LOW COMPLEXITY BLIND CONSTRAINED DATA-REUSING ALGORITHMS BASED ON MINIMUM VARIANCE AND CONSTANT MODULUS CRITERIA

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ABSTRACT

This work presents low complexity blind constrained data-reusing adaptive filtering algorithms based on the minimum variance and constant modulus cost functions. Constrained minimum variance (CMV) and constrained constant modulus (CCM) affine projection type algorithms are developed and investigated in a CDMA interference suppression scenario. Computer simulations are used to analyze the proposed techniques and compare them with existing stochastic gradient (SG) and recursive least-squares (RLS) type techniques. The results show that the new algorithms outperform previously reported SG techniques with small additional computational requirements and achieve a performance very close to RLS algorithms at greatly reduced complexity.

1. INTRODUCTION

Linearly constrained blind adaptive filtering algorithms are useful in several areas of communications and signal processing such as beamforming and interference suppression for code-division-multiple-access (CDMA) systems. In these applications, the linear constraints correspond to prior knowledge of certain parameters such as direction of arrival (DoA) of user signals in antenna array processing [1] and the signature sequence of the desired signal in CDMA interference suppression [2], [3].

An important research and development field is the implementation of blind algorithms in a computationally efficient way, while ensuring very good performance. In the literature of blind adaptive algorithms [4], stochastic gradient (SG) algorithms represent a simple and low complexity solution but result in slow convergence, depending on the eigenvalue spread of the covariance matrix of the observation data. Conversely, recursive least-squares (RLS) type algorithms have fast convergence, are independent of the eigenvalue spread of the covariance matrix of the observation data, but require significantly higher complexity than SG recursions. Due to their simplicity, low complexity, and good behavior in fixed point implementations, SG algorithms are preferred for practical deployment despite their convergence performance limitations.

Another important aspect in blind adaptation methods is the criterion adopted for optimization. Amongst the unsupervised adaptive filtering algorithms found in the literature, those based on the minimum variance (MV) and the constant modulus (CM) cost functions are some of the most promising techniques due to their simplicity and effectiveness for jointly optimizing the adaptive filter and the constraints. The constrained MV (CMV) criterion and its associated SG and RLS adaptive algorithms were studied for CDMA interference suppression in [5], where their global convergence was also established. The constrained CM (CCM) approach and the CCM-based algorithms were reported in [6] and [7], where some conditions for global convergence were established based on a convexity forcing parameter. The existing works on low complexity blind constrained techniques [5], [6] employ standard SG algorithms that are not efficient with respect to convergence and steady-state performance, whereas RLS techniques exhibit fast convergence but may have numerical problems when implemented in fixed-point arithmetic.

In wireless networks characterized by non-stationary environments, users frequently enter and exit the system, requiring adaptation methods with good tracking performance and low computational requirements. In this context, the affine projection (AP) algorithm is an efficient adaptive algorithm that can achieve a good trade-off between fast convergence and low computational complexity. By adjusting the number of projections or data reuses, the performance of the AP algorithm can range from that of the NLMS to that of the RLS algorithm [4]. To the best of our knowledge, there are no available general purpose blind constrained data-reusing algorithms based on the MV and CM cost functions in the literature.

The goal of this paper is to develop blind adaptive constrained data-reusing algorithms based on the MV and CM criteria. We consider a set of linear constraints and multiple data observations to derive the constrained minimum variance affine projection (CMV-AP) and constrained constant modulus affine projection (CCM-AP) adaptive recursions. The new proposed algorithms require a computational complexity and exhibit convergence performance between those of their corresponding SG and RLS type techniques. Computer simulations are used to analyze the proposed techniques and compare them with the existing stochastic gradient (SG) and recursive least-squares (RLS) type techniques based on the CMV and CCM design criteria in a CDMA interference suppression scenario.

