} \quad (8)$$

By using (7) and (8), we get the expression of the $SINR = \frac{|\hat{\mathbf{W}}^H\boldsymbol{\Phi}|^2}{\hat{\mathbf{W}}^H\mathbf{R}\hat{\mathbf{W}}}$:

$$SINR = \frac{\left|\boldsymbol{\phi}^H\boldsymbol{\phi}\sum_{i=1}^{M}\left(\frac{s_2 - ks_1 - (i-1)Ls_1}{s_2 - s_1^2}\right)\right|^2}{\boldsymbol{\phi}^H\boldsymbol{\phi}\sum_{i=1}^{M}\left(\frac{s_2 - ks_1 - (i-1)Ls_1}{s_2 - s_1^2}\right)^2}$$

And finally, after straightforward but tedious algebra manipulations we obtain expression (3) for the normalized SINR ($\rho = \frac{SINR}{\boldsymbol{\Phi}^H\mathbf{R}^{-1}\boldsymbol{\Phi}}$).

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