A STATE SPACE APPROACH TO THE INVERSE OF LPTV FILTERS

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ABSTRACT

In this paper, a state space approach for computing the inverse of a general single-input single-output linear periodically time-varying filter is proposed. The inverse of the filter is derived by first constructing the inverse of an LPTV filter. The result is then extended to obtain the inverse of the IIR filter. Compared with existing approaches using other techniques, the proposed approach is quite straightforward, hence, can reduce the computational complexity.

1. INTRODUCTION

Linear periodic time-varying (LPTV) filters are an important class of time-varying filters, which have been extensively studied and applied in digital signal processing and telecommunication; see for example,[8]- [10].

The inversion of LPTV filter discussed in [1] is based on the idea of converting the periodic time-varying filter to the block time-invariant filter. A method for computing an optimal approximate inverse is proposed in [2] which transforms the periodic time-varying system to a block time-invariant system. After obtaining the transfer matrix for the inverse filter, it follows the procedures given in [3] to convert the transfer matrix to an LPTV state space model. In [5], the LPTV FIR inverse of original LPTV FIR filter is presented in terms of blocked transfer matrices. The filter bank inversion is presented in [6], which uses the impulse response of the filter at each branch to form the block transmission polyphase matrix and find the inverse of it. This approach is suitable for the FIR filter bank FIR channels only. In [7], the transfer function in Z-domain and the state-space method are used to tackle the optimal design of channel equalizers presented in [6]. It forms the multi-input multi-output state-space realizations for finding the optimal inverse.

In the analysis of inverting LPTV filter, one commonly used technique is the lifting (or blocking) technique, which is a technique to transform the LPTV filter model into an equivalent linear time-invariant (LTI) filter model with greater input and output dimension. The past results on the inversion of LPTV filter are commonly based on the lifting technique, see for example, [1]-[4]. An advantage of the lifting is that the existing method for LTI filters can be applied to the lifted LTI model for the analysis and design of the LPTV filter. However, the problem occurs while converting the designed lifted model to the LPTV state-space model, where complicated procedure and computations are involved, especially when the inverse of an LPTV IIR filter is sought.

In this paper, we use a state-space approach to the analysis of single-input single-output (SISO) LPTV filters to propose a new inversion procedure. The proposed approach will not employ the lifting technique.And it will lead to reduction the computational complexity especially for the IIR filter inverse.

In section 2, we formulate the inversion problem for LPTV filters. The new inversion approach is proposed in section 3, where we first find the inverse of an SISO LPTV FIR filter, then extend it to the general SISO LPTV IIR inverse. An example is given to illustrate the computational procedures in section 4.

2. PROBLEM FORMULATION

To formulate the inversion problem, we define N-periodic matrices $M_a(n,k,a_{i,k}) \in \mathcal{R}^{n\times n}$, $M_b(n) \in \mathcal{R}^{n\times 1}$, $M_c(n,k,c_{i,k}) \in \mathcal{R}^{1\times n}$, and $M_d(k) \in \mathcal{R}$ as follows.

$$M_a(n,k,a_{i,k}) = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -a_{n,k} & -a_{n-1,k} & \cdots & -a_{1,k} \end{bmatrix} \tag{1}$$

$$M_b(n) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 1 \end{bmatrix}^T \tag{2}$$

$$M_c(n,k,c_{i,k}) = \begin{bmatrix} c_{n,k} & \cdots & c_{1,k} \end{bmatrix}, M_d(k,b_{0,k}) = b_{0,k} \tag{3}$$

Consider an nth order N-periodic time-varying SISO LPTV IIR digital filter $\mathcal{G}$ expressed as: $y_k + a_{1,k}y_{k-1} + \cdots + a_{n,k}y_{k-n} = b_{0,k}u_k + b_{1,k}u_{k-1} + \cdots + b_{n,k}u_{k-n}$, where $u_k, y_k \in \mathcal{R}$ are the input and output of the filter, $a_{i,k}$,$b_{i,k} \in \mathcal{R}$ for $0 \leq i \leq n$, are time-varying coefficients at each time k.