The paper is structured as follows. Section 2 describes the blind constrained adaptive filtering framework. Sections 3 and 4 present the proposed blind adaptive constrained algorithms. In Section 5 the proposed algorithms are applied to blind interference suppression in DS-CDMA systems. Section 6 is devoted to the presentation and discussion of numerical results, while Section 7 presents the conclusions.

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2. BLIND CONSTRAINED ADAPTIVE FILTERING PROBLEM

Next, we briefly describe the framework used to derive the proposed algorithms. We focus on the linearly constrained adaptive filtering problem using a chosen design criterion.

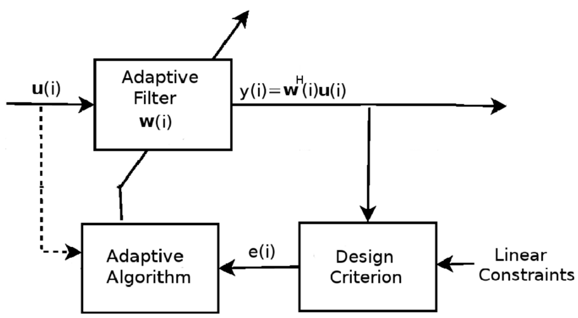

Fig. 1. Linearly Constrained Blind Adaptive Filtering Framework

The approach we follow in this paper is to obtain an M-dimensional parameter vector $\mathbf{w}$ that minimizes the cost function defined as the design criterion in order to retrieve a desired signal obtained from the $M \times 1$ observation vector $\mathbf{u}$. In many cases, to avoid the trivial solution $\mathbf{w} = 0$, it is necessary to impose some constraints on $\mathbf{w}$. In this work the constraints are given by a set of L equations given by $\mathbf{C}^H\mathbf{w} = \mathbf{g}$ where $\mathbf{C}$ is an $M \times L$, $L \leq M$, constraint matrix and $\mathbf{g}$ is an L-dimensional parameter vector to be determined. In the next sections we derive low complexity solutions based on the CMV and CCM approaches. In particular, we develop general purpose constrained adaptive algorithms that exploit data-reusing to speed up convergence while keeping low complexity.

3. CONSTRAINED MINIMUM VARIANCE AFFINE PROJECTION ALGORITHM

In this section, we develop a blind adaptive constrained data-reusing algorithm based on the MV criterion.

Define an error vetor $\mathbf{e}(i) = \mathbf{U}^H(i)\mathbf{w}(i)$ where $\mathbf{U}(i) = [\mathbf{u}(i) \ldots \ldots \mathbf{u}(i-P+1)]$ is a $M \times P$ matrix containing P observation vectors. The cost function considered is the sum of squared errors:

$$\begin{aligned} J_{\mathrm{MV}}[\mathbf{w}(i)] &= \mathbf{e}^H(i)\mathbf{e}(i) = \sum_{j=0}^{P-1}\Big[\mathbf{w}^H(i)\mathbf{u}(i-j)\Big]^2 \\ &= \mathbf{w}^H(i)\mathbf{U}(i)\mathbf{U}^H(i)\mathbf{w}(i). \end{aligned} \tag{1}$$

So, the Lagrangian cost function can be written as:

$$\mathcal{L}_{\mathrm{MV}} = \mathbf{w}^H(i)\mathbf{U}(i)\mathbf{U}^H(i)\mathbf{w}(i) + \Re\Big[(\mathbf{C}^H\mathbf{w}-\mathbf{g})^H\boldsymbol{\lambda}\Big] \tag{2}$$

where $\boldsymbol{\lambda}$ is a vector containg the Lagrange multipliers and $\Re(\cdot)$ selects the real part. Taking the gradient with respect to $\mathbf{w}$ we obtain the following equation to update $\mathbf{w}$:

$$\mathbf{w}(i+1) = \mathbf{w}(i) - \mathbf{U}(i)\mathbf{e}(i) - \mathbf{C}\boldsymbol{\lambda}. \tag{3}$$

