Design of FRM Digital Filters Over the CSD Multiplier Coefficient Space Employing Genetic Algorithms

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Abstract—**It is well known that the use of canonical signed-digit (CSD) multiplier coefficients in combination with sub-expression sharing and elimination leads to a substantial reduction in the hardware complexity of FIR digital filters. This paper presents a genetic algorithm for the design and optimization of frequency-response masking (FRM) FIR digital filters over the CSD multiplier coefficient space. This is based on designing a corresponding infinite-precision-coefficient digital filter seed (through a conventional continuous optimization), and on quantizing the resulting multiplier coefficients into CSD coefficients via a look-up table. The resulting digital filter is subsequently encoded into a chromosome which is perturbed to form an initial population for the genetic algorithm. The salient feature of the resulting genetic algorithm is that it automatically leads to legitimate CSD-coefficient offspring digital filters after the operations of crossover and mutation, i.e. without any recourse to gene repair. Application to the design of a bandpass FIR digital filter produces a CSD-coefficients digital filter with very close performance to that obtained by the corresponding continuous infinite-precision optimization.**

1. INTRODUCTION

The hardware implementation of higher-order FIR digital filters exhibiting sharp transition bands usually requires large chip area and involves high power consumption. Several techniques are available for reducing the hardware implementation complexity of FIR digital filters, e.g. through the use of the frequency-response masking (FRM) technique in combination with signed power-of-two (SPT) multiplier coefficients [1], [2]. FRM digital filters use lower order subfilters arranged in such a manner as to produce very sharp transition bands as are the characteristic of higher order digital filters, while keeping the multiplier coefficients very sparse so as to minimize the number of multipliers and adders. The use of SPT multiplier coefficients reduces the hardware implementation complexity by replacing the corresponding multiplications by series of shift and add operations. By using a subset of SPT multiplier coefficient representation called the canonical signed-digit (CSD) coefficient representation, further reduction in the hardware complexity can be achieved through sub-expression sharing or elimination [3], [4].

This paper presents a genetic algorithm for the design and optimization of FRM bandpass FIR digital filters incorporating CSD multiplier coefficients. A similar technique based on SPT multiplier coefficients was developed in [5] and was applied to the optimization of lowpass FIR digital filters.

2. FREQUENCY RESPONSE MASKING TECHNIQUE

Let us consider a linear-phase lowpass FIR digital filter characterized by a transfer function $H_b(z)$ of order M, and a corresponding magnitude frequency response $|H_b(e^{j\omega})|$ as shown in Fig. 1a, where z (ω, respectively) represents the discrete-time complex (real, respectively) frequency-variable. The transition bandwidth of $H_b(z)$ is $\Delta=\omega_s - \omega_p$. Moreover, let us consider a digital filter characterized by a transfer function $H_c(z)$ which is complementary to the transfer function $H_b(z)$ in accordance with

$$H_c(z) = z^{-(M-1)/2} - H_b(z).$$

By interpolating the digital filters $H_b(z)$ and $H_c(z)$ by a factor L, the magnitude frequency responses of the interpolated digital filters, having transfer functions $H_b(z^L)$ and $H_c(z^L)$, respectively, are obtained as shown in Figs. 1b and 1d. Consequently, by using a pair of masking digital filters $H_{Mb}(z)$ and $H_{Mc}(z)$, one can suppress the unwanted image bands of the interpolated filters $H_b(z^L)$ and $H_c(z^L)$ (c.f. Figs. 1b and 1d). This gives rise to the magnitude frequency responses $H_b(z^L)H_{Mb}(z)$ and $H_c(z^L)H_{Mc}(z)$ associated with the masked digital filters, as shown in Figs. 1c and 1e, respectively.

The desired digital filter will be obtained by adding two masked filters as shown in Fig. 2, having a transfer function

$$H(z) = H_b(z^L)H_{Mb}(z) + H_c(z^L)H_{Mc}(z)$$

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with a narrow transition bandwidth of Δ/L, and with a magnitude frequency response as shown in Fig. 1e.

