STEADY-STATE PERFORMANCE OF CONSTRAINED NORMALIZED ADAPTIVE FILTERS FOR CDMA SYSTEMS

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ABSTRACT

Constrained normalized adaptive filters are used as a computationally efficient class of receivers to decrease multiple access interference (MAI). Some receivers estimate the amplitude and/or use different normalization parameters in order to improve the convergence speed. In this paper, we derive the steady-state performance of this class of receivers, from which it is revealed that the normalization parameters that aim at increasing the convergence speed deteriorate the steady-state performance if the step-size is not changed. Additionally, we prove that an estimate of the desired user's amplitude can greatly improve the steady-state performance. Computer simulations show remarkably good agreement with our analysis.

1. INTRODUCTION

Blind adaptive receivers are strong candidates to mitigate multiple access interference (MAI) in DS/CDMA systems, as they offer manageable complexity and many proposals do not require more information than the information required by the rake receiver [1]–[5]. In this paper, we focus on a common class of constrained normalized adaptive receivers of which the adaptive filters geometrically belong to a hyperplane determined by the desired user's signature. Blind adaptive receivers of this type include, for example, the OPM-based gradient projection [3] (OPM-GP) algorithm, the generalized projection [3] (GP) algorithm, the space alternating generalized projection [3] (SAGP) algorithm and their versions with a modified normalization parameter that increases the convergence speed [4], [5]. Some of these blind adaptive receivers update the adaptive filter using estimates of the desired user's amplitude and symbols.

We derive the steady-state performance of the aforementioned algorithms by including the (possible) information about the desired user's amplitude into the energy-conservation relation [6, Ch. 6], which originally considered a different system model. Hence, it becomes clear that the algorithms that do not use any information about the desired user's signature have a steady-state performance that is strongly limited by the choice of the step-size. Additionally, we study how the steady-state performance is influenced by the angle between the desired user's signature and the input signal.

It is worthy to mention that the algorithms discussed in this paper are simple examples of a more general class of receivers based on the adaptive projected subgradient method [7]. Additionally, our results can be immediately extended to adaptive array antenna systems if the sources transmit signals with constant modulus. Due to the space limitation, the proofs presented in this paper are only outlined.

2. SYSTEM MODEL

For simplicity, we consider a synchronous BPSK DS-CDMA system with K active users and M chips per symbol, although an extension of our analysis to asynchronous systems with constant modulus symbols is possible. If the receiver is synchronized with the desired user, the received vector containing M samples of the symbol interval i is given by

$$\boldsymbol{r}[i] = A_1 b_1[i]\boldsymbol{s}_1 + \sum_{k=2}^{K} A_k b_k[i]\boldsymbol{s}_k + \boldsymbol{n}[i]. \tag{1}$$

where $A_k \in [0,\infty)$, $b_k[i] \in \{-1,+1\}$, and $\boldsymbol{s}_k \in \mathbb{R}^M$ ($\boldsymbol{s}_1,\ldots,\boldsymbol{s}_K$ linear independent) are the kth user's amplitude, transmitted bit, and received signature [2]. Without loss of generality, we can assume that the first user is the desired one and $\|\boldsymbol{s}_k\|^2 = \boldsymbol{s}_k^T\boldsymbol{s}_k = 1$ ($k = 1,\ldots,K$). The following common assumptions for the analysis of CDMA systems are used in this paper:

Assumption 1 *The noise vector $\boldsymbol{n}[i]$ is a zero-mean random vector with $E\{\boldsymbol{n}[i]\boldsymbol{n}[i]^T]\} = \sigma_n^2\boldsymbol{I}_M$, where $\boldsymbol{I}_M$ is the M-dimensional identity matrix.*

Assumption 2 $E\{b_k[i]\} = 0$, $k = 1,\ldots,K$ *and* $E\{b_q[i]b_r[i]\} = \begin{cases} 1, & q = r, \\ 0, & \text{otherwise.}\end{cases}$