Enforcing the constraints on $\mathbf{w}$ to be $\mathbf{C}^H\mathbf{w}(i+1) = \mathbf{g}(i)$ and solving for the Lagrange multipliers we obtain:

$$\mathbf{C}^H\mathbf{w}(i) - \mathbf{C}^H\mathbf{U}(i)\mathbf{e}(i) - \mathbf{C}^H\mathbf{C}\boldsymbol{\lambda} = \mathbf{g}(i)$$

$$\boldsymbol{\lambda} = (\mathbf{C}^H\mathbf{C})^{-1}\Big[\mathbf{C}^H\mathbf{w}(i) - \mathbf{C}^H\mathbf{U}(i)\mathbf{e}(i) - \mathbf{g}(i)\Big]. \tag{4}$$

Substituting (4) in (3) we arrive at the following recursion:

$$\mathbf{w}(i+1) = \boldsymbol{\Pi}\left[\mathbf{w}(i) - \mathbf{U}(i)\mathbf{e}(i)\right] + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}(i) \tag{5}$$

where

$$\boldsymbol{\Pi} = \Big[\mathbf{I} - \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{C}^H\Big]. \tag{6}$$

3.1. Normalizing the Step-Size

In order to devise a normalized version of the algorithm, we introduce a convenient $P \times P$ matrix step-size $\boldsymbol{\mu}$

$$\mathbf{w}(i+1) = \boldsymbol{\Pi}\left[\mathbf{w}(i) - \mathbf{U}(i)\boldsymbol{\mu}\mathbf{e}(i)\right] + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}(i) \tag{7}$$

and propose a normalized step size based on the minimization of the *a posteriori* cost function.

$$\begin{aligned} J_{\mathrm{MV}}[\mathbf{w}(i+1)] &= \Big\|\mathbf{U}^H(i)\Big\{\boldsymbol{\Pi}\left[\mathbf{w}(i) - \mathbf{U}(i)\boldsymbol{\mu}\mathbf{e}(i)\right] + \\ &\quad + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\mathbf{g}(i)\Big\}\Big\|^2. \end{aligned} \tag{8}$$

So the optimum step-size is given by:

$$\boldsymbol{\mu} = \min_{\boldsymbol{\mu}} J_{\mathrm{MV}}[\mathbf{w}(i+1)]. \tag{9}$$

Taking the gradient with respect to $\boldsymbol{\mu}$ and setting it to zero, we obtain the optimum step-size

$$\boldsymbol{\mu} = \mu_0\Big[\mathbf{U}^H(i)\boldsymbol{\Pi}\mathbf{U}(i)\Big]^{-1}, \tag{10}$$

where μ_0 is a constant. Note that for the special case of $P = 1$ the results are identical of those in [8].

4. CONSTRAINED CONSTANT MODULUS AFFINE PROJECTION ALGORITHM

In this section, we develop blind adaptive constrained data-reusing algorithms based on the CM criterion.

For the CM criteria, the jth component of the $P \times 1$ error vector $\mathbf{e}(i)$ is given by $e_j(i) = |\mathbf{w}^H(i)\mathbf{u}(i-j)|^2 - 1$. As in the previous section, the cost function considered is the sum of squared errors:

$$J_{\mathrm{CM}}[\mathbf{w}(i)] = \mathbf{e}^H(i)\mathbf{e}(i) = \sum_{j=0}^{P-1}[|\mathbf{w}^H(i)\mathbf{u}(i-j)|^2 - 1]^2. \tag{11}$$

The Lagrangian cost function can be written as:

$$\mathcal{L}_{\mathrm{CM}} = \sum_{j=0}^{P-1}[|\mathbf{w}^H(i)\mathbf{u}(i-j)|^2 - 1]^2 + \Re\Big[(\mathbf{C}^H\mathbf{w} - \nu\mathbf{g})^H\boldsymbol{\lambda}\Big] \tag{12}$$

where ν is a constant to ensure the convexity of the CM-based function. Denoting $z_j(i) = \mathbf{w}^H(i)\mathbf{u}(i-j)$ and $e_j(i) = |z_j(i)|^2 - 1$, substituting in (12) and taking the grandient with respect to the filter parameters we obtain the following recursion:

$$\mathbf{w}(i+1) = \mathbf{w}(i) + \sum_{j=0}^{P-1} e_j(i)z_j^*(i)\mathbf{u}(i-j) - \mathbf{C}\boldsymbol{\lambda}. \tag{13}$$

