IDENTIFICATION OF QUADRATIC NON LINEAR SYSTEMS USING HIGHER ORDER STATISTICS AND FUZZY MODELS

Jilali Antari[1], Radouane Iqdour[1], Said Safi[2], Abdelouhab Zeroual[1] & Abdelouahid Lyhyaoui[2]

[1]Department of Physics, Faculty of Sciences Semlalia Cadi Ayyad University
Po. Box: 2390, Marrakesh, Morocco
E-mail j.antari@ucam.ac.ma, antari_j@yahoo.fr
r.iqdour@ucam.ac.ma, zeroual@ucam.ac.ma
[2]National School of Applied Sciences, Tangier Principal Morocco
Po. Box. 1818, Fax 212 39 39 37 43
E-mail ssafi@ensat.ac.ma, abdel@tsc.uc3m.es

ABSTRACT

In this work we compare tow methods for the identification of non-linear systems. The first one uses a quadratic non linear model of which parameters are estimated using a new algorithm based on the fourth order cumulants. The second one is based on the Takagi-Sugeno fuzzy models. The simulation results show that the fuzzy models give the good results in noiseless and weak noise environment. However the quadratic model of which parameters are identified using the proposed algorithm works well in the high noise environment case.

1. INTRODUCTION

Applications of Volterra non linear system theory [1] have played an ever increasing role in the non linear system identification domain [2-5]. It is partially traced to the fact that the Volterra series has two important properties [3]. The first one is that the output of the Volterra system is linearly dependent on the system kernel parameters. The second one relies on the fact that the signals non linearity can be represented through multidimensional operators working on the products of the input samples. As a result any gently non linear system can be described with reasonable accuracy by a truncated version of the Volterra series, which considerably reduces the complexity of the problem and thus requires a limited amount of statistical knowledge of the input and output. In practice, many important non linear effects in engineering and science can be approximated by a Volterra series of second or third order, i.e., quadratic non linear systems or cubic non linear systems [5].
Other non linear systems are available in literature, among them, the fuzzy models identification is an effective tool for the approximation of non-linear systems on the basis of measured data [6]. Among the different fuzzy modelling techniques, the Takagi-Sugeno (TS) model [7] has attracted most attention. The advantages of the TS fuzzy systems with regard to other systems reside in the fact that it does not demands a previous knowledge of the system that we wish to identified and it is characterised by their robustness and tolerance to noise. This model consists of if -then rules with fuzzy antecedents and mathematical functions in the consequent part. The fuzzy sets partition the input space into a number of fuzzy regions, while the consequent functions describe the system's behaviour in these regions.
The construction of a TS model is usually done in two steps. In the first step, the fuzzy sets (membership functions) in the rule antecedents are determined. This can be done manually, using knowledge of the process, or by some data-driven techniques. In the second step, the parameters of the consequent functions are estimated. As these functions are usually chosen to be linear in their parameters, standard linear least-squares methods can be applied. The most important step in the construction procedure is the identification of the antecedent membership functions, which is a non-linear optimisation problem. Typically, the Levenberg-Marquardt optimisation technique is used.
In this paper, we proposed an algorithm based on Higher Order Cumulants technique for identification of quadratic non linear systems. The obtained results are compared with those gave by the TS fuzzy models in term of the Mean Square Error (MSE).

2. MODEL AND ASSUMPTION

The output, corrupted by an additive Gaussian noise, of non linear quadratic system excited by an unobservable input is described by:

$$y(n)=\sum_{i=0}^{q}\sum_{j=0}^{q}h(i,j)u(n-i)u(n-j) \qquad (1)$$

Where $\{u(n)\}$ is the input sequence, $\{h(i,j)\}$ the impulse response coefficients, $\{w(n)\}$ is the noise sequence, q is the order of non linear quadratic system and $\{y(n)\}$ is the non measurable sequences.

The observed output $\{S(n)\}$ is given by:

$$S(n)=y(n)+w(n) \qquad (2)$$

To simplify the construction of the algorithm we assume that the input sequence, $\{u(n)\}$, is independent and identically distributed (i.i.d) zero mean, $\sigma_u^2 \cong 1$, and non Gaussian. The system is causal, i.e.

$h(i,j)=0 \ \ for \ \ (i,j)<0 \ \ and \ \ (i,j)>q$, where $h(0,0)=1$

The measurement noise sequence $\{w(n)\}$ is assumed to be zero mean, i.i.d, Gaussian and independent of $\{u(n)\}$ with unknown variance.

3. IDENTIFICATION METHODS

3.1. Blind identification of non linear systems

3.1.1. Basic relationships

The second order cumulants (AutoCorrelation Function ACF) of the process $y(n)$ is described by the following expression:

$$C_{2y}(\tau)=Cum\{y(n)y(n+\tau)\} \qquad (3)$$

Where $Cum(y)$ represent the cumulant of processes $y(n)$, τ represent time lag of random sequence.

