Performance Assessment of a Computationally Efficient MMSE Receiver for Asynchronous MC-CDMA Systems over Multipath Fading Channels

S. J. Yi, C. C. Tsimenidis, B. S. Sharif and S. Y. Le Goff

School of Electrical Electronic & Computer Engineering, University of Newcastle upon Tyne, UK

ABSTRACT

In this paper, the performance of a computationally efficient adaptive MMSE receiver is investigated for asynchronous MC-CDMA systems over multipath fading channels. According to Wiener filter and LMS algorithm theories, we prove that the proposed computationally efficient MMSE receiver has the same BER and convergence performance as that of the conventional MMSE receiver, assuming the sampling rate per symbol is equal to the number of subcarriers. When the sampling rate per symbol is increased, the proposed receiver can outperform the conventional MMSE receiver without sacrificing the convergence speed. The theoretical analysis is verified by the results through Monte-Carlo simulation.

1. INTRODUCTION

In this paper, we extend the research in [1] to an asynchronous multicarrier code division multiple access (MC-CDMA) system over multipath channels. A least mean square (LMS) algorithm is employed by the adaptive computationally efficient MMSE (CE-MMSE) receiver, and its performance including the system BER and convergence characteristic is theoretically analyzed and investigated through Mont-Carlo simulations.

This paper is organized as follows. Section 2 describes the multipath channel model and an asynchronous MC-CDMA signal model. In Section 3, we briefly introduce the configuration and the adaptive implementation of the CE-MMSE receiver. Section 4 addresses the theoretical analysis of the performance for the CE-MMSE receiver in asynchronous MC-CDMA systems over multipath channel. And finally the simulation results and conclusions are given in Section 5 and 6, respectively.

Notation: Superscripts *, T, and H stand for conjugation, transpose and Hermitian transpose, respectively; and $E\{\ \}$ represents statistical expectation with the random variables within the curly braces. The operator diag( ) represents diagonal matrix, and the column vectors (matrices) are denoted by boldface lowercase (uppercase) letters.

2. SYSTEM MODEL

A. Multipath Channel Model

A wireless channel can be assumed to be a wide sense stationary uncorrelated scattering (WSSUS) channel and described by a truncated tapped delay line (TDL) model with random taps. The propagation medium of the channel acts as a time-varying linear filter and the impulse response of the channel can be written as [2]

$$h(\tau,t)=\sum_{l=1}^{L}h_l(t)\delta(\tau-n/W) \qquad (1)$$

where W is the signal bandwidth, and L is the number of resolved path-arrivals, which is defined as

$$L=\lfloor T_m W\rfloor+1 \qquad (2)$$

where the operator $\lfloor\cdot\rfloor$ denotes truncation to the nearest integer.

In a Rayleigh fading channel, the path gain $h_l(t)$ of the lth path is a complex-valued random Gaussian process with zero mean, and can be denoted as

$$h_n(t)=\beta_n(t)e^{-j\theta_n(t)} \qquad (3)$$

It is assumed that $\beta_n(t)$ are independent, WSS processes with time-invariant powers. The fading envelopes $\beta_n(t)$ are, therefore, Rayleigh-distributed and the phase distortion $\theta_n(t)$ is uniformly distributed over $[0,2\pi)$.

In order to keep the average energy constant, the power of each path is normalised so that the aggregate power from total L paths remains unit, that is

$$\sum_{n=1}^{L}\sigma_{h_n}^2=1, \qquad (4)$$

where $\sigma_{h_n}^2$ is the variance of fading process $\beta_n(t)$. It is noted that the sequence of variance $\sigma_{h_n}^2$, (n=1, 2, $\cdots$, L) represents the discrete multipath intensity profile (MIP) of the channel.