After applying the constraints and solving for the Lagrange multipliers we obtain the recursion for the filter coefficients:

$$\begin{aligned}\mathbf{w}(i+1) &= \mathbf{\Pi}\Big[\mathbf{w}(i) - \sum_{j=0}^{P-1} e_j(i) z_j^*(i)\mathbf{u}(i-j)\Big] + \\ &\quad + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\nu\mathbf{g}(i) \end{aligned} \tag{14}$$

which can be written in compact form as:

$$\mathbf{w}(i+1) = \mathbf{\Pi}\Big[\mathbf{w}(i) - \mathbf{U}(i)\mathbf{Z}(i)\mathbf{e}(i)\Big] + \mathbf{C}(\mathbf{C}^H\mathbf{C})^{-1}\nu\mathbf{g}(i) \tag{15}$$

where $\mathbf{Z}(i) = \mathrm{diag}\Big[z_0^*(i), \ldots, z_{P-1}^*(i)\Big]$ is a $P \times P$ diagonal matrix, $\mathbf{e}(i)$ is the error vector, and $\mathbf{\Pi}$ is given by (6).

4.1. Normalizing the Step-Size

In a procedure similar to the CMV we multiply the error vector $\mathbf{e}(i)$ in (15) by a $P \times P$ step-size matrix. The normalized matrix $\boldsymbol{\mu}$ proposed here for the CCM parameter estimation is an extension of the result obtained in [9] for $P = 1$ through the minimization of the *a posteriori* cost function and is given by

$$\boldsymbol{\mu} = \mu_0 \mathbf{M}\Big[\mathbf{U}^H(i)\mathbf{\Pi}\mathbf{U}(i)\Big]^{-1} \tag{16}$$

where $\mathbf{M} = \mathrm{diag}\left(\frac{1}{|z_0(i)|(|z_0(i)|-1)}, \ldots, \frac{1}{|z_{P-1}(i)|(|z_{P-1}(i)|-1)}\right)$ is a $P \times P$ diagonal matrix and μ_0 is a constant.

5. BLIND INTERFERENCE SUPPRESSION FOR DS-CDMA SYSTEMS

Here we apply the proposed algorithms to the problem of blind interference suppression in DS-CDMA systems. Consider the uplink connection of a BPSK DS-CDMA system with K users, N chips per symbol, and L_p paths. Assuming that the channel is constant during each symbol interval, the received signal after coherent demodulation and filtering by a chip-pulse matched filter and sampled at chip rate yields the $(M = N + L_\mathrm{p} - 1) \times 1$ received vector

$$\mathbf{u}(i) = \sum_{k=1}^{K} A_k b_k(i)\mathbf{C}_k\mathbf{h}_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i) \tag{17}$$

where $\mathbf{n}(i) = [n_1(i) \ \ldots \ n_M(i)]^T$ is a complex Gaussian noise vector with $E[\mathbf{n}(i)\mathbf{n}^H(i)] = \sigma^2\mathbf{I}$, where $(\cdot)^T$ and $(\cdot)^H$ denote transpose and Hermitian transpose, respectively. The operator $E[\cdot]$ stands for ensemble average, $b_k(i) \in \{\pm 1 + j0\}$ is the symbol for user k with $j^2 = -1$, $\boldsymbol{\eta}(i)$ represents the intersymbol interference (ISI), the amplitude of user k is A_k, the kth user channel vector is $\mathbf{h}_k(i) = [h_{k,0}(i) \ \ldots \ h_{k,L_\mathrm{p}-1}(i)]^T$, and the columns of the $M \times L_\mathrm{p}$ convolution matrix $\mathbf{C}_k$ contains one-chip shifted versions of the signature sequence for user k given by $\mathbf{s}_k = [a_k(1) \ldots a_k(N)]^T$.