The filter $\mathcal{G}$ can be described in the nth order state-space model $\{A_k, B_k, C_k, D_k\}$, which can be formed as:

$$\begin{array}{ll} A_k = M_a(n,k,a_{i,k}), & B_k = M_b(n) \\ C_k = M_c(n,k,c_{i,k}), & D_k = M_d(k,b_{0,k}) \end{array} \quad (4)$$

with

$$c_{i,k} = b_{i,k} - b_{0,k}a_{i,k}, 0 \le i \le n$$

The inverse filter $\hat{\mathcal{G}}$ is an $\hat{n}$th order N-periodic time-varying IIR filters in the form:

$$\begin{array}{rcl} \hat{x}_{k+1} & = & \hat{A}_k\hat{x}_k + \hat{B}_k y_k \\ \hat{u}_k & = & \hat{C}_k\hat{x}_k + \hat{D}_k y_k \end{array} \quad (5)$$

where $\hat{x}_k \in \mathcal{R}^{\hat{n}}$ is the state vector, $\hat{u}_k$ is the output of the inverse filter, and $\hat{A}_k \in \mathcal{R}^{\hat{n}\times\hat{n}}$, $\hat{B}_k \in \mathcal{R}^{\hat{n}\times 1}$, $\hat{C}_k \in \mathcal{R}^{1\times\hat{n}}$, and $\hat{D}_k \in \mathcal{R}$ are N-periodic matrices of the filter in the form:

$$\begin{array}{ll} \hat{A}_k = M_a(\hat{n},k,\hat{a}_{i,k}), & \hat{B}_k = M_b(\hat{n}) \\ \hat{C}_k = M_c(\hat{n},k,\hat{c}_{i,k}), & \hat{D}_k = M_d(k,\hat{b}_{0,k}) \end{array}$$

The inversion is to find values for $\hat{a}_{i,k}, \hat{c}_{i,k}$, and $\hat{b}_{0,k}$.

If $D_k \ne 0$ for all k, the inverse filter state-space model can be computed as follows.

$$\left[\begin{array}{cc} \hat{A}_k & \hat{B}_k \\ \hat{C}_k & \hat{D}_k \end{array}\right] = \left[\begin{array}{cc} A_k - B_k D_k^{-1} C_k & B_k D_k^{-1} \\ -D_k^{-1} C_k & D_k^{-1} \end{array}\right]. \quad (6)$$

The output of the inverse filter is identical to the input of the original filter, i.e. $\hat{u}_k = u_k$.

For certain filter $\mathcal{G}$ with $D_k = 0$ for some k, the direct inverse formula (5) is not applicable. We propose a state space approach in the following to the inversion of this kind of filters such that the cascaded filter $\mathcal{G}\mathcal{G}$ is equivalent to a constant delay system, i.e. output $\hat{u}_k = \hat{\mathcal{G}}\mathcal{G}u_k = u_{k-d}$ with minimum time delay d for all $k \ge d$.

3. INVERSION OF LPTV FILTERS

We will transform the LPTV filter $\mathcal{G}$, which has $D_k = 0$ for some k, into an equivalent LPTV filter $\tilde{\mathcal{G}}$ so that it can apply the inverse formula directly. It is achieved by cascading one LPTV FIR filter $\mathcal{T}$ to the original filter $\mathcal{G}$. We will derive the inversion of LPTV FIR filter first, and then extend the result to the LPTV IIR filter inversion.

3.1. SISO LPTV FIR Inverse

Given the impulse response of an n^{th} order LPTV FIR filter $\mathcal{G}$ as $g_{i,k}, 0 \le i \le n$, the filter $\mathcal{G}$ can be represented in the state-space model $\{A_k \in \mathcal{R}^{n\times n}, B_k \in \mathcal{R}^{n\times 1}, C_k \in \mathcal{R}^{1\times n}, D_k \in \mathcal{R}\}$ formed as follows.

$$\begin{array}{ll} A_k = M_a(n,k,0), & B_k = M_b(n) \\ C_k = M_c(n,k,g_{i,k}), & D_k = M_d(k,g_{0,k}) \end{array}$$

Let the impulse response of the N-periodic mth order FIR filter $\mathcal{T}$ be $h_{i,k}, 0 \le i \le m$. It can be represented in a state-space model $\{A_{c,k} \in \mathcal{R}^{m\times m}$, $B_{c,k} \in \mathcal{R}^{m\times 1}$, $C_{c,k} \in \mathcal{R}^{1\times m}$, $D_{c,k} \in \mathcal{R}\}$ formed as follows

$$\begin{array}{ll} Ac_k = M_a(m,k,0), & Bc_k = M_b(m) \\ Cc_k = M_c(m,k,h_{i,k}), & Dc_k = M_d(k,h_{0,k}) \end{array}$$

where the computation of the coefficients $h_{i,k}$ is given in the Appendix.

