MANEUVERING TARGET TRACKING USING THE NONLINEAR NON-GAUSSIAN KALMAN FILTER

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ABSTRACT

The problem of maneuvering target tracking is addressed in this paper. The main challenge in maneuvering target tracking stems from the nonlinearity and non-Gaussianity of the problem. The Singer model was used to model the maneuvering target dynamics and abrupt changes in the acceleration. According to this model, the heavy-tailed Cauchy distribution driving noise is used to model the abrupt changes in the target acceleration. The nonlinear, non-Gaussian Kalman filter was applied to this problem. The algorithm is based on the Gaussian mixture model for the posterior state vector. The nonlinear, non-Gaussian Kalman filter for this problem was tested using simulations, and it is shown that it outperforms both the particle filter and the extended Kalman filter.

1. INTRODUCTION

Target tracking is a basic problem in radar, sonar and infra-red (IR) applications. Since 1960 many methods [1]-[2] have been proposed to solve the tracking problem. Most of the tracking algorithms are model-based and use the knowledge about the target motion [3]. The most popular model-based approach is dynamic state-space (DSS) modeling. According to this approach, the state vector contains the time-varying dynamics of the target which is usually unobserved. The target tracking problem can be interpreted as estimation of the system state in a DSS model and thus the Bayesian approach can be effectively used. According to this approach the posterior probability density function (PDF) of the state is derived. The optimal estimator in the minimum-mean-square error (MMSE) sense can be found using the posterior PDF. The Kalman filter (KF), which is optimal in the MMSE sense for linear, Gaussian systems [4], is the most popular tracking algorithm. Typically, the observation model in tracking systems is nonlinear because the observations are given in polar coordinates. For nonlinear problems there is no general analytic expression for the posterior PDF and only approximated estimation algorithms are studied [5]. The extended Kalman filter (EKF) is the most popular approach for recursive nonlinear estimation [4]. The main idea of the EKF is first-order linearization of the estimation problem and the posterior PDF is assumed to be Gaussian. In nonlinear systems the PDF of the state may be multi-modal. The Gaussian approximation of this multi-modal distribution leads to poor tracking performance.

Real radar tracking systems are rarely Gaussian due to many other factors. For example, maneuvering target might abruptly change its acceleration by sudden break or steering [6]. Usually, a heavy-tailed distributed system noise is used to model the abrupt changes of the system state in a maneuvering target tracking applications. The Cauchy distribution is typically used to model heavy-tailed PDFs.

In 90's, a new class of filtering methods was proposed based on the sequential Monte Carlo (MC) approach for nonlinear non-Gaussian problems, as an alternative to linearized Kalman-type filters (see for example [5]). In these techniques the filtering is performed recursively generating MC samples of the state variables. The most popular realization of the MC approach is the particle filters (PF) which approximate the posterior distribution by a set of random samples with associated weights, rather than using an analytic model [5]. The PF is extensively used for maneuvering target tracking (e.g. [6]).

Recently, the non-Gaussian Kalman filter was proposed in [7]. This algorithm was shown to be optimal under the minimum-mean-square error (MMSE) criterion for non-Gaussian problem. The non-Gaussian linear DSS model in which the PDFs of the system initial state, system noise, and the posterior state PDFs are modeled by the Gaussian mixture model (GMM), was assumed. Using the property that any PDF can be approximated by a mixture of finite number of Gaussians [4], a recursive method based on the MMSE estimator for GMM-distributed random vector was derived. This algorithm estimates the posterior PDF of the system state by the GMM, and therefore it can be effectively used for maneuvering target tracking.

In this work, the non-Gaussian Kalman filter was generalized to nonlinear problems, resulting the nonlinear, non-Gaussian Kalman filter. The resulting nonlinear, non-Gaussian Kalman filter algorithm is applied to the problem of maneuvering target tracking with abrupt acceleration changes. The target tracking problem is stated in Section 2. The nonlinear, non-Gaussian Kalman filter is briefly presented in Section 3. The maneuvering target tracking performance of the proposed algorithm is evaluated and compared to the PF and the EKF in Section 4. Finally, our conclusions are drawn in Section 5.