Let us describe the design of blind linearly constrained detectors. Consider the received vector $\mathbf{u}(i)$, and $\mathbf{C}_k$ as the $M \times L_\mathrm{p}$ constraint matrix for user k. The receiver design determines an FIR filter $\mathbf{w}_k(i)$ with M coefficients, that provides an estimate of the desired symbol, as given by

$$\hat{b}_k(i) = \mathrm{sgn}\Big\{\Re\Big[\mathbf{w}_k^H(i)\mathbf{u}(i)\Big]\Big\} \tag{18}$$

subject to a set of multipath constraints given by $\mathbf{C}_k^H\mathbf{w}_k(i+1) = \mathbf{h}_k(i)$ for the CMV case, or $\mathbf{C}_k^H\mathbf{w}_k(i+1) = \nu\mathbf{h}_k(i)$ for the CCM case, where $\mathbf{h}_k(i)$ is the kth user channel vector ,sgn$(\cdot)$ is the signum function, and the receiver parameter vector $\mathbf{w}_k$ is optimized by the CMV or the CCM criterion, which assume knowledge of the channel. However, when multipath is present, these parameters are unknown and time-varying, requiring channel estimation. Here, we adopt the simple and effective blind adaptive SG channel estimation algorithm of [10].

6. SIMULATIONS AND RESULTS

The simulation results presented are for a BPSK synchronous DS-CDMA system that employs Gold sequences of length $N = 31$. Because we focus on uplink scenarios, users experimence different channel conditions. All channels assume that $L_\mathrm{p} = 3$. It is also assumed here that the channels experienced by different users are statistically independent and identically distributed. For fading channels, the sequence of channel coefficients for each user k ($k = 1, \ldots, K$), $h_{k,l}(i) = p_{k,l}\alpha_{k,l}(i)$ ($l = 0, 1, 2, \ \ldots, L_\mathrm{p} - 1$) is obtained with Clarke's model [11]. This procedure corresponds to the generation of independent sequences of correlated unit power complex Gaussian random variables $[E[|\alpha_{k,l}^2(i)|] = 1]$ with the path weights $p_{k,l}$ normalized so that $\sum_{l=1}^{L_\mathrm{p}} p_{k,l}^2 = 1$. In this work $p_1 = 0.7581$, $p_2 = 0.5307$ and $p_3 = 0.3790$. The phase ambiguity derived from the blind channel estimation method in [10] is eliminated in our simulations by using the phase of $h_{k,0}$ as a reference, and for fading channels we assume ideal phase tracking and express the results in terms of the normalized Doppler frequency $f_\mathrm{d}T$ (cycles/symbol).

In the experiments, we compare an improved [8] (normalized step-size) version of the blind receiver of Xu and Tsatsanis [5], denoted CMV-SG, an improved [9] (normalized step-size) version of the constrained constant-modulus of Xu and Liu [6], denoted CCM-SG, the RLS-like versions of CMV [5] and CCM [7], and the proposed AP-like versions, denoted CMV-AP and CCM-AP. For the proposed algorithms, we used $P = 2$ and $P = 3$ in the experiments. Note that $P = 4$ can theoretically increase convergence speed, even though we found in our studies that it did not lead to performance improvements, due to increased misadjustment. All experiments are averaged over 100 runs and the parameters of the algorithms are optimized for each scenario.

In Fig. 2 we assess the average BER performance of the analyzed algorithms under fading ($f_\mathrm{d}T = 10^{-4}$). We consider a non-stationary scenario where at a given time, users enter the system and the blind adaptive algorithms are subject to a sudden change in the environment. The system starts with $K = 8$ users whose power distribution follows a log-normal random variable with standard deviation (sd) equal to 1.5 dB. At 1000 symbols, 4 users enter the cell and the power control is loosened, resulting in a power distribution with sd equal to 3 dB for all users.

