STUDY OF EARLY STOPPING CRITERIA FOR TURBO DECODING AND THEIR APPLICATIONS IN WCDMA SYSTEMS

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ABSTRACT

This paper presents a systematic study of early stopping criteria for Turbo decoding. First, statistical analysis is carried out on numerous hard/soft variables that may be used in an early stopping criterion. Desirable variables are suggested based on their statistical properties. Simulation results show that any stopping criteria based on a single variable will have BER/FER performance loss. Two criteria, each of which uses two variables, are recommended in this paper and neither of them will result in any performance loss. It is also shown in this paper that the thresholds for these variables should be set to be proportional to the logarithm of the block size instead of being proportional to the block size.

1. INTRODUCTION

Turbo codes, invented in 1993 [1], have outstanding error correction performance, and have become one of the important research topics. Turbo encoder and decoder structures are shown in Fig. 1 (a) and (b), respectively. A block of information bits $\mathbf{u}$ enter the first Recursive Systematic Convolutional (RSC) encoder in sequential order while the interleaved sequence enter the second RSC encoder [1, 2]. A Turbo decoder contains two *Soft Input Soft Output (SISO)* decoders associated with the two encoders, and the interleavers and de-interleavers between them. Either the Maximum *a posteriori* Probability (MAP)algorithm or the *Soft Output Viterbi Algorithm (SOVA)* can be used for SISO decoders. Turbo decoding is an iterative process with the exchange of reliability information. Each decoder generates two outputs at each time instant. The *extrinsic information* output from one decoder is used as the intrinsic information for the other decoder after interleaving or de-interleaving. $L_{ex}^{i,j}(k)$ denotes the extrinsic information generated by the j^{th} decoder in i^{th} iteration. The *Log Likelihood Ratio (LLR)* output from the j^{th} decoder in i^{th} iteration is denoted as $L_{lr}^{i,j}(k)$.

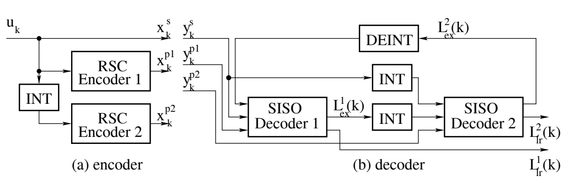

Fig. 1. Turbo encoder and decoder.

In order to achieve a satisfactory performance, a certain number of iterations have to be performed in Turbo decoding. This results in low throughput, long decoding latency and large energy consumption as well. In practice, a Turbo decoder may converge earlier (before the preset maximum number of iterations is reached) when the channel condition is good. Thus, a stopping criterion should be employed to reduce the average number of iterations and hence the decoding latency and power consumption. From an implementation standpoint, a good stopping criterion should save as many iterations as possible with no or negligible performance loss. In the meantime, the hardware overhead should be negligible.

In this paper, a systematic study of early stopping criteria for Turbo decoding is presented. First, statistical analysis is carried out on numerous hard/soft variables to be used in early stopping criteria. Good variables are suggested according to their statistical properties. Simulation results show that any criteria based on a single variable will have BER/FER performance loss. Two criteria, consisting of two variables each, are recommended in this paper which do not lead to performance loss. It is also shown that the thresholds for these variables should be set to be proportional to the logarithm of the block size instead of being proportional to the block size. The simulation results presented in this paper are performed for Turbo codes used in WCDMA systems.

2. LITERATURE OVERVIEW

Various criteria have been proposed in recent years for the early stopping function in Turbo decoding. Many of them claim to perform better than others based on some simulations. In this section, we give a brief overview of these methods while their detailed analysis is considered in the next section.

Cross Entropy (CE) Criterion. Hagenauer et al. [3] used a threshold value on the Cross entropy between the output distributions of the two SISO decoders. For a Turbo decoder, it is shown in [3] that the CE of iteration i can be approximated by,

$$T(i) \approx \sum_{k=1}^{K} \frac{L_{ex}^{i,2}(k) - L_{ex}^{i-1,2}(k)}{e^{|L_{lr}^{i,1}(k)|}} \tag{1}$$

where K is the block size. The decoding process is stopped after iteration i for $i \geq 2$, if

$$\frac{T(i)}{T(1)} < \theta \tag{2}$$

where $T(1)$ is the approximated CE after the first iteration.

