A METHOD FOR UNBIASED IDENTIFICATION OF THE FEEDBACK PATH IN HEARING AIDS

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ABSTRACT

It is well known that the conventional adaptive algorithm for cancellation of the effect of the feedback path in hearing aids is biased. In this paper the stationary point of a modified adaptive algorithm by splitting the delay in the conventional one is derived by a direct method considering the causality constraint and an explicit expression of the bias is obtained. Also, a stability condition is mentioned. Then, when the input signal to hearing aids is a moving average (MA) process or an autoregressive (AR) process, unbiased algorithms for identification of the feedback path are proposed. It is shown by simulations that the new algorithm for an AR process converges considerably faster than that of the previously proposed two-channel adaptive algorithm.

1. INTRODUCTION

There have been several works concerning the analysis of some adaptive algorithms in hearing aids [1]–[3]. In hearing aids there is an acoustic feedback path from the receiver to the microphone and this causes annoying effects such as whistling and howling. An adaptive filter is used to model this acoustic feedback path and cancel its effect. In [1], based on the time domain approach, an expression of the bias in the weight vector of the conventional adaptive filter algorithm has been derived by assuming that the incoming signal is an AR process. But this expression is not a closed form one so that it is not easy to interpret and utilize. In [2] via the frequency domain technique a corresponding formula and the convergence condition have been presented.

In [3] an unbiased algorithm using a two-channel identification scheme has been proposed for the case where the incoming signal is an AR process of order $q-1$ with its identifiability condition that the length of the delay is greater than or equal to q. In [4] the convergence analysis of this algorithm using the ODE (ordinary differential equation) method has been presented together with the expression of the steady-state mean square parameter estimation errors.

In this paper, first by a method which is more direct than that in [2] but is still considering the causality constraint, the explicit expression of this bias in [2] is rederived. Also, the stability condition in [2] is mentioned. But it is not possible to separate the parameters of the feedback path from the stationary point of the adaptive filter based on this expression. In [5], a modified scheme by splitting the delay in the forward path has been considered. Here, based on this scheme unbiased identification algorithms are proposed for the case where the incoming signal is an MA process or an AR process. Finally, the new algorithm and the algorithm in [3] are compared under the condition that the steady-state relative mean square estimation errors are same. It is seen by simulations that the new algorithm converges considerably faster.

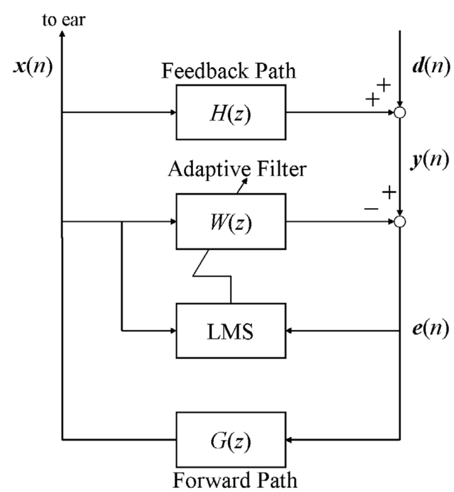

Fig. 1. Block diagram of the conventional hearing aids.

2. DERIVATION OF THE STATIONARY POINT

Fig.1 shows the block diagram of a hearing aid plant with the conventional adaptive filter where $d(n)$ is a zero-mean stationary incoming signal and the transfer functions of the forward path and the feedback path are $G(z)$ and $H(z)$, respectively. The forward path transfer function $G(z)$ is the desired characteristic of the hearing aids and is fixed and known but $H(z)$ is the transfer function from the receiver to the microphone which is unknown and may be slowly time-varying. The conventional LMS algorithm for cancellation of the effect of the feedback path is

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu \boldsymbol{x}(n) e(n) \tag{1}$$

with

$$\boldsymbol{x}(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^T \tag{2}$$

$$\boldsymbol{w}(n) = [w_0(n), w_1(n), \ldots, w_{L-1}(n)]^T \tag{3}$$

where $x(n)$ and $w_i(n)$ are the input signal and the i-th weight of the adaptive filter, respectively and μ is the positive step size. The signals $x(n)$ and $e(n)$ are expressed as

$$x(n) = G(z)e(n) \tag{4}$$
$$\begin{aligned} e(n) &= y(n) - W(z)x(n) \\ &= d(n) + (H(z) - W(z))G(z)e(n) \end{aligned} \tag{5}$$

where $y(n)$ is the output of the microphone and z^{-1} denotes the unit time delay operator with

$$W(z) = w_0 + w_1 z^{-1} + \cdots + w_{L-1} z^{-L+1} \tag{6}$$
$$H(z) = h_0 + h_1 z^{-1} + \cdots + h_{M-1} z^{-M+1}. \tag{7}$$