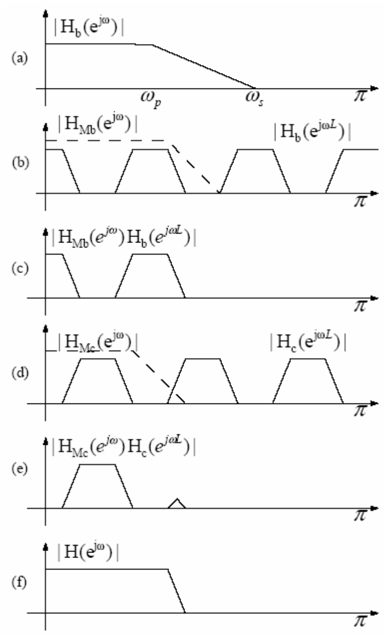


Fig. 1. Illustration of frequency response masking technique

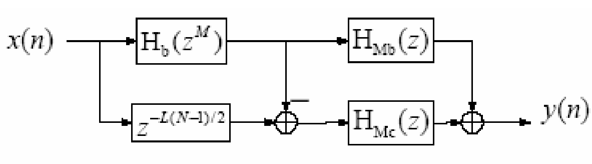


Fig. 2. Block diagram of the FRM FIR digital filter

3. GENETIC ALGORITHMS

Genetic algorithms simulate a natural evolution process for the purpose of finding the optimal solution to a potentially complex problem. They are based on three main operations, namely, selection, crossover, and mutation.

Genetic algorithms begin with a collection of potential solutions called the population pool. Often, a single seed solution is obtained and encoded as a binary string called a chromosome, which is then randomly perturbed to generate the initial population pool. In the case of digital filters, digital representations of the constituent multiplier coefficients are concatenated to form a chromosome. In the course of genetic optimization, a certain number of chromosomes in the population pool are randomly selected to join the mating pool. In the mating pool, the chromosomes will pair up at random via parent selection to produce offspring chromosomes by crossover. However, to simulate the natural evolution process more accurately, it is required to ensure that better solutions have higher probabilities of having their offspring enter into the population pool in the next generation. In this way, the fitness of a chromosome is used as a factor in parent selection within the mating pool.

The crossover operation randomly interchanging the bits of two parent chromosomes to be released into the subsequent population pool. Mutation operation encourages population diversity by randomly flipping certain bits in a chromosome before including the chromosome to the subsequent population pool. The genetic operations are repeated until a pre-specified stopping criterion is reached.

4. DESIGN TECHNIQUE

A) Chromosome Encoding: The encoding of an optimization problem into a chromosome suitable for genetic algorithms can prove to be a difficult problem. In digital filter design, usually the digital representation of the constituent multiplier coefficients or the constituent poles and zeros are concatenated in the form of a chromosome [6-8]. This method may become computationally inefficient if the multiplier coefficients are represented as CSD numbers, mainly due to the fact that careful consideration much be given to ensure that the underlying genetic operations lead to CSD offspring multiplier coefficients only.

In this paper, a look-up table similar to that shown in [5] is used to ensure that the resulting multiplier coefficients remain as CSD numbers by the underlying genetic algorithm operations. The proposed look-up table comprises five columns, including columns for table entry values, CSD multiplier coefficient values, CSD multiplier coefficient sign bits, the decimal equivalents of the CSD multiplier coefficient values, and the wordlengths of the CSD multiplier coefficients.

In order to properly produce a look-up table, certain practical restrictions must be taken into account. For example, if each table entry value is to be genetically encoded into a 12-bit CSD number, then 4096 table entries are required to ensure that the underlying genetic operations lead to legitimate CSD numbers. The range of the decimal equivalents of the CSD numbers must also be taken into account, which is directly related to the number of nonzero CSD digits and their respective exponents. For the purposes of this paper, a table of size 4096 provides an adequate set of multiplier coefficient values. In this way, a maximum of three non-zero CSD digits were allocated to a range from 2^{-13} to 2^3, to generate an exhaustive look-up table. In order to keep the table size limited to 4096, the smallest values were removed from the table, with the remaining values being in the range ±9.125 to ±0.000366.

Using the above look-up table, the multiplier coefficients of an infinite precision FRM seed FIR digital filter are quantized to their nearest CSD counterparts in the table, the table entry values are encoded into binary numbers, and these binary numbers are concatenated to form a chromosome. Note that due to the linear phase property, only the first half of the multiplier coefficients need to be encoded.