Sign-Change Ratio (SCR) Criterion. Based on the concept of CE, Shao et al. [4, 5] presented two simple and effective criteria, known as SCR and hard-decision aided (HDA), respectively. SCR evaluates the number of sign changes in the extrinsic information between successive iterations, and the decoding process is stopped after iteration i for $i \geq 2$, if

$$\frac{1}{K} \sum_{k=1}^{K} \left(\mathbb{S}\left(L_{ex}^{i,2}(k)\right) \oplus \mathbb{S}\left(L_{ex}^{i-1,2}(k)\right) \right) < \theta \tag{3}$$

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where $\mathbb{S}(x)$ denotes the sign part of x and $\oplus$ denotes the *XOR* operation.

Hard-Decision Aided (HDA) Criterion. This criterion is proposed in [4, 5]. It compares the decoded bits of the two successive iterations. The decoding process is stopped after iteration i for $i \geq 2$, if

$$\mathbb{S}(L_{lr}^{i,2}(k)) = \mathbb{S}(L_{lr}^{i-1,2}(k)), \forall k \in 1...K. \tag{4}$$

HDA2 Criterion. The idea of HDA criterion is extended in [6] and three new similar hard-decision aided criteria are proposed. Among these three criteria, only one criterion has similar implementation complexity and the other two criteria require double or triple implementation complexity. Therefore, only the first criterion is discussed in this paper. The decoding process is stopped after iteration i for $i \geq 2$, if

$$\mathbb{S}(L_{lr}^{i,1}(k)) \quad = \quad \mathbb{S}(L_{lr}^{i,2}(k)), \quad \forall k \in 1...K. \tag{5}$$

HDA-DHDD Criterion. In [7], the original HDA criterion is combined with another stopping criterion proposed in [8]. After iteration i, the HDA criterion is accessed first. If the HDA criterion is satisfied, the decoding process is stopped. If the HDA criterion is not satisfied, the hard-decision-decrease (HDD) will be evaluated,

$$HDD^i = \sum_{k=1}^{K} \left(\mathbb{S}(L_{lr}^{i,2}(k)) \oplus \mathbb{S}(L_{lr}^{i-1,2}(k)) \right) \tag{6}$$

and the process will be stopped, based on difference of hard-decision-decrease (DHDD), after iteration i for $i \geq 3$, if

$$DHDD^i = HDD^{i-1} - HDD^i < \theta. \tag{7}$$

This criterion should have larger performance loss than using HDA only.

Sign Difference Ratio (SDR) Criterion. Extending the SCR method, a new criterion called SDR is proposed in [9]. SDR evaluates the number of sign differences between the intrinsic information and the extrinsic information for the same SISO decoder in the same iteration, and the decoding process is stopped after iteration i for $i \geq 1$, if

$$\frac{1}{K} \sum_{k=1}^{K} \left(\mathbb{S}(L_{in}^{i,j}(k)) \oplus \mathbb{S}(L_{ex}^{i,j}(k)) \right) < \theta. \tag{8}$$

SDR2 Criterion. A similar criterion, proposed in [10], evaluates the number of sign differences between the LLR and the sum of the intrinsic and channel information, and the decoding process is stopped after iteration i, for $i \geq 1$, if

$$\sum_{k=1}^{K} \left(\mathbb{S}(L_{lr}^{i,2}(k)) \oplus \mathbb{S}(L_{in}^{i,2}(k) + Y_s(k)) \right) = 0. \tag{9}$$

Min-LLR Criterion. The minimum of the absolute values of the LLRs is first used in an early stopping criterion in [11, 12] and is later presented in [6, 13].The decoding process is stopped after iteration i for $i \geq 1$, if

$$\min_{1 \leq k \leq K} |L_{lr}^{i,2}(k)| > \theta. \tag{10}$$

Mean-LLR Criterion. The stopping criterion based on the mean of the absolute values of the LLRs is presented in [6, 14, 15]. The decoding process is stopped after iteration i for $i \geq 1$, if

$$\frac{1}{K} \sum_{k=1}^{K} |L_{lr}^{i,2}(k)| > \theta. \tag{11}$$

Sum-LLR Criterion. In [16], the sum of the absolute values of the LLRs is calculated to avoid a costly division operation in the Mean-LLR criterion,

$$S_i = \sum_{k=1}^{K} |L_{lr}^{i,2}(k)|. \tag{12}$$

The decoding process is stopped after iteration i for $i \geq 2$, if

$$S_i - S_{i-1} \leq 0. \tag{13}$$

Comb. Min-LLR & Sum-LLR Criterion. Min-LLR and Sum-LLR criteria are combined in [16]. The decoding process is stopped after iteration i for $i \geq 2$, if

$$(S_i - S_{i-1} \leq 0) \,||\, \left(\min_{1 \leq k \leq K} |L_{lr}^{i,2}(k)| > \theta \right) \tag{14}$$

where $||$ denotes the *OR* operation.