2. TARGET TRACKING PROBLEM FORMULATION

Maneuvering target dynamics can be modeled according to the DSS approach as

$$\begin{aligned} \mathbf{s}[n] &= \mathbf{a}(\mathbf{s}[n-1], \mathbf{u}[n]) , \quad (1) \\ \mathbf{x}[n] &= \mathbf{h}(\mathbf{s}[n], \mathbf{w}[n]) , \quad (2) \end{aligned}$$

where the transition function $\mathbf{a}(\cdot,\cdot)$, and the observation function, $\mathbf{h}(\cdot,\cdot)$, are known, and $\{\mathbf{s}[n],\ n = 0, 1, 2, \ldots\}$ and $\{\mathbf{x}[n],\ n = 0, 1, 2, \ldots\}$ are the state vector and the observation sequences. The state vector consists of target position $[r_x\ r_y]^T$, velocity $[\dot{r}_x\ \dot{r}_y]^T$, and acceleration $[\ddot{r}_x\ \ddot{r}_y]^T$:

$$\mathbf{s}[n] = [r_x[n]\ r_y[n]\ \dot{r}_x[n]\ \dot{r}_y[n]\ \ddot{r}_x[n]\ \ddot{r}_y[n]] \ .$$

The Singer model [8] is widely used in the literature for maneuvering target modeling [3]. This model assumes that the target acceleration is a zero-mean first order stationary Markov process. According to the discrete-time Singer model the transition function in (1) is

$$\begin{aligned} \mathbf{a}(\mathbf{s}[n-1], \mathbf{u}[n]) &= \begin{bmatrix} 1 & 0 & T & 0 & c_1 & 0 \\ 0 & 1 & 0 & T & 0 & c_1 \\ 0 & 0 & 1 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & c_2 \\ 0 & 0 & 0 & 0 & e^{-\alpha T} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\alpha T} \end{bmatrix} \mathbf{s}[n-1] \\ &+ \begin{bmatrix} b_1 & 0 \\ 0 & b_1 \\ b_2 & 0 \\ 0 & b_2 \\ b_3 & 0 \\ 0 & b_3 \end{bmatrix} \mathbf{u}[n] , \end{aligned}$$

where

$$\begin{aligned} b_1 &= \frac{T^2}{2\alpha} - \frac{T}{\alpha^2} + \frac{1 - e^{-\alpha T}}{\alpha^3} \\ c_1 = b_2 &= \frac{T}{\alpha} - \frac{1 - e^{-\alpha T}}{\alpha^2} \\ c_2 = b_3 &= \frac{1 - e^{-\alpha T}}{\alpha} , \end{aligned}$$

T is the sampling interval and $\alpha = \frac{1}{\tau}$ is reciprocal of the maneuver time constant τ. According to this model, the target position change is determined by its velocity. The target velocity change is determined by its acceleration and the acceleration change is driven by the system noise. The system noise, $\mathbf{u}[n]$, was assumed to be zero-mean Gaussian with covariance matrix $\mathbf{\Gamma_u}[n] = \begin{bmatrix} \sigma^2_{\mathbf{u}1} & 0 \\ 0 & \sigma^2_{\mathbf{u}2} \end{bmatrix}$.

The abrupt changes of the maneuvering target acceleration were modeled by the driving noise with heavy-tailed distribution, usually chosen to be i.i.d. with zero-mean Cauchy distribution:

$$f_{u[n]}(u) = \frac{1}{\pi} \frac{\gamma/2}{u^2 + (\gamma/2)^2} \ . \quad (3)$$

Assuming that the radar is placed at the origin $[0\ 0]^T$, the radar measurements: range $r[n] = (r_x^2[n] + r_y^2[n])^{\frac{1}{2}}$ and bearing $\beta[n] = \arctan\left(\frac{r_y[n]}{r_x[n]}\right)$ of the target are described by the measurement function

$$\mathbf{h}\left(\begin{bmatrix} \mathbf{s}[n] \\ \mathbf{w}[n] \end{bmatrix}\right) = \begin{bmatrix} (r_x^2[n] + r_y^2[n])^{\frac{1}{2}} \\ \arctan\left(\frac{r_y[n]}{r_x[n]}\right) \end{bmatrix} + \mathbf{w}[n] \ .$$