Denote the impulse response of the cascaded FIR system $\mathcal{T}\mathcal{G}$ impulse responses as $\bar{b}_{i,k}$, which are computed as follows. If $m \le n$,

$$\bar{b}_{i,k} = \left\{ \begin{array}{l} \sum_{l=0}^{i} h_{l,k}g_{i-l,k-l},\ 0 \le i \le m \\ \sum_{l=0}^{m} h_{l,k}g_{i-l,k-l},\ m < i \le n \\ \sum_{l=i-n}^{m} h_{l,k}g_{i-l,k-l},\ n < i \le (m+n) \end{array} \right.$$

Else,

$$\bar{b}_{i,k} = \left\{ \begin{array}{l} \sum_{l=0}^{i} h_{l,k}g_{i-l,k-l},\ 0 \le i \le n \\ \sum_{l=i-n}^{i} h_{l,k}g_{i-l,k-l},\ n < i \le m \\ \sum_{l=i-n}^{m} h_{l,k}g_{i-l,k-l},\ m < i \le (m+n) \end{array} \right.$$

If $\bar{b}_{i,k} = 0, 0 \le i \le (d-1)$ for all k, where d is the minimum delay, we can separate a time-invariant constant d delay from $\mathcal{T}\mathcal{G}$ and assign the first non-zero value $\bar{b}_{d,k}$ of the impulse response of $\mathcal{T}\mathcal{G}$ as $\tilde{b}_{0,k}$. This yields a filter $\tilde{\mathcal{G}}$ with impulse response:

$$\tilde{b}_{i,k} = \bar{b}_{i+d,k},\ 0 \le i \le (m+n-d)$$

The order of the equivalent filter $\tilde{\mathcal{G}}$ is $\tilde{n} = m+n-d$ and it has a state-space model $\{\tilde{A}_k, \tilde{B}_k, \tilde{C}_k, \tilde{D}_k\}$ given by:

$$\begin{array}{ll} \tilde{A}_k = M_a(\tilde{n},k,0), & \tilde{B}_k = M_b(\tilde{n}) \\ \tilde{C}_k = M_c(\tilde{n},k,\tilde{b}_{i,k}), & \tilde{D}_k = M_d(k,\tilde{b}_{0,k}) \end{array}$$

Let $\bar{\mathcal{G}}$ be the inverse filter of $\tilde{\mathcal{G}}$ with state matrices $\{\bar{A}_k,\ \bar{B}_k,\ \bar{C}_k,\ \bar{D}_k\}$. Then it can be computed as:

$$\begin{array}{rcl} \bar{A}_k & = & \tilde{A}_k - \tilde{B}_k\tilde{D}_k^{-1}\tilde{C}_k \\ \bar{B}_k & = & \tilde{B}_k\tilde{D}_k^{-1} \\ \bar{C}_k & = & -\tilde{D}_k^{-1}\tilde{C}_k \\ \bar{D}_k & = & \tilde{D}_k^{-1} \end{array} \quad (7)$$

The order of the inverse of the original filter $\hat{n} = max\{m, \tilde{n}\}$. Then the state-space model of the inverse filter $\hat{\mathcal{G}}$ can be obtained as:

$$\begin{array}{ll} \hat{A}_k = M_a(\hat{n},k,\hat{a}_{i,k}), & \hat{B}_k = M_b(\hat{n}) \\ \hat{C}_k = M_c(\hat{n},k,\hat{c}_{i,k}), & \hat{D}_k = M_d(k,\hat{b}_{0,k}) \end{array} \quad (8)$$

with

$$\hat{a}_{i,k} = \left\{ \begin{array}{l} \tilde{b}_{i,k}/\tilde{b}_{0,k}, 0 \le i \le \tilde{n} \\ 0, otherwise \end{array} \right.$$

$$\hat{b}_{i,k} = \left\{ \begin{array}{l} h_{i,k}/\tilde{b}_{0,k}, 0 \le i \le m \\ 0, otherwise \end{array} \right. \quad (9)$$

$$\hat{c}_{i,k} = \hat{b}_{i,k} - \hat{b}_{0,k}\hat{a}_{i,k}$$

3.2. SISO LPTV IIR Inverse

Since an LPTV IIR filter $\mathcal{G}$ can be decomposed into an all-pole filter followed by an all-zero filter, we can apply the inversion procedure in Subsection 3.1 to the all-zero (FIR) filter to form an LPTV IIR flter $\tilde{\mathcal{G}}$.