Decoding Metrics Criterion. Decoding metrics is proposed in [11, 12]. It consists of three variables: the minimum of the absolute values of the LLRs, the minimum of the absolute values of the extrinsic information, and the number of the non-matching bits (NMb). The idea of NMb is very similar to the SDR and SDR2. It evaluates the number of sign differences between the LLRs and the extrinsic information for the same SISO decoder in the same iteration. The decoding process is stopped after iteration i for $i \geq 1$, if

$$\left(\min_{1 \leq k \leq K} |L_{lr}^{i,j}(k)| > \theta_1 \right) \& \left(\min_{1 \leq k \leq K} |L_{ex}^{i,j}(k)| > \theta_2 \right) \\ \& \left(\sum_{k=1}^{K} \left(\mathbb{S}(L_{lr}^{i,j}(k)) \oplus \mathbb{S}(L_{ex}^{i,j}(k)) \right) < \theta_3 \right) \tag{15}$$

where $\&$ denotes the *AND* operation.

Cyclic Redundancy Check (CRC) Criterion. CRC is introduced as a stopping criterion in several papers. As this criterion is not generally applicable, it is not discussed in this paper.

3. WHAT VARIABLES CAN BE USED FOR EARLY STOPPING CRITERION?

In order to find suitable variables for designing a good early stopping criterion, the relationship between the LLR, the extrinsic information and the intrinsic information should be studied first.

As shown in Eq. 16 and Eq. 17, the LLR is composed of three parts: $L_{ex}^{i,j}(k)$ is the extrinsic information to be sent to the other decoder, $L_{in}^{i,j}(k)$ is the intrinsic information received from the other decoder, and $Y_s(k)$ is the *channel information*. The relationship between the LLR and the extrinsic information is shown in the following equations:

$$L_{lr}^{i,1}(k) = L_{ex}^{i,1}(k) + L_{in}^{i,1}(k) + Y_s(k) \tag{16}$$

$$L_{lr}^{i,2}(k) = L_{ex}^{i,2}(k) + L_{in}^{i,2}(k) + Y_s(k) \tag{17}$$

where $Y_s(k) = L_c \cdot y_s(k)$ with L_c being the signal-to-noise ratio (SNR). Among these three parts, $Y_s(k)$ is fixed for every iteration, while $L_{ex}^{i,j}(k)$ and $L_{in}^{i,j}(k)$ are updated from iteration to iteration. The following two equations show the simple relationship between extrinsic information and the intrinsic information according to the iterative decoding process:

$$L_{in}^{i,1}(k) = L_{ex}^{i-1,2}(k) \tag{18}$$

$$L_{in}^{i,2}(k) = L_{ex}^{i,1}(k) \tag{19}$$

Substituting Eq. 18 and Eq. 19 into Eq. 16 and Eq. 17, the following two equations can be derived:

$$L_{lr}^{i,1}(k) = L_{ex}^{i,1}(k) + L_{ex}^{i-1,2}(k) + Y_s(k) \tag{20}$$

$$L_{lr}^{i,2}(k) = L_{ex}^{i,2}(k) + L_{ex}^{i,1}(k) + Y_s(k). \tag{21}$$

Two types of variables, hard variables and soft variables, can be constructed using the above equations. *Hard variables* refer to the variables that can be categorized as the number of mismatched sign bits. Table 1 summarizes some possible combinations for constructing these variables. As shown in this table, six variables have already been used in the literatures, and two brand-new variables are constructed by using combinatorics. Hard variables can also be functions of the number of mismatched sign bits as the DHDD used as an early stopping criterion [8], or as a part of the early stopping criterion [7]. *Soft variables* refer to the variables that are functions of the soft information, where the soft information can be cross entropy, LLR, intrinsic information or extrinsic information. Table 2 summarizes some soft variables that have been used in the literatures.