In (5) and (6) a fixed weight vector $\boldsymbol{w}$ is used. Then, from (4) and (5) $e(n)$ is expressed as

$$e(n) = Q(z)d(n) \tag{8}$$

with

$$Q(z) = \frac{1}{1 + G(z)(W(z) - H(z))}. \tag{9}$$

where we assume that this is stable. The stationary point satisfies

$$\mathrm{E}[x(n-l)e(n)] = 0 \quad (l = 0, \ldots, L-1). \tag{10}$$

From (4) and (8) the left hand side of (10) can be expressed as

$$\begin{aligned} &\frac{1}{2\pi}\int_0^{2\pi} e^{-jlw} Q(e^{j\omega})G(e^{j\omega})Q(e^{-j\omega})S(e^{j\omega})d\omega \\ &= \frac{1}{2\pi}\int_0^{2\pi} z^l Q(z^{-1})G(z^{-1})Q(z)S(z^{-1})d\omega \end{aligned} \tag{11}$$

where $S(e^{j\omega})$ is the power spectrum of $d(n)$ and we put $z = e^{-j\omega}$. Hence, (10) can be expressed as

$$\int_0^{2\pi} z^l Q(z^{-1})G(z^{-1})Q(z)S(z^{-1})d\omega = 0 \tag{12}$$

Although (12) holds for $l = 0, \ldots, L-1$, if L is large enough, it is reasonable to find a solution that satisfies (12) for all $l \geq 0$. For this, the integrand must be a series of positive powers of z. Hence, we have

$$[Q(z^{-1})G(z^{-1})Q(z)S(z^{-1})]_+ = 0 \tag{13}$$

where $[\cdot]_+$ denotes the extraction of the causality part, that is, the constant term and negative powers of z. Let the spectral factorization of $S(z)$ be

$$S(z) = R(z)R(z^{-1})\sigma^2 \tag{14}$$

where $R(z)$ is of minimum phase and its constant term is 1. Since $Q(z)$ is stable, $R(z^{-1})Q(z^{-1})$ is purely non-causal and can be factored out from $[\cdot]_+$ in (13). So, we have

$$\left[\frac{G(z^{-1})R(z)}{1 + G(z)(W(z) - H(z))}\right]_+ = 0. \tag{15}$$

To obtain an explicit solution of (15) we assume that the transfer function of the forward path is expressed as

$$G(z) = z^{-q}G_c(z) \tag{16}$$

where $G_c(z)$ is of minimum phase. Then, from (15) and (16) we have

$$\left[\frac{z^q R(z)}{1 + z^{-q}G_c(z)B(z)}\right]_+ = 0 \tag{17}$$

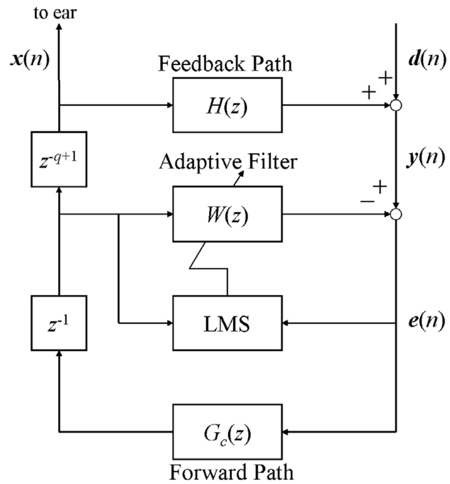

Fig. 2. Block diagram of a modified scheme by splitting the delay.