Basing assumption of $\{u(n)\}$ sequence, Brillinger and Rosenblatt have establishes the kth order cumulants of the excitation signal is given by formula [8]:

$$C_{ku}(\tau_1,\ldots,\tau_{k-1})=\begin{cases}\gamma_{ku} & if \quad \tau_1=\tau_2=\ldots=\tau_{k-1}=0\\ 0 & otherwise\end{cases} \qquad (4)$$

Using Eqs. (3) and (4) the ACF can be written as follows:

$$C_{2y}(\tau)=\gamma_{4u}\sum_{i=0}^{q}h(i,i)h(i+\tau,i+\tau)\,, \qquad (5)$$

Fourth order cumulants are defined by:

$$C_{4y}(\tau_1,\tau_2,\tau_3)=Cum\{y(n)y(n+\tau_1)y(n+\tau_2)y(n+\tau_3)\} \qquad (6)$$

Using Eqs. (4) and (6) the fourth order cumulatns can be written as follows:

$$C_{4y}(\tau_1,\tau_2,\tau_3)=\gamma_{8u}\sum_{i=0}^{q}h(i,i)h(i+\tau_1,i+\tau_1)$$
$$\times h(i+\tau_2,i+\tau_2)h(i+\tau_3,i+\tau_3) \qquad (7)$$

The Fourier transform of the Eqs. (5) and (7) are given respectively by the following relationships:

$$S_{2y}(\omega)=TF\{C_{2y}(\tau)\}=\gamma_{4u}H(-\omega,-\omega)H(\omega,\omega) \qquad (8)$$

And:

$$S_{4y}(\omega_1,\omega_2,\omega_3)=TF\{C_{4y}(\tau_1,\tau_2,\tau_3)\}$$
$$S_{4y}(\omega_1,\omega_2,\omega_3)=\gamma_{8u}H(-\omega_1-\omega_2-\omega_3,-\omega_1-\omega_2-\omega_3)$$
$$\times H(\omega_1,\omega_1)H(\omega_2,\omega_2)H(\omega_3,\omega_3) \qquad (9)$$

with $H(\omega,\omega)=\sum_{i=0}^{+\infty}h(i,i)\exp(-j\omega i)$

If we suppose that $\omega=\omega_1+\omega_2+\omega_3$, the Eq. (8) becomes:

$$S_{2y}(\omega_1+\omega_2+\omega_3)=\gamma_{4u}H(-\omega_1-\omega_2-\omega_3,\omega_1-\omega_2-\omega_3)$$
$$\times H(\omega_1+\omega_2+\omega_3,\omega_1+\omega_2+\omega_3) \qquad (10)$$

then, from the Eqs. (9) and (10) we obtain the following equations

$$S_{4y}(\omega_1,\omega_2,\omega_3)H(\omega_1+\omega_2+\omega_3,\omega_1+\omega_2+\omega_3)=$$
$$\mu_{(8,4)}H(\omega_1,\omega_1)H(\omega_2,\omega_2)H(\omega_3,\omega_3)S(\omega_1+\omega_2+\omega_3)$$
$$(11)$$

With $\mu_{(8,4)}=\left(\gamma_{8u}/\gamma_{4u}\right)$.

The inverse Fourier transformation of the Eq. (11) given by following equation

$$\sum_{i=0}^{q}C_{4y}(t_1-i,t_2-i,t_3-i)h(i,i)=$$
$$\mu_{(8,4)}\sum_{i=0}^{q}h(i,i)h(t_2-t_1+i,t_2-t_1+i)h(t_3-t_1+i,t_3-t_1+i)C_{2y}(t_1-i)$$
$$(12)$$

3.1.2. Proposed algorithm using fourth order cumulants

if we take $t_1=t_3$ into Eq. (12) we obtain:

$$\sum_{i=0}^{q}C_{4y}(t_1-i,t_2-i,t_1-i)h(i,i)=$$
$$\mu_{(8,4)}\sum_{i=0}^{q}h(i,i)h(t_2-t_1+i,t_2-t_1+i)h(i,i)C_{2y}(t_1-i) \qquad (13)$$

if we use the property of the ACF of the stationary process, such as $C_{2y}(t)\neq 0$ only for $-q\leq t\leq q$ and vanish elsewhere.