Table 1. Possible choices of hard variables.

sign bit 1	iter. no.	dec. no.	sign bit 2	iter. no.	dec. no.	Note
$\mathbb{S}(L_{ex})$	i	2	$\mathbb{S}(L_{ex})$	$i-1$	2	SCR [4, 5]
$\mathbb{S}(L_{lr})$	i	2	$\mathbb{S}(L_{lr})$	$i-1$	2	HDA [4, 5]
$\mathbb{S}(L_{lr})$	i	1	$\mathbb{S}(L_{lr})$	i	2	HDA2 [6]
$\mathbb{S}(L_{ex})$	i	j	$\mathbb{S}(L_{in})$	i	j	SDR [9]
$\mathbb{S}(L_{lr})$	i	j	$\mathbb{S}(L_{in}+Y_s)$	i	j	SDR2 [10]
$\mathbb{S}(L_{lr})$	i	j	$\mathbb{S}(L_{ex})$	i	j	NMb [11, 12]
$\mathbb{S}(L_{lr})$	i	j	$\mathbb{S}(L_{in})$	i	j	new! SDR3
$\mathbb{S}(L_{ex})$	i	j	$\mathbb{S}(L_{in}+Y_s$	i	j	new! SDR4

Table 2. Possible choices of soft variables.

func.	soft info.	iter. no.	dec. no.	Note
mean	L_{lr}	i	j	Mean-LLR [6, 14, 15]
sum.	L_{lr}	i	j	Sum-LLR [16], Comb.Min-LLR & Sum-LLR [16]
min.	L_{lr}	i	j	Min-LLR [6, 13], Comb.Min-LLR & Sum-LLR [16], part of Decoding Metrics [11, 12]
min.	L_{ex}	i	j	part of Decoding Metrics [11, 12]
-	CE	i	2	CE [3]

4. STATISTICAL ANALYSIS OF TESTING VARIABLES

In order to determine which variables perform better, a statistical analysis is carried out on ten variables, hard or soft. First, the values of these variables at the *ideal stopping point* are collected. Then, the mean μ and standard deviation σ are computed based on the collected values. The results are summarized in Table 3, Table 4 and Table 5 for different block sizes K, covering short, medium and long block sizes, respectively. Neither Mean-LLR nor Sum-LLR is listed in these tables as their variances are significantly larger than that of Min-LLR.

The standard deviation of a variable is used as a major indicator of overall early stopping performance. It should be as small as possible, since a large standard deviation causes more variation in the observed variable and hence makes it difficult to set a tighter threshold, which makes the variable less useful. In Table. 3, Table. 4 and Table. 5, the following observations have been made:

- The standard deviation of HDA2 variable is always the smallest for all block sizes, indicating its good performance regardless of the block size. It has been verified through our extensive simulation that using HDA2 generally outperforms all other stopping criteria using single variable.
- The standard deviations of NMb and Min-LLR variables are also very small for all block sizes, indicating they will also result in good early stopping performance, which again has been verified through our simulation.
- The standard deviation of CE variable decrease rapidly with the increase in the block size K, indicating that it is only suitable as an early stopping criterion for larger block sizes, which is also confirmed in our simulation.
- The standard deviations of all other variables are pretty similar and worse than the variables that have been discussed above. This explains their poorer performance from our simulation.

The implementation complexity should also be taken into account. The most complex stopping variable is CE, followed by SCR and HDA2 since they require to save the signs (of either L_{ex} or L_{lr}) of the previous iteration or half iteration. All the other variables have relatively low complexity.

Fig. 2 and Fig. 3 show the frame-error rate (FER) and bit-error rate (BER) performance with various stopping criterion when block size is 40. The unrealizable ideal stopping criterion is included for performance comparison purpose. It stops the iterative decoding process as soon as it finds out that all the bits in a data block have been correctly decoded, based on the foreknowledge of the transmitted bits. HDA2 criterion is included to show the FER and BER performance of the best single-variable criterion. As shown in these two figures, there is still performance loss compared to the ideal stopping criterion. When HDA2 variable is combined with NMb variable, there is no observable performance loss. Combining NMb variable with Min-LLR variable also results in no performance loss from the ideal criterion. These two example criteria, (HDA2, NMb) and (Min-LLR, NMb), clearly show that a good early stopping criterion has to be constructed with more than one variables. However, most criteria in the literature used only one variable [5, 4, 6, 9, 10, 14, 15, 13].

Table 3. Statistics of various variables ($K = 200$).

	SCR	SDR	SDR2	SDR3	SDR4
μ	18.8	4.9	1.2	4.2	3.6
σ	14.6	5.8	2.1	4.8	4.9
	NMb	**HDA2**	**CE**	**Min-LLR**	**Min-Lex**
μ	0.72	0.03	5.36	2.12	8.62
σ	2.1	0.34	11.1	1.85	4.25

Table 4. Statistics of various variables ($K = 1000$).