The results show that the proposed CCM-AP and CMV-AP outperform their SG counterparts and perform near their RLS-like versions. Note that for moderate loads, the proposed CCM-AP for $P = 2$ performs even better than the CMV-RLS. For higher loads, the proposed algorithms, despite a slight performance degradation in relation to the RLS-like versions, still perform much better than their SG versions.

In Fig. 3 we assess the SINR (signal-to-interference-plus noise ratio) performance in a 12-user, moderate near-far scenario and under faster ($f_\mathrm{d}T = 10^{-3}$) fading. We assume that the user of interest is User 1. One interferer has a power level 10 dB above and another has 7 dB above the desired user. The remaining 9 interferers have the same power as the desired user, which corresponds to $E_{\mathrm{b}}/N_0 = 15$ dB. The results show that the proposed algorithms mantain good performance under different fading scenarios.

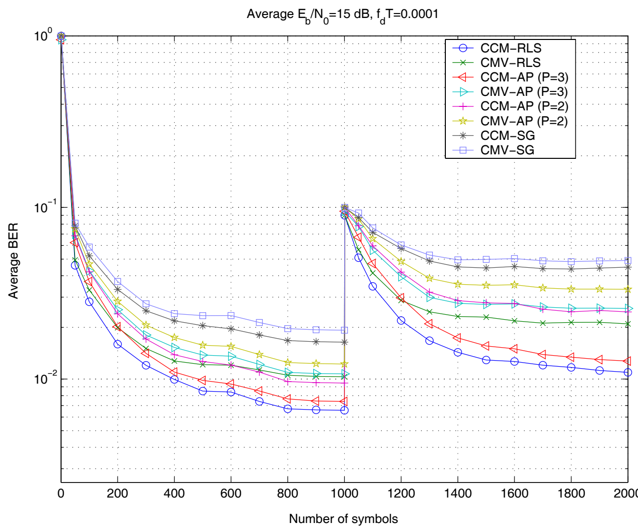

Fig. 2. Bit error rate versus number of symbols for a non-stationary scenario ($f_{\mathrm{d}}T = 10^{-4}$).

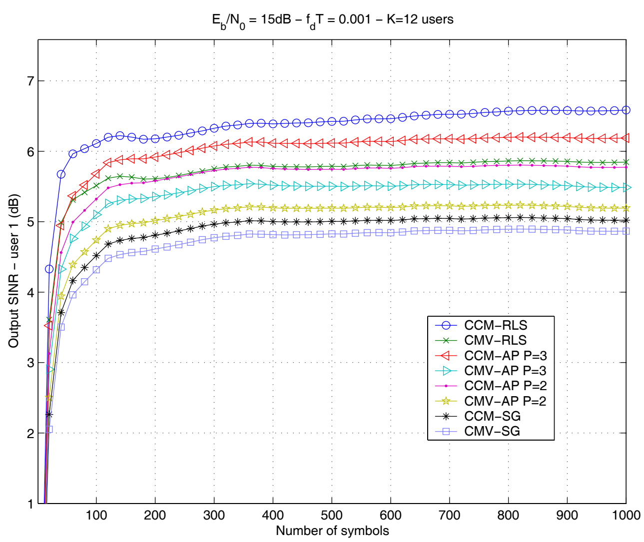

Fig. 3. Signal-to-interference-plus-noise ratio versus number of symbols for a faster fading scenario ($f_{\mathrm{d}}T = 10^{-3}$).

7. CONCLUSIONS

This work presented low complexity blind constrained data-reusing adaptive filtering algorithms based on the minimum variance and constant modulus cost functions. The proposed techniques were evaluated through computer simulationss and comparisons with existing SG and RLS implementations in a CDMA interference suppression scenario were performed. The proposed algorithms have shown performance close to the RLS implementations at significantly lower complexity.

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