Assumption 3 $b_k[i]$, $k = 1,\ldots,K$ *and* $\boldsymbol{n}[i]$ *are mutually independent.*

The detector estimates the received bits with the aid of a properly designed filter $\boldsymbol{h} \in \mathbb{R}^M$ by

$$\hat{b}_1[i] = \mathrm{sgn}(\boldsymbol{h}^T\boldsymbol{r}[i]), \tag{2}$$

where $\mathrm{sgn}(x) = \begin{cases} +1, & \text{for } x \geq 0, \\ -1, & \text{otherwise.}\end{cases}$

Remark 1 *Although the model in (1) represents a synchronous system over a flat-fading channel, an extension to asynchronous systems over frequency-selective channels is possible [1, Sec. II].*

3. OPTIMAL CONSTRAINED FILTERS

The linear minimum mean-square error (MMSE) filter is expressed by

$$\boldsymbol{h}_{\mathrm{opt}}^{\mathrm{mmse}} \in \arg\min_{\boldsymbol{h}} E\{|\boldsymbol{h}^T\boldsymbol{r}[i] - b_1[i]|^2\}. \tag{3}$$

Under Assumptions 1–3, we readily verify that $\boldsymbol{h}_{\mathrm{opt}}^{\mathrm{mmse}} = A_1\boldsymbol{R}^{-1}\boldsymbol{s}_1$, where $\boldsymbol{R} = E[\boldsymbol{r}[i]\boldsymbol{r}[i]^T] = \sum_{k=1}^{K} A_k^2\boldsymbol{s}_k\boldsymbol{s}_k^T + \sigma_n^2\boldsymbol{I}_M$. Additionally, from (2), we conclude that the bit-error-rate (BER) is

Table 1. Characterization of the adaptive filters under study

Algorithm	α	$\hat{b}_1[i]$	$g(\boldsymbol{r}[i])$
GP [3]	A_1	$\mathrm{sgn}(\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i])$	$\|\boldsymbol{r}[i]\|^2$
SAGP [3]	$\hat{A}_1$	$\mathrm{sgn}(\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i])$	$\|\boldsymbol{r}[i]\|^2$
OPM-GP [3]	0	-	$\|\boldsymbol{r}[i]\|^2$
Modified GP	A_1	$\mathrm{sgn}(\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i])$	$\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]$
Modified SAGP [5]	$\hat{A}_1$	$\mathrm{sgn}(\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i])$	$\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]$
Modified OPM-GP [4]	0	-	$\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]$

the same for $\boldsymbol{h}_{\mathrm{opt}}^{\mathrm{mmse}}$ and any filter of the form $\boldsymbol{h} = \beta\boldsymbol{R}^{-1}\boldsymbol{s}_1$, provided that $\beta > 0$. On the other hand, we can see that $\boldsymbol{h}_{\mathrm{opt}} = (\boldsymbol{R}^{-1}\boldsymbol{s}_1)/(\boldsymbol{s}_1^T\boldsymbol{R}^{-1}\boldsymbol{s}_1)$ is a scaled version of $\boldsymbol{h}_{\mathrm{opt}}^{\mathrm{mmse}}$ and the solution to the optimization problem

$$\boldsymbol{h}_{\mathrm{opt}} \in \arg\min_{\boldsymbol{h}\in C_s} E\{|\boldsymbol{h}^T\boldsymbol{r}[i] - \alpha b_1[i]|^2\}, \quad \forall\alpha \in [0,\infty), \tag{4}$$

where $C_s := \{\boldsymbol{h} \in \mathbb{R}^M \mid \boldsymbol{h}^T\boldsymbol{s}_1 = 1\}$. It is clear that by imposing the constraint C_s, the solution does not change irrespective of the choice of α.