	SCR	SDR	SDR2	SDR3	SDR4
μ	36.1	8.53	1.64	7.8	6.03
σ	23.9	8.81	3.24	7.9	7.17
	NMb	**HDA2**	**CE**	**Min-LLR**	**Min-Lex**
μ	0.72	0.00	1.29	1.72	10.3
σ	1.57	0.08	3.20	1.69	4.33

Setting the threshold for the variables is an important issue which has not been addressed so far. In the past, researchers have used threshold that is proportional to the block size, such as SCR [4, 5] and SDR [9]. We found out that the mean of the testing variable should be taken into consideration when setting the threshold. As shown in Table 3, Table 4 and Table 5, the means of SCR and SDR are approximately proportional to the logarithm of the block size. This suggests that the thresholds for these two criteria should be set to be proportional to the logarithm of the block size. Similar analysis has been performed for all other variables. For the two considered criteria, (HDA2, NMb) and (NMb, Min-LLR), the thresholds for individual variables are set as follows:

$$\theta_{NMb} = \lfloor \log_2(N/256) \rfloor \tag{22}$$

$$\theta_{HDA2} = 0 \tag{23}$$

$$\theta_{MIN-LLR} = (15.0 - \lfloor \log_2(N) \rfloor)/4.0. \tag{24}$$

Next, the average savings in the number of iterations are shown in Fig. 4 as a percentage of the maximum number of iterations, which is set to be 8 in our simulations. On the average, all these stopping criteria can save more than 60% computations. In particular, the (HDA2, NMb) criterion has a stopping delay, as compared to the ideal criterion, of almost always half an iteration, that is 6.25% when the maximum number of iterations is 8. The HDA2 criterion saves about 1% more iterations than the (HDA2, NMb) criterion, whereas the (Min-LLR, NMb) criterion saves about 5% less iterations than the (HDA2, NMb) criterion.

The two criteria (HDA2, NMb) and (Min-LLR, NMb), are suggested in this paper as they cause no performance loss. In addition, they achieve optimal tradeoffs between implementation complexity and average savings in the number of iterations. The (HDA2, NMb) criterion is able to save more iterations. However, it has higher complexity as it needs to store all the sign bits of the LLRs computed

Table 5. Statistics of various variables ($K = 5000$).

	SCR	SDR	SDR2	SDR3	SDR4
μ	70.1	12.1	1.56	11.2	7.21
σ	52.5	13.1	3.3	12.0	11.0
	NMb	**HDA2**	**CE**	**Min-LLR**	**Min-Lex**
μ	0.94	0.00	0.28	1.43	12.1
σ	1.58	0.00	0.70	1.47	4.42

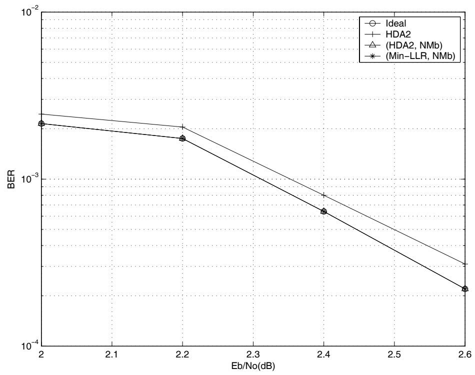

Fig. 2. BER for various stopping criteria.

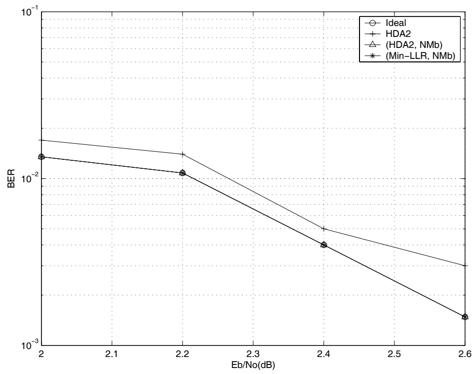

Fig. 3. FER for various stopping criteria.

by SISO decoder 1. In order to make a final decision on the stopping criterion, the block size and the application-specific constraints should be studied as well.

5. CONCLUSIONS

A systematic study of early stopping criteria for Turbo decoders has been presented. Statistical analysis has shown that some variables have better statistical properties than others. It has been pointed out that any criteria based on a single variable will have BER/FER performance loss. Two criteria have been recommended in this paper and neither of them will result in any performance loss. It has also been proposed that the thresholds for these variables are better set to be proportional to the logarithm of the block size instead of being proportional to the block size.