We suppose that $t_1=2q$ the Eq. (13) becomes:

$$\sum_{i=0}^{q}C_{4y}(2q-i,t_2-i,2q-i)h(i,i)=$$
$$\mu_{(8,4)}h^2(q,q)h(t_2-q,t_2-q)C_{2y}(q) \qquad (14)$$

According to the assumption ($h(i,i)=0$ for $i<0$ and $i>q$), the choice of t_2 impose that ($t_2 \geq q$). So, this implies that $q \leq t_2 \leq 2q$

If we take $t_1 = t_2 = -q$ into the equation (13) we obtain:

$$\sum_{i=0}^{q} C_{4y}(-q-i,-q-i,-q-i)h(i,i) =$$

$$\mu_{(8,4)} \sum_{i=0}^{q} h(i,i)h(i,i)h(i,i)C_{2y}(-q-i) \qquad (15)$$

According to the ACF property the relation (15) is valid if i=0 from where:

$$C_{4y}(-q,-q,-q)h(0,0) = \mu_{(8,4)} h^3(0,0)C_{2y}(-q) \qquad (16)$$

using the property of the cumulants we write: $C_{4y}(t_1,t_2,t_3) = C_{4y}(-t_1,t_2-t_1,t_3-t_1)$, and from Eqs. (14) and (16) we obtain:

$$\sum_{i=0}^{q} C_{4y}(2q-i,t_2-i,2q-i)h(i,i) =$$

$$h^2(q,q)h(t_2-q,t_2-q)C_{4y}(q,0,0) \qquad (17)$$

to simplify the Eq. (17) we considering equation (7) we obtain with $\tau_1 = \tau_2 = q$:

$$C_{4y}(q,q,\tau) = \mu_{(8,4)} h^2(q,q)h(\tau,\tau) \qquad (18)$$

If $\tau_1 = \tau_2 = q$ and $\tau_3 = 0$

$$C_{4y}(q,q,0) = \mu_{(8,4)} h^2(q,q)h(0,0) \qquad (19)$$

The Eqs (18) and (19) give $h(\tau_3,\tau_3) = \dfrac{C_{4y}(q,q,\tau_3)}{C_{4y}(q,q,0)}$

if $\tau_3 = q$ we obtain the relationship:

$$h(q,q) = \frac{C_{4y}(q,q,q)}{C_{4y}(q,q,0)} \qquad (20)$$

We take:

$h^2(q,q)C_{4y}(q,0,0) = \left(\dfrac{C_{4y}(q,q,q)}{C_{4y}(q,q,0)}\right)^2 \times C_{4y}(q,0,0) = \alpha$, into Eq.(17) we obtain the proposed algorithm based on fourth order cumulants:

$$\sum_{i=0}^{q} C_{4y}(2q-i,t_2-i,2q-i)h(i,i) = \alpha\ h(t_2-q,t_2-q) \quad (21)$$

With $\alpha = \left(\dfrac{C_{4y}(q,q,q)}{C_{4y}(q,q,0)}\right)^2 \times C_{4y}(q,0,0)$

The system equation (21) can be written as follows:

$$A\phi = b \qquad (22)$$

with A the matrix of size $(q+1,q)$ element, ϕ denote the vector that carries all filter coefficients of the model, is the size $(q,1)$ and b is a vector column of size $(q+1,1)$ element.

So, the solution will be written under the following form:

$$\hat{\phi} = (A^T A)^{-1} A^T b \qquad (23)$$

With $(.)^{\,T}$ represents the transpose of $(.)$.

3.2. Fuzzy systems

The TS fuzzy system is given in the following form of If – Then rules:

R_k : if x_t is A_k then $\hat{y}_{t,k} = P_k^{(d)}(x_t)$, k = 1,2,…,c (24)

Where x_t is the input variable ($x_t \in \Re^n$), A_k is a fuzzy set of R_k and $P_k^{(d)}(x_t)$ is a polynomial of order d in the components $x_{t,i}$ of x_t. In the sequel, we will suppose d=1. For convenience, we will write the conclusion of rule R_k relatively to input x_t as:

$$\hat{y}_{t,k} = x_t{}'\ \beta_k \qquad (25)$$

Where $\beta_k = (\beta_1,\ldots,\beta_n)'$. An intercept is allowed in the conclusion $\hat{y}_{t,k}$ if we suppose $x_{t,1}$ =1 (bias term).

Output $\hat{y}_t$ relative to input x_t obtained after aggregating a set of c TS rules can be written as a weighted sum of the individual conclusions:

$$\hat{y}_t = \sum_{k=1}^{c} \pi_k(x_t)\hat{y}_{t,k}, \text{ with } \pi_k(x_t) = \frac{\mu_{A_k}(x_t)}{\sum_{j=1}^{c} \mu_{A_j}(x_t)} \qquad (26)$$

Where μ_{A_k} is the membership function related to the fuzzy set A_k.

The membership functions are selected gaussian types:

$$\mu_{A_k}(x_t) = \exp\left(\frac{-1}{2}\left\|x_t - m_k\right\|_{S_k}^2\right) \qquad (27)$$

$$\left\|x_t - m_k\right\|_{S_k}^2 = (x_t - m_k)' S_k' S_k (x_t - m_k) \qquad (28)$$

The membership parameters S_k and m_k are estimated using the LM method.