4. ADAPTIVE FILTERS FOR MAI SUPPRESSION

The adaptive filters of our interest, which track the solution $\boldsymbol{h}_{\mathrm{opt}}$ in (4), are commonly expressed by

$$\boldsymbol{h}_i = \boldsymbol{P}\left[\boldsymbol{h}_{i-1} - \mu(\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i] - \hat{b}_1[i]\alpha)\frac{\boldsymbol{r}[i]}{g(\boldsymbol{r}[i])}\right] + \boldsymbol{s}_1, \tag{5}$$

where $\boldsymbol{h}_0 = \boldsymbol{s}_1$ and $\mu \in (0, 2]$ is the step-size. The parameters α, $\hat{b}_1[i]$, and the function $g(\boldsymbol{r}[\boldsymbol{i}])$ vary according to the algorithms shown in Table 1. In this table, the parameter $\hat{A}_1$ is an estimate of A_1. The original algorithms and their modified versions differ in the choice of the normalization parameter $g(\boldsymbol{r}[i])$. The matrix $\boldsymbol{P} = (\boldsymbol{I}_M - \boldsymbol{s}_1\boldsymbol{s}_1^T)$ is the orthogonal projection onto $(\mathrm{span}\{\boldsymbol{s}_1\})^\perp$. It is clear that $\boldsymbol{P}$ has the following properties: (a) $\boldsymbol{P}^2 = \boldsymbol{P}$, (b) $\boldsymbol{P}\boldsymbol{s}_1 = \boldsymbol{0}$ and (c) $\|\boldsymbol{P}\boldsymbol{x}\| \le \|\boldsymbol{x}\|$, $\boldsymbol{x} \in \mathbb{R}^M$. We newly introduce the modified GP algorithm, so that the GP algorithm has its modified counterpart. Additionally, we have $\boldsymbol{h}_i^T\boldsymbol{s}_1 = \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{s}_1 = 1$, $i = 0, 1, 2, \ldots$.

For mathematical tractability, in our analysis we assume $\hat{b}_1[i] = b_1[i]$ for the algorithms using $\alpha \neq 0$ (NOTE: This assumption does not always hold in our simulations). Hence, for these blind algorithms with $\alpha \neq 0$, our analysis for the steady-state signal-to-interference-plus-noise ratio (SINR) shows slightly better agreement for our simulations in high SNR environments than in low SNR environments, although very good agreement is achieved even in low SNR environments. We study the performance of these algorithms by fixing α to a possibly erroneous estimate of A_1. The analysis under these assumptions reveals how well the amplitude estimation algorithm should perform in order to achieve a desired SINR level.

For notational simplicity, we define the following:

$$\tilde{\boldsymbol{h}}_i := \boldsymbol{h}_i - \boldsymbol{h}_{\mathrm{opt}} \in (\mathrm{span}\{\boldsymbol{s}_1\})^\perp \tag{6}$$

$$e_{\mathrm{p}}[i] := \boldsymbol{r}[i]^T\boldsymbol{P}\tilde{\boldsymbol{h}}_i = \boldsymbol{r}[i]^T\tilde{\boldsymbol{h}}_i \tag{7}$$

$$e_{\mathrm{a}}[i] := \boldsymbol{r}[i]^T\boldsymbol{P}\tilde{\boldsymbol{h}}_{i-1} = \boldsymbol{r}[i]^T\tilde{\boldsymbol{h}}_{i-1} \tag{8}$$

$$v(\alpha)[i] := \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{r}[i] - b_1[i]\alpha \tag{9}$$

$$u(\alpha)[i] = \boldsymbol{h}_{i-1}^T\boldsymbol{r}[i] - b_1[i]\alpha, \tag{10}$$

where subscripts a and p stand for *a priori* and *posteriori*, respectively. With this notation, which includes the available information about the desired user's amplitude, we adopt the following assumptions in order to simplify our analysis in Sec. 5.