The initial state $\mathbf{s}[-1]$, a measurement noise $\mathbf{w}[n]$ and the system noise $\mathbf{u}[n]$ are assumed to be independent. The random vector $\mathbf{s}[-1]$ and the white measurement noise have the following distributions:

$$\begin{aligned} &\mathbf{s}[-1] \sim GMM(\alpha_{\mathbf{s}l}[-1], \boldsymbol{\mu}_{\mathbf{s}l}[-1], \mathbf{\Gamma}_{\mathbf{s}l}[-1]; l = 1, \ldots, L) , \\ &\mathbf{w}[n] \sim \mathcal{N}(\mathbf{0}, \mathbf{\Gamma_w}[n]) , \quad (4) \end{aligned}$$

where $GMM(\alpha_m, \boldsymbol{\mu}_m, \mathbf{\Gamma}_m, m = 1, \ldots, M)$ denotes an Mth-order complex Gaussian mixture distribution with weights, $\{\alpha_m\}_{m=1}^M$, mean vectors, $\{\boldsymbol{\mu}_m\}_{m=1}^M$, and covariance matrices, $\{\mathbf{\Gamma}_m\}_{m=1}^M$. The PDF of a GMM-distributed random vector $\mathbf{y} \sim GMM(\alpha_{\mathbf{y}m}, \boldsymbol{\mu}_{\mathbf{y}m}, \mathbf{\Gamma}_{\mathbf{y}m}; m = 1, \ldots, M)$ is given by

$$f_{\mathbf{y}}(\mathbf{y}) = \sum_{m=1}^{M} \alpha_{\mathbf{y}m} \Phi(\mathbf{y}; \boldsymbol{\theta}_{\mathbf{y}m}) ,$$

where $\Phi(\mathbf{y}; \boldsymbol{\theta}_{\mathbf{y}m})$ is a complex Gaussian PDF and $\boldsymbol{\theta}_{\mathbf{y}m}$ contains the mean vector, $\boldsymbol{\mu}_{\mathbf{y}m}$ and the covariance matrix, $\mathbf{\Gamma}_{\mathbf{y}m}$.

3. SUMMARY OF THE NONLINEAR, NON-GAUSSIAN KALMAN FILTER

Implementation of the nonlinear, non-Gaussian Kalman filter, involves the following recursion.
1. Initialization:

Initialize the L-order GMM parameters of the state vector at time instance $n = -1$.

$$
\begin{aligned}
\alpha_{\mathbf{s}}[-1|-1,\eta_{\mathbf{s}l}[-1]] &= \alpha_{\mathbf{s}l}[-1] \ , \\
\boldsymbol{\mu}_{\mathbf{s}}[-1|-1,\eta_{\mathbf{s}l}[-1]] &= \boldsymbol{\mu}_{\mathbf{s}l}[-1] \ , \\
\boldsymbol{\Gamma}_{\mathbf{s}}[-1|-1,\eta_{\mathbf{s}l}[-1]] &= \boldsymbol{\Gamma}_{\mathbf{s}l}[-1] \ .
\end{aligned}
$$

Set $n = 0$.

2. Mixture parameters of nonlinear function:

- Generate an artificial data set $\mathcal{D}$ from the conditional distribution of $\begin{bmatrix} \mathbf{s}[n-1] \\ \mathbf{u}[n] \\ \mathbf{w}[n] \end{bmatrix}$, given $\mathcal{X}[n-1]$, according to the PDF of $\mathbf{s}[n-1]|\mathcal{X}[n-1]$ from the previous step and PDFs of $\mathbf{u}[n]$ and $\mathbf{w}[n]$ given in (3) and (4).
- Apply the nonlinear function $\mathbf{G}(\cdot) = \begin{bmatrix} a(\cdot,\cdot) \\ h(a(\cdot,\cdot),\cdot) \end{bmatrix}$ on $\mathcal{D}$ and obtain a new artificial data set $\mathcal{D}' = \mathbf{G}(\mathcal{D})$.
- Model the conditional distribution of $\begin{bmatrix} \mathbf{s}[n] \\ \tilde{\mathbf{x}}[n] \end{bmatrix}$ given $\mathcal{X}[n-1]$ using the new artificial data $\mathcal{D}'$ by GMM of order M, obtained using a model order selection algorithm such as the minimum description length (MDL) [9]. The following parameters are obtained in this process