In this research, we consider an exponentially distributed MIP, and the variance of each path is given as

$$\sigma_{h_n}^2 = \begin{cases} 1-e^{-1/L}/1-e^{-1}, & n=1 \\ \sigma_{h_1}^2 e^{-\frac{n-1}{L}}, & n=2,3,\cdots,L \end{cases} \quad (5)$$

B. Asynchronous MC-CDMA Signal Model

An asynchronous MC-CDMA system is considered in this paper [3], [4]. The bipolar data symbol $b_k(i)$ of the kth user is modulated using binary phase shift-keying (BPSK) and transmitted in parallel over M subcarriers, each multiplied with a different chip of the kth user's spreading code $\mathbf{c}_k = [c_{k1}, c_{k2}, \cdots, c_{kM}]^T$. We assume that there are K active users and the frequency separation between adjacent subcarriers is $1/T_b$, where T_b is the symbol duration of the transmitted signal.

It is also assumed that the length of each transmission packet is ($2N_s$+1) data bits. For the kth user, the complete transmitted signal packet is given as follows

$$s_k(t) = \sum_{i=-N_s}^{N_s} A_k b_k(i) \sum_{m=1}^{M} c_{km} e^{j2\pi m(t-iT_b)/T_b} p(t-iT_b) \quad (6)$$

where $p(t)$ is a rectangular pulse having unit energy defined over [0, T_b]. After the multipath channel, the received signal of kth user is given as

$$r_k(t) = s_k(t-\tau_k) * h_k(\tau, t) \quad (7)$$

where the notation * represents the linear convolution operator; τ_k is the transmission delay of the kth user, which is uniformly distributed over [0, T_b).

Therefore, the received signal at the front end of the receiver can be written as

$$r(t) = \sum_{k=1}^{K} r_k(t) + n(t) \quad (8)$$

where $n(t)$ denotes a complex-valued AWGN process with zero mean and variance σ_n^2.

3. ADAPTIVE CE-MMSE RECEIVER

For the kth user, the adaptive CE-MMSE receiver proposed for asynchronous MC-CDMA systems has the same structure as that for synchronous MC-CDMA systems. Without loss of generality, it is assumed that the kth user is of interest and the receiver timing is strictly synchronized with the first signal arrival. Let the sampling rate (SR) be equal to N samples per symbol duration, thus, the input vector of the MMSE filter during the ith symbol interval can be denoted by an N-by-1 sampling vector $\mathbf{r}(i) = [r_1, r_2, \cdots, r_N]^T$, which is the sampled version of the continuous-time received signal $r(t)$. In contrast, the input to the conventional MMSE receiver is an M-by-1input vector $\mathbf{x}(i)$, which is composed by the outputs of the chip matched filters for each subcarrier. As described in [1], compared to the conventional MMSE receiver, the CE-MMSE receiver removes the FFT operation and the sampling sequence from the received signal is directly passed to the filter. This change simplifies the receiver structure and saves the computational overhead of the FFT operation.

The adaptive CE-MMSE is implemented by the standard normalized LMS algorithm. The weight vector $\mathbf{w}$ for the kth user is updated as follows [5]

$$\mathbf{w}_k(i+1) = \mathbf{w}_k(i) + \frac{\mu}{\lambda + \|\mathbf{r}(i)\|^2} \mathbf{r}(i) \left[b_k(i) - \mathbf{w}(i)^H \mathbf{r}(i) \right]^* \quad (9)$$

where $\mu \in (0,2)$ and $\lambda > 0$ are real constants and $\| . \|^2$ denotes the squared vector norm. In the training mode, $b_k(i)$ is replaced by the training sequence, whereas in the decision-directed mode $b_k(i)$ is substituted by the hard decision output $\hat{b}_k(i)$.

4. PERFORMANCE ANALYSIS

For simplicity, the symbol index i is dropped in the remaining analysis. The relationship between the input vector $\mathbf{r}$ to the CE-MMSE filter and the input vector $\mathbf{x}$ to the conventional MMSE filter is

$$\mathbf{x} = \mathbf{F}_N \mathbf{r} \quad (10)$$

where $\mathbf{F}_N$ is the FFT matrix and defined as

$$\mathbf{F}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{-j2\pi/N} & \cdots & e^{-j2\pi(N-1)/N} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j2\pi(N-1)/N} & \cdots & e^{-j2\pi(N-1)(N-1)/N} \end{bmatrix}_{N \times N} \quad (11)$$

As explained in [1], the matrix $\mathbf{F}_N$ is constant since it depends only on system specifications, e.g., sampling rate and the number of subcarriers. When an adaptive algorithm is employed to adjust the coefficients of the filter, the constant matrix $\mathbf{F}_N$ can be absorbed within the filter the N the need for the FFT operation required by the conventional MMSE receiver is eliminated.