Assumption 4 *At steady-state, $e_{\mathrm{a}}[i]$ and $g(\boldsymbol{r}[i])$ (for the functions in Table I) are independent.*

Noticing that (i) in CDMA systems the signatures are designed to satisfy $\boldsymbol{s}_k^T\boldsymbol{s}_j \approx 0$, $j \neq k$, (ii) the model in (1) utilizes symbols with constant modulus, we have that $g(\boldsymbol{r}[i]) \approx \mathrm{constant}$ when the noise power approaches zero, and thus we justify the above assumption for the noiseless case. However, even in more general situations, it is known that at steady-state $e_{\mathrm{a}}[i]$ is often less sensitive to the input $\boldsymbol{r}[i]$ [6, p. 293]. Therefore, this assumption provides good approximations for our steady-state analysis even in low SNR environments with random signatures, as verified by our simulations in Sec. 6.

Assumption 5 *$v^2(\alpha)[i]$ is independent of $\boldsymbol{r}[i]$.*

This assumption holds asymptotically as the noise variance approaches zero. In the noiseless case, the decorrelating filter and the filter in (4) become identical [8, Lemma 2], and thus $\boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{r}[i] = A_1b_1[i]$, which implies that $v^2(\alpha)[i]$ is constant. Nonetheless, in more general situations, Assumption 5 simplifies our steady-state analysis, which shows good agreement with our simulations.

5. STEADY-STATE PERFORMANCE

In this section, we calculate the steady-state SINR of the adaptive filters in (5). We start by the following Lemma:

Lemma 1 *The adaptive filters in (5) satisfy $\lim_{i\to\infty} E\{\boldsymbol{h}_{i-1}\} \approx \boldsymbol{h}_{\mathrm{opt}}$ for $\mu \in (0, 2]$.*

Proof (sketch): The main idea is to use the approximations $E\left\{\frac{\boldsymbol{r}[i]\boldsymbol{r}[i]^T}{g(\boldsymbol{r}[i])}\right\} \approx \frac{E\{\boldsymbol{r}[i]\boldsymbol{r}[i]^T\}}{E\{g(\boldsymbol{r}[i])\}}$ and $E\left\{\frac{b_1[i]\boldsymbol{r}[i]}{g(\boldsymbol{r}[i])}\right\} \approx \frac{E\{b_1[i]\boldsymbol{r}[i]\}}{E\{g(\boldsymbol{r}[i])\}} = \frac{A_1\boldsymbol{s}_1}{E\{g(\boldsymbol{r}[i])\}}$, which are justified by the discussion after Assumption 4, and then proceed in a similar way to the one for nonnormalized algorithms [2, Sec. IV-A]. ■

Proposition 1 *For the model in (1) and under Assumptions 1–5, the SINR,*

$$\mathrm{SINR}(\boldsymbol{h}_{i-1}) = \frac{A_1^2}{E\{|\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i] - A_1b_1[i]|^2\}}, \tag{11}$$

of the adaptive filters $\boldsymbol{h}_{i-1} \in C_s$ in (5) is given at steady-state by

$$\lim_{i\to\infty} \mathrm{SINR}(\boldsymbol{h}_{i-1}) \approx \frac{A_1^2}{\dfrac{\sigma_v^2(\alpha)\mu}{2b/a - \mu} + \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}} - A_1^2}, \tag{12}$$

where $a = E\left\{\frac{\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]}{g^2(\boldsymbol{r}[i])}\right\}$, $b = E\left\{\frac{1}{g(\boldsymbol{r}[i])}\right\}$, *and* $\sigma_v^2(\alpha) = E\{v^2(\alpha)[i]\} = \alpha^2 - 2\alpha A_1 + \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}}$.

Proof (sketch): With the aid of Lemma 1, the denominator of (11) at steady-state can be approximated by

$$E\{|\boldsymbol{h}_{i-1}^T\boldsymbol{r}[i] - A_1b_1[i]|^2\} \approx E\{e_{\mathrm{a}}^2[i]\} + \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}} - A_1^2, \quad i \to \infty. \tag{13}$$

Hence, in order to obtain the steady-state SINR, we need to evaluate $E\{e_{\mathrm{a}}^2[i]\}$, $i \to \infty$.