$$
\begin{aligned}
&\boldsymbol{\theta}_{\tilde{\mathbf{x}}m}[n]=\{\boldsymbol{\mu}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]],\boldsymbol{\Gamma}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\} \ , \\
&\boldsymbol{\theta}_{\hat{\mathbf{x}}m}[n|n-1]=\{\boldsymbol{\mu}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]],\boldsymbol{\Gamma}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\} \ , \\
&\boldsymbol{\theta}_{\mathbf{s}m}[n|n-1]=\{\boldsymbol{\mu}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]],\boldsymbol{\Gamma}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]]\} \ , \\
&\alpha_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \ , \alpha_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \ , \\
&\boldsymbol{\Gamma}_{\mathbf{s}\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]].
\end{aligned}
$$

3. Innovation:

The measurement prediction is calculated using these parameters as follows

$$
\hat{\mathbf{x}}[n|n-1] = \sum_{m=1}^{M} \alpha_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\boldsymbol{\mu}_{\hat{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]] \ ,
$$

and the innovation is calculated according to

$$
\tilde{\mathbf{x}}[n] = \mathbf{x}[n] - \hat{\mathbf{x}}[n|n-1] \ .
$$

4. Kalman gain: $\mathbf{K}_m[n] \triangleq \boldsymbol{\Gamma}_{\mathbf{s}\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\boldsymbol{\Gamma}_{\tilde{\mathbf{x}}}^{-1}[n|n-1,\tilde{\eta}_m[n]]$.

5a. Estimated state mixture parameters:

$$
\begin{aligned}
\alpha_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] &= \frac{\alpha_{\mathbf{x}}[n|n-1,\tilde{\eta}_m[n]]\Phi(\tilde{\mathbf{x}}[n];\boldsymbol{\theta}_{\mathbf{x}m}[n])}{\sum_{m'=1}^{M}\alpha_{\mathbf{x}}[n|n-1,\tilde{\eta}_{m'}[n]]\Phi(\tilde{\mathbf{x}}[n];\boldsymbol{\theta}_{\mathbf{x}m'}[n])} \ , \\
\boldsymbol{\mu}_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] &= \boldsymbol{\mu}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]] \\
&\quad + \mathbf{K}_m[n]\left(\tilde{\mathbf{x}}[n] - \boldsymbol{\mu}_{\tilde{\mathbf{x}}}[n|n-1,\tilde{\eta}_m[n]]\right) \ , \\
\boldsymbol{\Gamma}_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]] &= \boldsymbol{\Gamma}_{\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]] \\
&\quad - \mathbf{K}_m[n]\boldsymbol{\Gamma}_{\tilde{\mathbf{x}}\mathbf{s}}[n|n-1,\tilde{\eta}_m[n]] \ , \\
&\quad \forall \ \ m = 1,\ldots,M \ .
\end{aligned}
$$

5b. Estimation: $\hat{\mathbf{s}}[n|n] = \sum_{m=1}^{M} \alpha_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]]\boldsymbol{\mu}_{\mathbf{s}}[n|n,\tilde{\eta}_m[n]]$.

6. Set $n \rightarrow n+1$, go to step 2.

The nonlinear, non-Gaussian Kalman filter algorithm is schematically presented in Fig. 1.

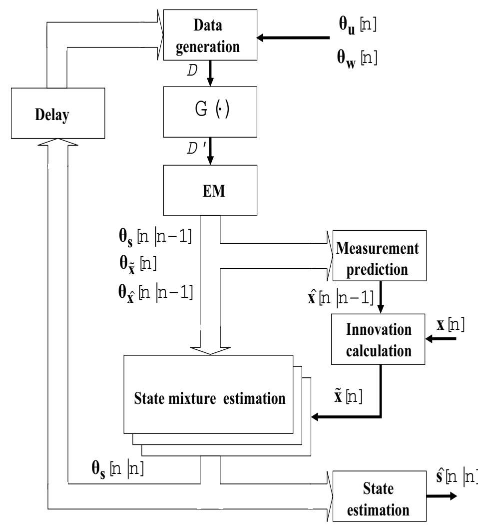

Fig. 1. The nonlinear, non-Gaussian Kalman filter schematic diagram.