When the sampling rate per symbol is equal to the number of subcarriers M, $\mathbf{F}_N$ becomes a unitary matrix. The correlation matrix $\mathbf{R}_{rr}$ of the input vector $\mathbf{r}$ to the CE-MMSE receiver is given as

$$\mathbf{R}_{rr} = E\{\mathbf{r}\mathbf{r}^H\} \quad (12)$$

and the steering vector of the CE-MMSE filter is given as

$$\mathbf{p}_r = E\{\mathbf{r} b_k^*\} \quad (13)$$

For the conventional MMSE receiver, the correlation matrix of the input vector $\mathbf{x}$ and the steering vector are respectively given by

$$\mathbf{R}_{xx} = E\{\mathbf{x}\mathbf{x}^H\} \tag{14}$$

and

$$\mathbf{p}_x = E\{\mathbf{x}b_k^*\} \tag{15}$$

According to the MMSE filter theory, the achievable minimum MSE $J_{\min,r}$ of the CE-MMSE is given as [5]

$$J_{\min,r} = \sigma_d^2 - \mathbf{p}_r^H \mathbf{R}_{rr}^{-1}\mathbf{p}_r \tag{16}$$

where σ_d^2 denotes the desired signal energy.

Similarly, the achievable minimum MSE $J_{\min,x}$ of the conventional MMSE receiver is given as

$$J_{\min,x} = \sigma_d^2 - \mathbf{p}_x^H \mathbf{R}_{xx}^{-1}\mathbf{p}_x \tag{17}$$

By using (14)-(17) and taking advantage of the property of unitary matrix $\mathbf{F}_N$, it is easy to derive that

$$\begin{aligned} J_{\min,r} &= \sigma_d^2 - \left(E\{\mathbf{r}b_k^*\}\right)^H \left(E\{\mathbf{r}\mathbf{r}^H\}\right)^{-1} E\{\mathbf{r}b_k^*\} \\ &= \sigma_d^2 - \left(E\{\mathbf{r}b_k^*\}\right)^H \mathbf{F}_N^H\mathbf{F}_N \left(E\{\mathbf{r}\mathbf{r}^H\}\right)^{-1} \mathbf{F}_N^H\mathbf{F}_N E\{\mathbf{r}b_k^*\} \\ &= \sigma_d^2 - \left(E\{\mathbf{F}_N\mathbf{r}b_k^*\}\right)^H \left(E\{\mathbf{F}_N\mathbf{r}\mathbf{r}^H\mathbf{F}_N^H\}\right)^{-1} E\{\mathbf{F}_N\mathbf{r}b_k^*\} \\ &= \sigma_d^2 - \mathbf{p}_x^H \mathbf{R}_{xx}^{-1}\mathbf{p}_x = J_{\min,x} \end{aligned} \tag{18}$$

This indicates that the CE-MMSE receiver can provide the same performance as the conventional MMSE receiver when the SR per symbol is equal to the number of subcarriers.