The identification process of TS model from sets of numerical data, generally, is subdivided in two categories: the structure identification and the parametric identification. In the first one, the number of the fuzzy rules c and the antecedent fuzzy sets (A_k, k = 1,…,c) are identified. In the second one, the model parameters (linear and non linear) are estimated. The goal of the parameters optimisation is to find the "best" approximation $\hat{y}_t$ to the measured output y_t . The linear parameters β_k are identified using the Weighted Least Squares (WLS) algorithm, while the Levenberg-Marquardt (LM) algorithm are using to estimate the non-linear parameters (S_k and m_k)

4. SIMULATION RESULTS AND COMPARISON

The non linear model considered here is a quadratic system of second order:

$$y(n) = u^2(n) - 0.35u^2(n-1) - 0.95u^2(n-2) + w(n)$$

Using Eq. (23) for the estimation of the parameters $\hat{h}(i,i)$, and from these parameters we computed the output system $\hat{y}(n)$.

The identified TS fuzzy model has three inputs and one output. Each input has two membership functions and the output has a linear membership function.

The comparison criterion is the MSE defined as:

$$MSE = \frac{1}{N}\sum_{n=1}^{N}[y(n) - \hat{y}(n)]^2 \qquad (29)$$

With $\hat{y}(n)$ is the output of the non linear models considered here and $y(n)$ is the output of the non linear system to be identified.

To measure the strength of noise, we define the signal-to-noise ratio (SNR) as:

$$SNR = 10\log_{10}\left(\frac{E(y^2(n))}{E(w^2(n))}\right) \qquad (30)$$

The Table 1 shows the results of comparison between the quadratic non linear model of which the parameters are estimated using the proposed algorithm and the identified TS fuzzy model over 30 Monte Carlo runs.

From these results we note that, in noiseless case the TS fuzzy model is more accurate than the quadratic model (proposed algorithm).

In noise case, two remarks can be established: The first one is that when the SNR=0 or SNR=15dB, the quadratic model (proposed algorithm) is most advantageous. The second one when the SNR=30dB we note that TS fuzzy systems gives better precision.

SNR	Models	MSE ± SD
0	quadratic model (proposed algorithm)	0.0108 ± 0.0100
	TS fuzzy model	1.5209 ± 0.1194
15	quadratic model (proposed algorithm)	0.0100 ± 0.0031
	TS fuzzy model	0.0492 ± 0.0022
30	quadratic model (proposed algorithm)	0.0070 ± 0.0056
	TS fuzzy model	$0.0015 \pm 8.12\ 10^{-12}$
Noiseless	quadratic model (proposed algorithm)	0.0066 ± 0.0057
	TS fuzzy model	$2.71\ 10^{-14} \pm 1.07\ 10^{-14}$

Table.1. Values of MSE in noiseless and noise environment over 30 Monte Carlo runs and for N=1000.

5. CONCLUSION

In this work, we have presented an algorithm based on fourth order cumulants for the identification of non linear quadratic system. A comparison in term of MSE criterion is performed with the TS fuzzy systems in noiseless and noise environment cases. The obtained results show that the quadratic model of which the parameters are identified using the proposed algorithm is able to estimate the non linear system in high noise environment. While the TS fuzzy model gives the better results in noiseless and weak noise variance cases.

6. REFERENCES

[1] J. W. Rugh," Nonlinear System Theory : The Volterra/ Wiener Approach, Baltimore", Md : Johns Hopkins University Press,1981.

[2] P. Bondon," On the identifiability of a quadratic stochastic system", IEEE Signal Processing Workshop on Higher Order Statistics, pp. 46-50, 1993.

[3] P. Koukoulas and N. Kalouptsidis, "Nonlinear system identification using Gaussian inputs", IEEE Trans. On Signal Processing, 43, pp. 1831-1841, 1995.

[4] G. L. Sicuranza. Quadratic filters for signal processing, Proc. Of IEEE , 80, pp. 1263-1285, 1992.

[5] C. H. Tseng and E. J. Powers. "Adaptive identification of third order frequency domain Volterra kernels", Int. J. Adaptive Control and Signal Processing, 10, pp. 319-342, 1996.

[6] T. Takagi and M. Sugeno. "Fuzzy identification of systems and its application to modelling and control". IEEE Transactions on Systems, Man and Cybernetics, 15(1), pp. 116-132, 1985.

[7] Hellendoorn, H and D. Driankov, editors. Fuzzy Model Identification: Selected Approaches. Springer, Berlin, Germany, 1997.

[8] D. R. Brillinger, and M. Rosenblatt, "Computation and interpretation of Kth-order spectra" in: B. Harris, ed., Spectral Analysis of Time Series, Wiley, New York, pp. 189-232, 1987.