4. SIMULATION RESULTS

In this section, the performance of the nonlinear, non-Gaussian Kalman filter, applied to the problem of maneuvering target tracking, is evaluated and compared to the PF and the EKF. The root-mean-square error (RMSE) of the estimate of the system state vector was used for performance evaluation.

The maneuvering target tracking was simulated for $N = 20$ time instances with sampling interval $T = 1$ sec. The driving noise was modeled by Cauchy distribution with parameter $\gamma = 0.3$. The covariance matrix of the zero-mean Gaussian measurement noise was assumed to be $\boldsymbol{\Gamma}_{\mathbf{u}}[n] = 0.1\mathbf{I}$. The Singer model parameter was assumed $\alpha = 0.1$, which corresponds to evasive maneuver. For estimation performance evaluation, each test was performed over 100 trials. The initial system state was assumed

$$
\mathbf{s}[-1|-1] \ = \ [10000\ 10000\ 1\ 1\ 0.1\ -0.1]^T \ .
$$

For the nonlinear, non-Gaussian Kalman filter, the conditional distribution of the state vector $\mathbf{s}[n]$, given $\mathcal{X}[n]$ was assumed to be GMM of order $M = 12$. The nonlinear, non-Gaussian Kalman filter is initialized at time instance $n = -1$ for $l = 1, \ldots, L$ with

$$\begin{aligned} \alpha_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}l}[-1]] &= \frac{1}{L} , \\ \boldsymbol{\mu}_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}l}[-1]] &= \mathbf{0} , \\ \boldsymbol{\Gamma}_{\mathbf{s}}[-1|-1, \eta_{\mathbf{s}l}[-1]] &= 1000\mathbf{I} . \end{aligned}$$

In this example, the standard sampling importance resampling (SIR) PF was used [5] with 10000 particles. For the EKF, the conditional distribution of the state vector $\mathbf{s}[n]$, given $\mathcal{X}[n]$ was assumed to be Gaussian. The parameters of this PDF were initialized with

$$\boldsymbol{\mu}_{\mathbf{s}}[-1|-1] = \mathbf{0} ,$$
$$\boldsymbol{\Gamma}_{\mathbf{s}}[-1|-1] = 1000\mathbf{I} .$$

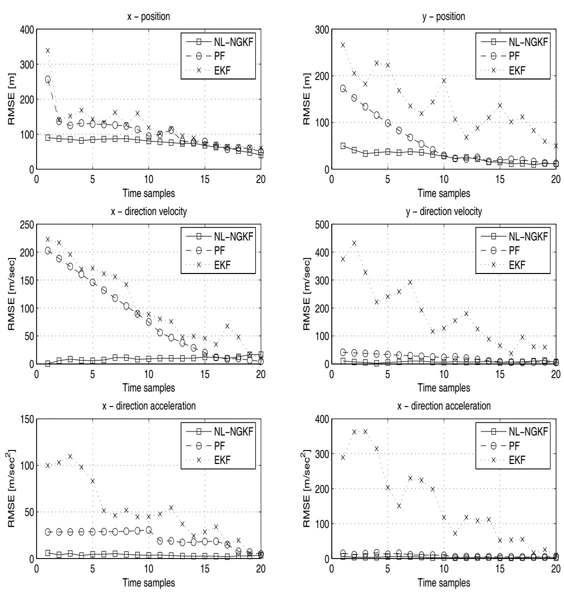

Fig. 2. the nonlinear, non-Gaussian Kalman filter vs. PF and EKF maneuvering target tracking performance.

The tracking performance of the nonlinear, non-Gaussian Kalman filter, PF and EKF in terms of RMSE are presented in Fig. 2. This figure shows the estimation performance of the maneuvering target's position, velocity and acceleration in two-dimensional space. It can be observed that the PF outperforms the EKF, and the nonlinear, non-Gaussian Kalman filter outperforms both of them.

5. CONCLUSION

In this work, the nonlinear, non-Gaussian Kalman filter was applied to the problem of maneuvering target tracking with abrupt acceleration changes. These abrupt changes are modeled by the heavy-tailed Cauchy distribution. The Singer model was used for maneuvering target modeling. Tracking performance of the nonlinear, non-Gaussian Kalman filter was compared to the PF and the EKF via simulations and it is shown that the nonlinear, non-Gaussian Kalman filter outperforms both the PF and the EKF.

6. REFERENCES

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