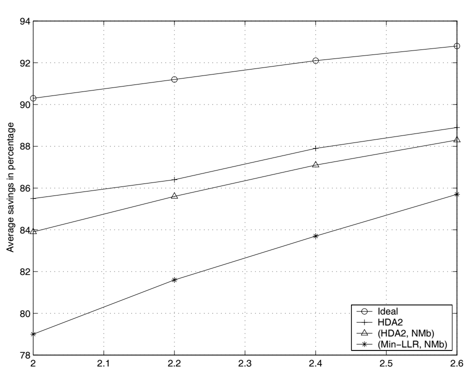

Fig. 4. Average savings in percentage for various stopping criteria.

6. REFERENCES

[1] C. Berrou, A. Glavieux, and P.Thitimajshima, "Near shannon limit error-correcting coding and decoding: Turbo-codes," in *IEEE International Conference on Communication (ICC'93)*, May 1993, vol. 2, pp. 1064–1070.

[2] C.Berrou and A. Glavieux, "Near optimium error correcting coding and decoding: Turbo codes," *IEEE Trans. on Communications*, vol. 44, no. 10, pp. 1261–1271, Oct. 1996.

[3] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes," *IEEE Trans. Inform. Theory*, vol. 42, no. 2, pp. 429 – 445, March 1996.

[4] R. Y. Shao, S. Lin, and M.P.C.Fossorier, "Two simple stopping criteria for turbo decoding," *IEEE Trans. Comm.*, vol. 47, no. 8, pp. 1117 – 1120, Aug. 1999.

[5] R. Y. Shao, S. Lin, and M. P. C. Fossorier, "Two simple stopping criteria for turbo decoding," in *Proc. IEEE International Symposium on Information Theory*, Aug. 1998, p. 279.

[6] A. Matache, S. Dolinar, and F. Pollara, "Stopping rules for turbo decoders," Tech. Rep., Jet Propulsion Laboratory, Pasadena, California, Aug. 2000.

[7] W. Lee, S. Lin, C. Tsai, T. Lee, and Y. Hwang, "A new low-power turbo decoder using hda-dhdd stopping iteration," in *Proc. of IEEE ISCAS*, May 2005, pp. 1040 – 1043.

[8] I. A. Al-Mohandes and M. I. Elmasry, "A new efficient dynamic-iterative technique for turbo decoders," in *Proc. IEEE Midwest Symposium on Circuits and Systems*, Aug. 2002, vol. 3, pp. 180 – 183.

[9] W. J. Ebel Y. Wu, B. D. Woerner, "A simple stopping criterion for turbo decoding," *IEEE Comm. Letters*, vol. 4, no. 8, pp. 258 – 260, Aug. 2000.

[10] T. M. N. Ngatched and F. Takawira, "Simple stopping criterion for turbo decoding," *Electronics Letters*, vol. 37, no. 22, pp. 1350 – 1351, Oct. 2001.

[11] Z. Wang, H. Suzuki, and K. K. Parhi, "Vlsi implementation issues of turbo decoder design for wireless applications," in *Proc. of 1999 IEEE Workshop on Signal Processing Systems (SIPS'99)*, Oct. 1999, pp. 503–512.

[12] Z. Wang and K. K. Parhi, "Decoding metrics and their applications in vlsi turbo decoders," in *Proc. ICASSP*, 2000, pp. 3370 – 3373.

[13] N. Y. Yu, M. G. Kim, Y. S. Kim, and S. U. Chung, "Efficient stopping criterion for iterative decoding of turbo codes," *Electronics Letters*, vol. 39, no. 1, pp. 73–75, Jan. 2003.

[14] F. Zhai and I. J. Fair, "New error detection techniques and stopping criteria for turbo decoding," in *Proc. 2000 IEEE Canadian Conference on Electrical and Computer Engineering*, March 2000, pp. 58–62.

[15] P. A. Hoeher I. Land, "Using the mean reliability as a design and stopping criterion for turbo codes," in *Proc. IEEE Inform. Theory Workshop (ITW)*, Sept. 2001, pp. 27 – 29.

[16] F. Gilbert, F. Kienle, and N. Wehn, "Low complexity stopping criteria for umts turbo-decoders," in *Proc. IEEE VTC 2003-Spring*, April 2003, vol. 4, pp. 2376 – 2380.