For the adaptive filters in (5), by $\boldsymbol{P}(\boldsymbol{h}_i) = \boldsymbol{P}(\boldsymbol{h}_i - \boldsymbol{s}_1) = \boldsymbol{h}_i - \boldsymbol{s}_1$, we have $\boldsymbol{P}\boldsymbol{h}_i = \boldsymbol{P}\left[\boldsymbol{h}_{i-1} - \mu u(\alpha)[i]\boldsymbol{r}[i]/g(\boldsymbol{r}[i])\right].$

From this equation, we find the following variance-relation for steady-state performance [6, Ch. 6]:

$$E\left\{\mu u^2(\alpha)[i]\frac{\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]}{g^2(\boldsymbol{r}[i])}\right\} = 2E\left\{\frac{e_{\mathrm{a}}[i]u(\alpha)[i]}{g(\boldsymbol{r}[i])}\right\}. \tag{14}$$

Using the relation $u(\alpha)[i] = e_{\mathrm{a}}[i] + v(\alpha)[i]$ in the last equation and from Assumptions 4 and 5, we get

$$E\{e_{\mathrm{a}}^2[i]\} = \frac{\sigma_v^2(\alpha)\mu}{2b/a - \mu}. \tag{15}$$

Therefore, from (11), (13) and (15), we arrive at the desired result. ■

Next, we compare the steady-state performance of the filters in (5) by studying the ratio a/b and we also study the influence of the parameter α.

5.1. Algorithms based on $g(\boldsymbol{r}[i]) = \boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]$

This case is the simplest one, since $a = b$. Hence, from Proposition 1, the steady-state SINR for the modified algorithms can be approximated by

$$\mathrm{SINR}_{\mathrm{modified}} \approx \frac{A_1^2}{\dfrac{\sigma_v^2(\alpha)\mu}{2-\mu} + \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}} - A_1^2} \tag{16}$$

In the last equation, the global maximum is reached at $\alpha = A_1$. This implies that a good estimate of the amplitude A_1 is expected to improve the steady-state performance.

5.2. Algorithms based on $g(\boldsymbol{r}[i]) = \|\boldsymbol{r}[i]\|^2$

For these original algorithms, by Cauchy-Schwartz's inequality we have

$$\begin{aligned} a &= E\left\{\frac{\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]}{g^2(\boldsymbol{r}[i])}\right\} \\ &\leq E\left\{\frac{\|\boldsymbol{r}[i]\|\|\boldsymbol{P}\boldsymbol{r}[i]\|}{g^2(\boldsymbol{r}[i])}\right\} \leq E\left\{\frac{\|\boldsymbol{r}[i]\|^2}{g^2(\boldsymbol{r}[i])}\right\} = E\left\{\frac{1}{\|\boldsymbol{r}[i]\|^2}\right\} = b. \end{aligned}$$

Therefore, at steady-state we conclude that

$$\begin{aligned} \mathrm{SINR}_{\mathrm{original}} &\approx \frac{A_1^2}{\dfrac{\sigma_v^2(\alpha)\mu}{2\dfrac{b}{a} - \mu} + \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}} - A_1^2} \\ &\geq \frac{A_1^2}{\dfrac{\sigma_v^2(\alpha)\mu}{2-\mu} + \boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}} - A_1^2} = \mathrm{SINR}_{\mathrm{modified}} \end{aligned} \tag{17}$$

As in the case of modified filters, the original filters with amplitude estimation can perform potentially better than those that rely on $\alpha = 0$. It is also clear that the original filters provide higher SINR at steady-state than their modified versions when they have the same step-size. Nevertheless, care must be taken when comparing these algorithms. If a modified algorithm employs a smaller step-size than its original version, it may happen that both algorithms achieve the same steady-state performance, with the modified algorithm having superior convergence rate. If the bound is tight, the above inequality is useful to calculate the performance of the original algorithms, as $\mathrm{SINR}_{\mathrm{original}}$ can be well approximated by $\mathrm{SINR}_{\mathrm{modified}}$, which does not require the determination of the ratio b/a in (12).