where we put the bias as

$$B(z) = W(z) - H(z). \tag{18}$$

From (17) we have

$$[z^q R(z)]_+ = \frac{R(z)G_c(z)B(z)}{1 + z^{-q}G_c(z)B(z)}. \tag{19}$$

Putting

$$A(z) = \frac{[z^q R(z)]_+}{R(z)} \tag{20}$$

the bias is obtained as

$$B(z) = \frac{A(z)}{G_c(z)\big(1 - z^{-q}A(z)\big)}. \tag{21}$$

But this expression is not useful for separating the impulse response of the feedback path. In Fig.2 a modified scheme treated in [5] is shown where the delay z^{-q} is split into two parts, that is, z^{-1} is in the forward path and z^{-q+1} is in the feedback path. So in this case $G(z) = z^{-1}G_c(z), B(z) = W(z) - z^{-q+1}H(z)$ and

$$A(z) = \frac{[zR(z)]_+}{R(z)} \tag{22}$$
$$B(z) = \frac{A(z)}{G_c(z)\big(1 - z^{-1}A(z)\big)}. \tag{23}$$

Hence, the stationary point of the adaptive filter in Fig.1 is given by

$$W_{opt}(z) = \frac{A(z)}{G_c(z)\big(1 - z^{-1}A(z)\big)} + z^{-q+1}H(z). \tag{24}$$

3. THE STABILITY CONDITION

Here we use the ODE method to obtain a (local) stability condition of the stationary point of (1). The linearized ODE near the stationary point $\boldsymbol{w}_{opt}$ is described as

$$\dot{\bar{\boldsymbol{w}}}(t) = -\Phi(\bar{\boldsymbol{w}}(t) - \boldsymbol{w}_{opt}) \tag{25}$$

with

$$\Phi = -\frac{\partial}{\partial \boldsymbol{w}} E[\boldsymbol{x}(n)e(n)]\Big|_{\boldsymbol{w}=\boldsymbol{w}_{opt}}$$

where from (11) the (k, l)th elemet of Φ is given by

$$\Phi_{lk} = -\frac{\partial}{\partial w_k}\frac{1}{2\pi}\int_0^{2\pi} z^l Q(z^{-1})G(z^{-1})Q(z)S(z^{-1})d\omega.$$

From (9)

$$\frac{\partial}{\partial w_k}Q(z) = -z^k G(z)Q^2(z)$$

hence,

$$\Phi_{lk} = -\frac{1}{2\pi}\int_0^{2\pi}\big\{z^{l+k}G^2(z^{-1})S(z^{-1})Q^2(z^{-1})Q(z) + z^{l-k}G(z^{-1})S(z^{-1})Q^2(z)G(z)Q(z^{-1})\big\}d\omega. \quad (26)$$

At the stationary point from (13) $Q(z^{-1})G(z^{-1})Q(z)S(z^{-1})$ is a series of positive powers of z and so is the first term of the integrand of (26). Hence the first term of (26) is zero. If $\Phi + \Phi^H$ is positive definite, the Lyapunov function $V(\boldsymbol{w}) = ||\boldsymbol{w} - \boldsymbol{w}_{opt}||^2$ is decreasing, since $\dot{V}(\boldsymbol{w}) \le 0$ where the equality holds only at $\boldsymbol{w} = \boldsymbol{w}_{opt}$. Hence, $\boldsymbol{w}_{opt}$ is a stable stationary point. From (26) for any vector $\boldsymbol{\xi} = (\xi_0 \dots \xi_{L-1})^T$

$$\boldsymbol{\xi}^H(\Phi + \Phi^H)\boldsymbol{\xi} = \frac{1}{2\pi}\int_0^{2\pi}\big|\sum_i \xi_i z^{-i}\big|^2\big|G(z)\big|^2 \times S(z^{-1})\big|Q(z)\big|^2 \text{Re}\, Q(z^{-1})d\omega > 0 \quad (27)$$

if from (23) the positive real condition

$$\text{Re}\, Q(e^{j\omega})\big|_{\boldsymbol{w}=\boldsymbol{w}_{opt}} = \text{Re}(1 - e^{-j\omega}A(e^{j\omega})) > 0 \quad \text{for all } \omega \quad (28)$$

holds for the scheme in Fig.2.