B) Genetic Operations: Once a seed digital filter is encoded into a chromosome, a population pool is generated by randomly flipping the bits in the seed chromosome in accordance with the probability $0.8 \cdot (1/2)^{b+1-i}$, where b is the number of bits representing each multiplier coefficient, and i is the current bit location within the coefficient. In this way, the LSB of each coefficient has a high probability of flipping, while its MSB has a low probability of flipping. This encourages a population pool that is sufficiently diverse, and yet reasonable in terms of acceptability of solutions.

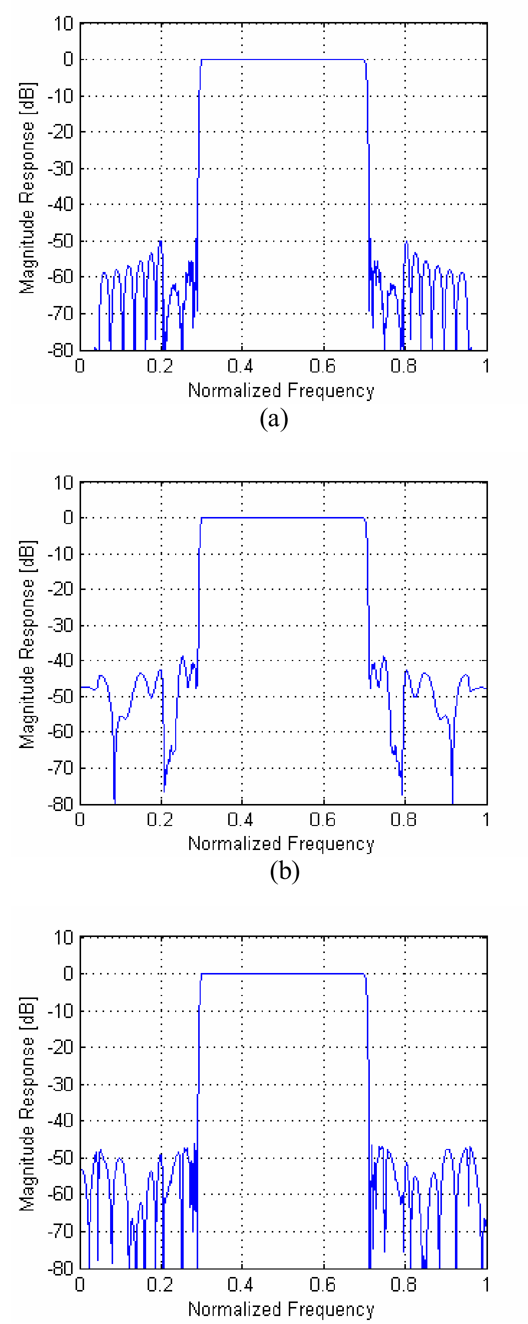
The proposed genetic algorithm is based on a population pool of N=500 chromosomes. The fitness function used is a slightly modified version of that in [5]:

$$fitness = -20\log_{10}\left[\max\left(\begin{array}{l}\max\limits_{0<\omega<\omega_{s1}} |H(e^{j\omega})|,\\ k\cdot\max\limits_{\omega_{p1}<\omega<\omega_{p2}} |H(e^{j\omega})-1|,\\ \max\limits_{\omega_{s2}<\omega<\pi} |H(e^{j\omega})|\end{array}\right)\right]+\zeta$$

where $H(e^{j\omega})$ is the frequency response of the digital filter, where k is the desired passband to stopband ratio, and where ζ is a constant to ensure all fitness functions remain positive (typically ζ is 30 or 40 and $k=1$). In this way, each member of the population is assigned a fitness level and ranked with respect to the rest of the population.

A mating pool of 150 chromosomes is generated by searching the population pool and by randomly selecting chromosomes, with preference given to chromosomes with better fittnes. The probability of selection is given by the exponential function $p(x) = k^{1-x} \cdot Z^x$, where $k = \left(Z^N/0.01\right)^{1/(N-1)}$, where Z is the probability of selection of the fittest chromosome (here $Z=0.8$), and where x is the fitness rank of the particular chromosome. Since natural systems tend to evolve slowly before a rapid jump in the overall fitness level of the population occurs, a pre-determined number of non-elite (low fitness) chromosomes are released into the mating pool to encourage mating diversity. In this case, the population pool is searched by using the given probability function until 110 chromosomes are selected. In the event that the entire population pool has been searched and the sought after 110 chromosomes have not yet been found, the search is renewed within a population pool where the previously selected chromosomes have been removed. The remaining 40 slots in the mating pool are selected from the lower ranks of the population pool.