For the original algorithms, it is possible to check in which cases the lower bound in (17) is tight. By $\boldsymbol{P}\boldsymbol{r}[i] = \boldsymbol{r}[i] - (\boldsymbol{r}[i]^T\boldsymbol{s}_1)\boldsymbol{s}_1$, we get

$$\begin{aligned} \frac{a}{b} &= \frac{E\left\{\dfrac{\boldsymbol{r}[i]^T\boldsymbol{P}\boldsymbol{r}[i]}{\|\boldsymbol{r}[i]\|^4}\right\}}{E\left\{\dfrac{1}{\|\boldsymbol{r}[i]\|^2}\right\}} \\ &= 1 - \frac{E\left\{\dfrac{\|\boldsymbol{r}[i]\|^2\|\boldsymbol{s}_1\|^2\cos^2\theta[i]}{\|\boldsymbol{r}[i]\|^4}\right\}}{E\left\{\dfrac{1}{\|\boldsymbol{r}[i]\|^2}\right\}} = 1 - \frac{E\left\{\dfrac{\cos^2\theta[i]}{\|\boldsymbol{r}[i]\|^2}\right\}}{E\left\{\dfrac{1}{\|\boldsymbol{r}[i]\|^2}\right\}}, \end{aligned} \tag{18}$$

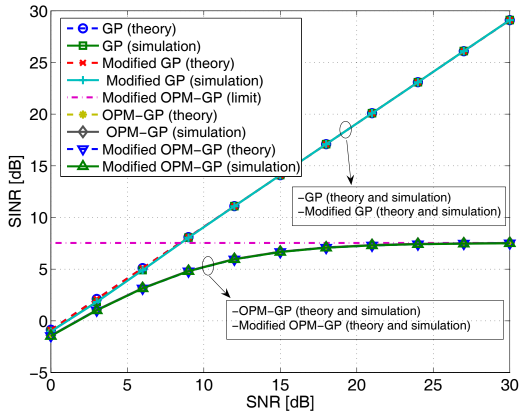
where $\cos\theta[i] = \boldsymbol{r}[i]^T\boldsymbol{s}_1/\|\boldsymbol{r}[i]\|$. From the ratio a/b derived above, we see that, if $\cos\theta[i]$ is small on average, we may expect that $a/b \approx 1$, and thus the bound in (17) is tight. A small $\cos\theta[i]$ has the equivalent geometrical interpretation of $\boldsymbol{r}[i]$ being nearly orthogonal to $\boldsymbol{s}_1$. Therefore, by also checking the model in (1), a tight bound happens when good spreading codes are used ($\boldsymbol{s}_k^T\boldsymbol{s}_j \approx 0, k \neq j$), such as Gold codes, and the interfering users' power is high as compared to the noise power and the desired user's power.

Remark 2 *As σ_n approaches 0, we have that $\sigma_v^2(\alpha)|_{\alpha=0} = (\boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{r}[i])^2 = A_1^2$ and $\boldsymbol{h}_{\mathrm{opt}}^T\boldsymbol{R}\boldsymbol{h}_{\mathrm{opt}} = A_1^2$ (see the discussion after Assumption 5). Hence, the steady-state SINR of the modified OPM-GP algorithm is simply given by $\mathrm{SINR}_{\mathrm{modifiedOPM-GP}} = (2-\mu)/\mu$. This is the maximum SINR that this adaptive filter can provide for a given step-size μ, as the presence of noise cannot increase the SINR.*