4. UNBIASED IDENTIFICATION OF THE FEEDBACK PATH

Based on the expression (23) methods for separating the parameters of the feedback path transfer function $H(z)$ are proposed. We assume that the stationary process $d(n)$ is expressed as an innovation representation

$$d(n) = \varepsilon(n) + r_1\varepsilon(n-1) + r_2\varepsilon(n-2) + \cdots, \quad (29)$$

where $\varepsilon(n)$ is white noise with zero mean and variance σ^2. Hence, from (14)

$$R(z) = 1 + r_1 z^{-1} + r_2 z^{-2} + \cdots \quad (30)$$

and $[zR(z)]_+ = r_1 + r_2 z^{-1} + \cdots$. So from (23) we have

$$B(z) = \frac{r_1 + r_2 z^{-1} + \cdots}{G_c(z)}. \quad (31)$$

If $d(n)$ is a $(q-1)$ th order MA process, that is, $d(n) = \varepsilon(n) + r_1\varepsilon(n-1) + \cdots + r_{q-1}\varepsilon(n-q+1)$, then

$$W_{opt}(z) = \frac{r_1 + r_2 z^{-1} + \cdots + r_{q-1}z^{-q+2}}{G_c(z)} + z^{-q+1}H(z). \quad (32)$$

Moreover, if $G_c(z)$ is constant, the parameters of $H(z)$ in (7) are readily identified from the $(q-1)$ th to $(q+M-2)$ th order coefficients of $W_{opt}(z)$. This fact was stated in [5] by simulation results without any theoretical explanations. If $G_c(z)$ is not constant, we write (32) as

$$W_{opt}(z)G_c(z) = r_1 + r_2 z^{-1} + \cdots + r_{q-1}z^{-q+2} + z^{-q+1}H(z)G_c(z). \quad (33)$$

This means that filtering $W_{opt}(z)$ by $G_c(z)$ gives the MA parameters by its zeroth to $(q-2)$ th coefficients and filtering the remaining part, that is, from the $(q-1)$ th to $(q+M-2)$ th coefficients by $1/G_c(z)$ gives the parameters of $H(z)$.

When $d(n)$ is a $(q-1)$ th order AR process

$$d(n) = P^{-1}(z)\varepsilon(n), \quad (34)$$

where

$$P(z) = 1 + p_1 z^{-1} + \cdots + p_{q-1}z^{-q+1}, \quad (35)$$

then from (29) it follows that

$$R(z) = \frac{1}{P(z)}. \quad (36)$$

In this case, we have

$$W_{opt}(z)G_c(z) = r_1 + \cdots + r_{q-1}z^{-q+2} + r_q z^{-q+1} + \cdots + r_{q+M-1}z^{-q-M+2} + \cdots + z^{-q+1}H(z)G_c(z). \quad (37)$$

Again, we obtain $r_1, \dots, r_{q-2}$ as the zero th to $(q-2)$ th coefficients of $W_{opt}(z)G_c(z)$ and then from (36) $p_1, \dots, p_{q-1}$ are obtained by

$$p_1 = -r_1 \quad (38)$$

$$p_i = -r_i - \sum_{k=1}^{i-1} p_k r_{i-k} \qquad (2 \le i \le q-1). \quad (39)$$

Using these $p_1, \dots, p_{q-1}, r_i (i = q, \dots, q+M-1)$ are determined by

$$r_i = -\sum_{k=1}^{q-1} p_k r_{i-k} \qquad (q \le i). \quad (40)$$

Next subtracting these from the q th to $(q+M-1)$ th coefficients of $W_{opt}(z)G_c(z)$ and finally filtering the resulting coefficients by $1/G_c(z)$ gives the coefficients of $H(z)$. The stability condition in (28) is just the positive realness of $R(e^{j\omega})$ for the MA case or $P(e^{j\omega})$ for the AR case.

5. SIMULATION RESULTS

To see the convergence characteristics of the above algorithms and the two-channel algorithm in [3] for the AR case, some simulation results are presented. The latter algorithm is based on the idea of using the two-channel signals $x(n)$ and $y(n)$ in Fig.1. By minimizing the mean square of $C(z)y(n) - D(z)x(n) = C(z)(1/P(z)\varepsilon(n) + H(z)x(n)) - D(z)x(n)$ with respect to the polynomials $C(z)$ and $D(z)$ where $C(z)$ is monic, i.e., the constant term is 1, the optimal ones are given by $C_{opt}(z) = P(z)$ and $D_{opt}(z) = C_{opt}(z)H(z)$.