Parent selection is performed within the mating pool using roulette-wheel selection, which encourages chromosomes with a high fitness factor to produce more offsprings. This is executed by summing the entire mating pool fitness factors, by choosing a random point between zero and the sum, and by selecting a parent where the sum of all the preceding fitness factors is greater than or equal to the random number. In this way, 230 pairs of parents are selected to produce 460 offsprings. The offsprings are produced by two-point crossover, where two random points within the length of a chromosome are selected and the bits of the chromosomes between these two points are interchanged. The remaining 40 slots of the population pool are filled by 40 non-elite chromosomes from the previous generation population pool in order to encourage additional diversity.



(c)

Fig. 3. FRM FIR digital filter magnitude frequency responses

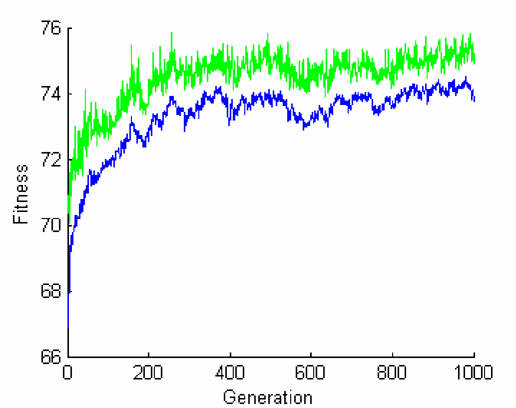


Fig. 4. Fitness Evolution

Finally, mutation is performed on the entire population pool to encourage even more diversity, and to eliminate convergence to a potentially poor solution. This is performed in a similar fashion to the technique used to generate the initial population pool, except that the above probability is multiplied by 0.08 instead of 0.8 in order to limit the most possible radical changes. The newly mutated population pool can now be transferred to the next generation and the genetic algorithm can begin another iteration until a stopping criterion is reached. In this paper, a certain number of iterations has been used (typically between 1000 and 5000).

4. DESIGN EXAMPLE

By using a Matlab least squares optimization, a seed bandpass FRM FIR digital filter consisting of infinite-precision multiplier coefficients was designed satisfying the following specifications: $\omega_{s1} = 0.31\pi$, $\omega_{p1} = 0.3\pi$, $\omega_{p2} = 0.7\pi$, and $\omega_{s2} = 0.71\pi$. From [1] and by using other considerations, $M=68$, $L=8$, and $N_{mb}=33$, $N_{mc}=61$ for a desired stopband attenuation of 50dB, where N_{mb} and N_{mc} are the orders of $H_{Mb}(z)$ and $H_{Mc}(z)$, respectively. The resulting magnitude frequency response is shown in Fig. 3a, having a stopband attenuation of 49.2dB and a passband ripple of 0.033dB. Truncating the infinite-precision multiplier coefficients into three nonzero-digit CSD coefficients results in the magnitude response in Fig. 3b, having a stopband attenuation of 38.9dB and a passband ripple of 0.058dB.

The magnitude response obtained from the genetic algorithm after 1000 generations is shown in Fig. 3c, having a stopband attenuation 46.5dB and a passband ripple of 0.048dB. Finally, Fig. 4 shows the fitness evolution of the best chromosome in the population (top curve) and the average of the top 50% of the population (lower curve).

5. CONCLUSION

This paper has presented a genetic algorithm for the design of FRM FIR digital filters over the CSD multiplier coefficients space. This had been based on designing an infinite-precision coefficient seed digital filter through continuous optimization, and on quantizing the resulting multiplier coefficients into three nonzero-digit CSD coefficients via a look-up table. The resulting digital filter is encoded into a chromosome suitable for use in a genetic algorithm. The salient feature of the proposed genetic algorithm is that it automatically leads to legitimate offspring digital filters after the operations of crossover and mutation without any recourse to gene repair. Application to the design of a bandpass FRM FIR digital filter has produced a CSD-coefficient digital filter with very close performance to the corresponding infinite-precision digital filter obtained by continuous optimization. The proposed genetic algorithm can be generalized [9] to other types of FIR digital filters as well as to other types of multiplier coefficient spaces [10-13].

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