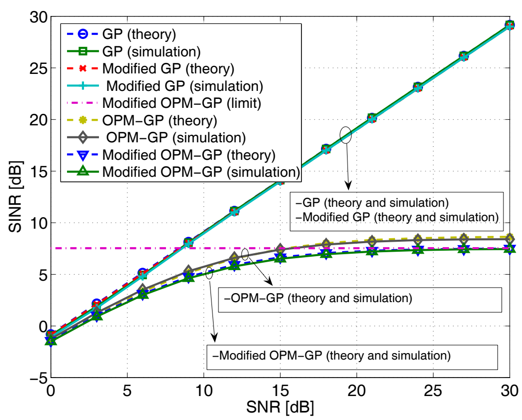
6. SIMULATION RESULTS

Figure 1(a) shows the theoretical and simulated curves for the OPM-GP algorithm, the GP algorithm and their modified counterparts. The system has 20 users and the signatures are chosen from length-31 Gold spreading codes. The desired user's amplitude is $A_1 = 1$. Each interferer has 9.54 dB power advantage as compared to the desired user. The step-size for all algorithms is set to $\mu = 0.3$. The simulated values are obtained by averaging the last 1000 out of 3000 iterations of the ensemble-average curve, which in turn is obtained by averaging the SINR curve over 1000 realizations. The ratio b/a, which is necessary to calculate the SINR of the original algorithms, is obtained through simulations. The SINR limit for the modified OPM-GP is calculated according to Remark 2. As the MAI is high and Gold sequences are employed, the original algorithms and their modified versions show very close performance, as predicted [see (17)]. Moreover, even for low SNR, the SINR of the algorithms based on estimations of the received bits is consistent with our analysis, which corroborates our assumptions.

In Fig. 1(b) we decrease the number of users to five. Additionally, the signatures are changed to random sequences and all users have the same power. With these changes, we increase $\cos\theta[i]$ in (18) on average. Other parameters are the same as in Fig. 1(a). It is clear that the gap in terms of steady-state performance between the original algorithms and their modified counterparts (with the same



(a)



(b)

Fig. 1. Simulated and theoretical SINR curves as a function of SNR. (a) High MAI and Gold sequences. (b) Low MAI and random sequences.

step-size) is more accentuated for the OPM-GP algorithm, as $\sigma_v^2(\alpha)$ is larger. This observation is also predicted by (17).

In Fig. 2 we consider the modified GP algorithm with amplitude mismatches, i.e., $\alpha \neq A_1$, for different step-sizes under SNR 15 dB, which gives an insight into the performance of the SAGP algorithm. Other parameters are the same as in Fig. 1(a). As the input signal presents low correlation with $\boldsymbol{s}_1$, the performance of the GP algorithm is similar to the performance of its modified counterpart, and thus the simulated curve for the original GP algorithm is omitted for visual clarity. Due to wrong estimates of the received bits, the algorithm is unable to reduce MAI for a large value of the parameter α. However, for $\alpha = 0$, which corresponds to the modified OPM-GP algorithm, and for α not much different from the desired user's amplitude, our analysis and the simulation results show remarkably good agreement. It is also clear that the algorithm is robust to amplitude mismatches for small step-sizes.

7. CONCLUSION

We have studied the steady-state performance of the OPM-GP algorithm, the GP algorithm, the SAGP algorithm, and their versions

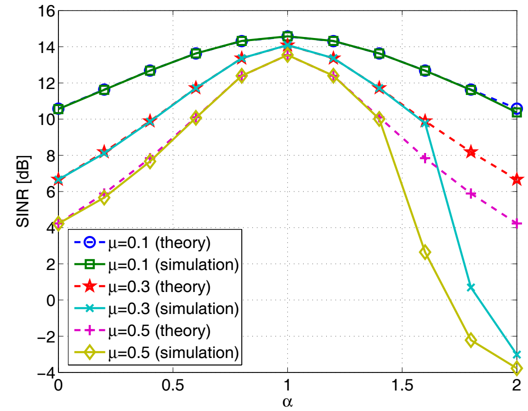


Fig. 2. SINR as a function of α for different values of μ. SNR 15 dB, $A_1 = 1$.

with a modified normalization parameter. Our results are valuable to help designers choose the most suitable algorithm for a given DS/CDMA system.

We have shown that the SAGP and GP algorithms improve the performance of the OPM-GP algorithm by utilizing information about the desired user's amplitude. Geometrical considerations of a CDMA system show the conditions under which the original algorithms and their modified versions perform almost equally at steady-state when they use the same step-size.

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