Another important performance criterion is the convergence characteristic of the adaptive implementation. It is well known that the eigenvalues of the correlation matrix $\mathbf{R}$ of the input vector to the MMSE filter affects the convergence speed and the maladjustment of the NLMS algorithm [5]. Using a rule in matrix algebra, it is easy to prove that

$$\begin{aligned} \mathrm{tr}\left(\mathbf{R}_{xx}\right) &= \mathrm{tr}\left(E\{\mathbf{x}\mathbf{x}^H\}\right) = \mathrm{tr}\left(\mathbf{F}_N E\{\mathbf{r}\mathbf{r}^H\}\mathbf{F}_N^H\right) \\ &= \mathrm{tr}\left(E\{\mathbf{r}\mathbf{r}^H\}\mathbf{F}_N\mathbf{F}_N^H\right) = \mathrm{tr}\left(E\{\mathbf{r}\mathbf{r}^H\}\right) \\ &= \mathrm{tr}\left(\mathbf{R}_{rr}\right) \end{aligned} \tag{19}$$

This reveals the fact that if the SR is fixed at the number of subcarriers, the CE-MMSE receiver has the same convergence speed as the conventional MMSE receiver when the LMS algorithm is employed by both receivers.

When the SR per symbol N is larger than the number of subcarriers, for the conventional MMSE receiver, the input vector $\mathbf{x}$ only takes the first M elements of the product $\mathbf{F}_N\mathbf{r}$, the remaining $(N-M)$ elements are the noise components on the $(N-M)$ virtual subcarriers. In this case (18) and (19) do not hold true anymore. However, since the noise components are white Gaussian and mutually independent, their impact on the eigenvalue spread of the correlation matrix $\mathbf{R}_{rr}$ is very small and almost negligible, consequently, it is supposed that the CE-MMSE receiver has almost the same convergence speed as the conventional

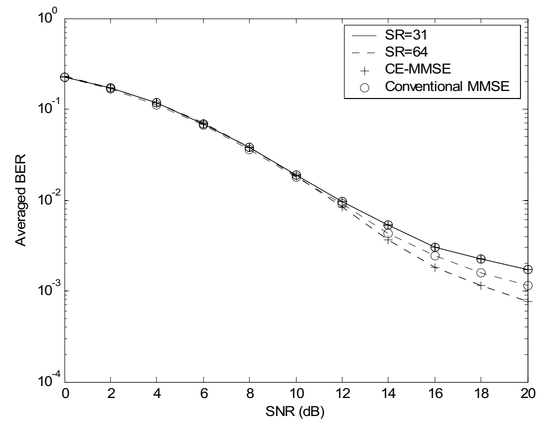

Fig. 1 Comparison of BER vs. SNR between the CE-MMSE and conventional MMSE receivers for a system with 20 active users over an AWGN channel

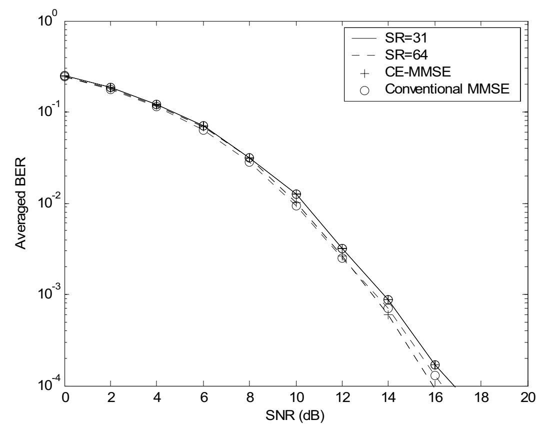

Fig. 2 Comparison of BER vs. SNR between the CE-MMSE and conventional MMSE receivers for a system with 10 active users over a multipath channel

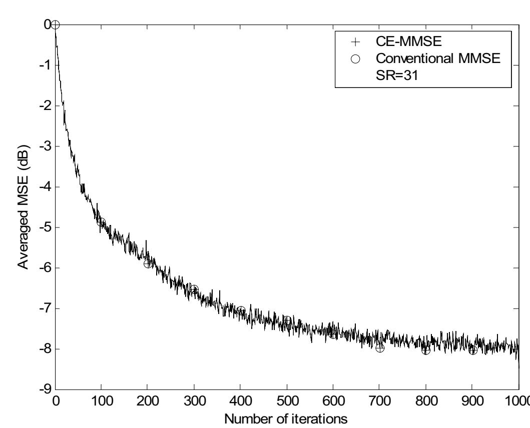

Fig. 3 Comparison of MSE convergence curves between the CE-MMSE and conventional MMSE receivers for a system with 20 active users over a multipath channel when SR=31.

chip level LMMSE receiver. This conclusion will be verified by simulations in the next section. On the other hand, when we increase the SR per symbol, the CE-MMSE receiver is expected to outperform the conventional MMSE receiver due to its higher spectral resolution.