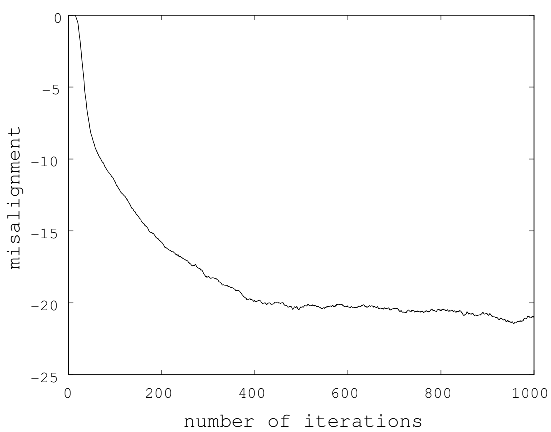

Fig. 3. The misalignment of the proposed algorithm for the MA case.

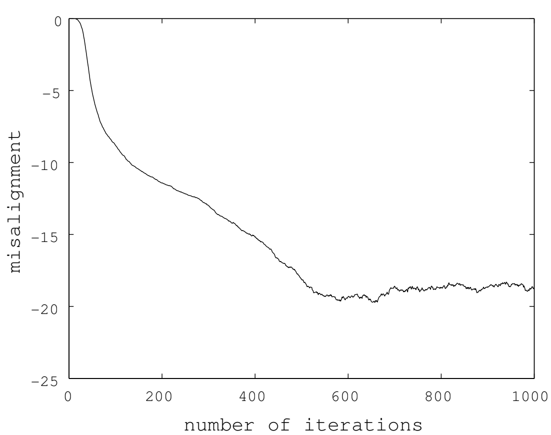

Fig. 4. The misalignment of the proposed algorithm for the AR case.

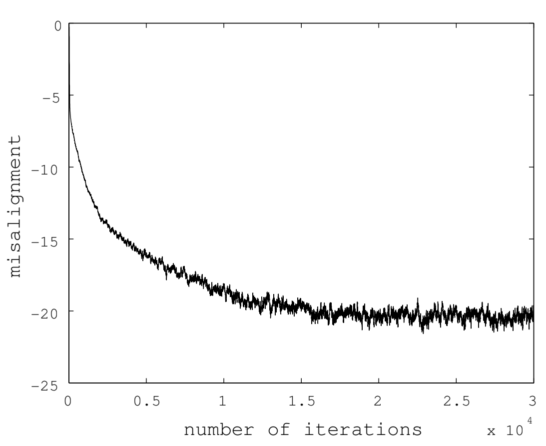

Fig. 5. The misalignment of two-channel algorithm for the AR case.

Hence, from $D_{opt}(z)/C_{opt}(z) = P(z)H(z)/P(z), H(z)$ is obtained. This shows that the two-channel algorithm relies on the pole-zero cancellation which may slow down the speed of convergence.

We use the following performance index called as the misalignment

$$\zeta = \sum_{i=0}^{M-1} (h_i - \hat{h}_i)^2 \Bigg/ \sum_{i=0}^{M-1} h_i^2 \tag{41}$$

where $\hat{h}_i$ is the estimate of h_i. The step size μ for each algorithm is taken so that the stead-state ζ is -20db. Fig.3 shows the plots of ζ of the proposed algorithm versus the iteration number averaged over 100 realizations for a 4 th order MA process with a 5 th order feedback path $(q = 5, M = 5)$ with $\mu = 1.0 \times 10^{-3}$. Fig.4 shows the corresponding result for a 4 th order AR process with the same feedback path and $\mu = 1.0 \times 10^{-3}$. Fig.5 shows the result of the two-channel algorithm in [3] for the same situation with that of Fig.4 with $\mu = 1.0 \times 10^{-3}$. Comparing Figs 4 and 5, the proposed algorithm converges considerably faster than the algorithm in [3].

6. CONCLUSION

We have presented a direct method considering the causality constraint for obtaining a stationary point and the corresponding stability condition of the conventional adaptive filter for cancellation of the effect of the feedback path in hearing aids. This method can be used for analyzing other adaptive algorithms in feedback systems. Then, a method for separating the impulse response of the feedback path from the adaptive filter weights has been proposed when the incoming signal is an MA or an AR process. It has been shown by simulations that the proposed algorithm converges considerably faster than the existing algorithm.

7. REFERENCES

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