5. SIMULATION RESULTS

The performance of the proposed adaptive CE-LMMSE receiver is evaluated by numerical simulations, and compared with that of the conventional MMSE receiver in terms of average BER performance and average MSE convergence speed. An asynchronous uplink MC-CDMA system using Gold codes of length 31 is considered in simulations. A quasi-static multipath fading channel is adopted and it is assumed that the maximum delay spread is of half-symbol duration and the four-ray multipath channel's MIP is exponentially distributed. A normalized LMS algorithm is used to adjust the weight coefficients in all simulations. The BER is calculated by averaging the results of 1000 independent runs over 1000 symbols per user. The first 200 symbols transmitted in one packet are used as training sequence. In order to improve the convergence speed, a gear-shift step size strategy is adopted, that is, during the training period, the step-size parameter μ is 0.8, whereas, the step-size is 0.06 in the decision-directed mode.

The performance of BER versus SNR is first investigated for both AWGN and multipath channels. Figs. 1-2 compare the BER performance between the CE-MMSE and conventional MMSE receivers for an asynchronous MC-CDMA system with different system loads over AWGN and multipath channels, respectively. It is noticed that when the SR is equal to the number of subcarriers, i.e. 31, the performance of the CE-MMSE receiver is identical to that of the conventional MMSE receiver. When we increase the SR to 64 points per symbol, the CE-MMSE receiver outperforms the conventional MMSE receiver in both AWGN and multipath channels. It is also worth noting that the performance of both CE-MMSE and conventional MMSE receivers improves with the increase of the SR per symbol. This verifies the conjecture concluded in the previous section.

In order to examine the convergence characteristic of the CE-MMSE receiver, we compare the instant MSE convergence curves of both CE-MMSE and conventional MMSE receivers for an asynchronous MC-CDMA system with 20 active users over a multipath channel when the operating SNR is fixed at 16 dB. The MSE values are averaged over all users, taken on 1000 independent simulations. When the SR per symbol is equal to the number of subcarriers, i.e. 31, it is shown in Fig. 3 that the CE-MMSE receiver's convergence characteristic is exactly the same as that of the conventional MMSE receiver,

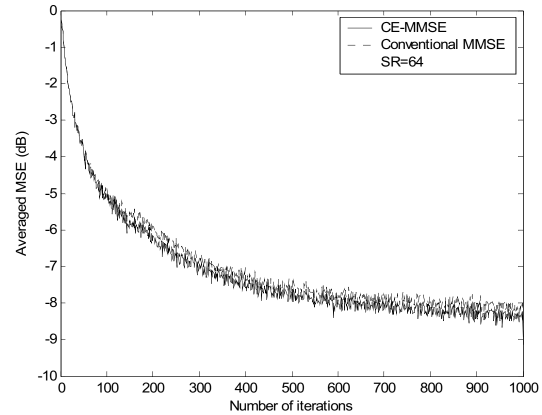

Fig. 4 Comparison of MSE convergence curves between the CE-MMSE and conventional MMSE receivers for a system with 20 active users over a multipath channel when SR=64.

whereas, as shown in Fig. 4, when the SR is equal to 64, its convergence characteristic is virtually the same as that of the conventional MMSE receiver.

6. CONCLUSIONS

In this paper, we extend the computationally efficient MMSE receiver to asynchronous MC-CDMA systems over multipath fading channels. Theoretical analysis proves that the proposed receiver has the same BER performance and convergence speed as that of the conventional MMSE receiver when the sampling rate per symbol is equal to the number of subcarriers. When the sampling rate is increased, the proposed MMSE receiver outperforms the conventional MMSE receiver due to the higher spectral resolution.

7. REFERENCES

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