Consider the nth order N-periodic time-varying SISO LPTV IIR digital filter in the form (4) with coefficients $a_{i,k}, b_{i,k}$ for $0 \leq i \leq n$. Follow the procedure in section 3.1, we can find a filter $\tilde{\mathcal{G}}$ with coefficients:

$$\tilde{a}_{i,k} = \begin{cases} a_{i,k}, 0 \leq i \leq n \\ 0, otherwise \end{cases}$$
$$\tilde{b}_{i,k} = \begin{cases} \bar{b}_{i+d,k}, 0 \leq i \leq (m+n-d) \\ 0, otherwise \end{cases}$$

The order of $\tilde{\mathcal{G}}$ order is $\tilde{n} = max\{n, (m+n-d)\}$. And its state matrices $\{\tilde{A}_k \in \mathcal{R}^{\tilde{n}\times\tilde{n}}, \tilde{B}_k \in \mathcal{R}^{\tilde{n}\times 1}, \tilde{C}_k \in \mathcal{R}^{1\times\tilde{n}}, \tilde{D}_k \in \mathcal{R}\}$ are given as:

$$\begin{array}{ll} \tilde{A}_k = M_a(\tilde{n}, k, \tilde{a}_{i,k}), & \tilde{B}_k = M_b(\tilde{n}) \\ \tilde{C}_k = M_c(\tilde{n}, k, \tilde{c}_{i,k}), & \tilde{D}_k = M_d(k, \tilde{b}_{0,k}) \end{array}$$

with

$$\tilde{c}_{i,k} = \tilde{b}_{i,k} - \tilde{b}_{0,k}\tilde{a}_{i,k}, 0 \leq i \leq \tilde{n}$$

The inverse ($\bar{\mathcal{G}}$) of $\tilde{\mathcal{G}}$ can be computed directly following from (7).

Let $\hat{\mathcal{G}}$ be the inverse filter of $\mathcal{G}$, its order be $\hat{n} = \max\{\tilde{n}, (m+n)\}$, and its state matrices $\{\hat{A}_k, \hat{B}_k, \hat{C}_k, \hat{D}_k\}$ be

$$\begin{array}{ll} \hat{A}_k = M_a(\hat{n}, k, \hat{a}_{i,k}), & \hat{B}_k = M_b(\hat{n}) \\ \hat{C}_k = M_c(\hat{n}, k, \hat{c}_{i,k}), & \hat{D}_k = M_d(k, \hat{b}_{0,k}) \end{array} \quad (10)$$

with

$$\hat{c}_{i,k} = \hat{b}_{i,k} - \hat{b}_{0,k}\hat{a}_{i,k}, 0 \leq i \leq \hat{n} \quad (11)$$

The coefficients $\hat{a}_{i,k}, \hat{b}_{i,k}$ for $0 \leq i \leq \hat{n}$ of the inverse filter $\hat{\mathcal{G}}$ can be computed as:

$$\hat{a}_{i,k} = \begin{cases} \tilde{b}_{i,k}/\tilde{b}_{0,k}, 0 \leq i \leq \tilde{n} \\ 0, otherwise \end{cases} \quad (12)$$

The values for $\hat{b}_{i,k}$ are determined as follows. If $m \leq n$,

$$\hat{b}_{i,k} = \begin{cases} \sum_{l=0}^{i} h_{l,k}\tilde{a}_{i-l,k-l}/\tilde{b}_{0,k}, \ 0 \leq i \leq m \\ \sum_{l=0}^{m} h_{l,k}\tilde{a}_{i-l,k-l}/\tilde{b}_{0,k}, \ m < i \leq n \\ \sum_{l=i-n}^{m} h_{l,k}\tilde{a}_{i-l,k-l}/\tilde{b}_{0,k}, \ n < i \leq (m+n) \end{cases} \quad (13)$$

else,

$$\hat{b}_{i,k} = \begin{cases} \sum_{l=0}^{i} h_{l,k}\tilde{a}_{i-l,k-l}/\tilde{b}_{0,k}, \ 0 \leq i \leq n \\ \sum_{l=i-n}^{i} h_{l,k}\tilde{a}_{i-l,k-l}/\tilde{b}_{0,k}, \ n < i \leq m \\ \sum_{l=i-n}^{m} h_{l,k}\tilde{a}_{i-l,k-l}/\tilde{b}_{0,k}, \ m < i \leq (m+n) \end{cases} \quad (14)$$

We can now summarize the procedure for computing the general inverse of the LPTV FIR or IIR filters in the following.

Step 1: *Find coefficients $h_{i,k}$ for $0 \leq i \leq m$ of the cascaded LPTV FIR filter $\mathcal{T}$ following from the formula given in Appendix.*

Step 2: *Formulate the state-space model of the filter $\tilde{\mathcal{G}}$ as presented in Subsection 3.1 for FIR filter or in Subsection 3.2 for IIR filter.*

Step 3: *Apply the direct inverse formula (7) to find the inverse of $\tilde{\mathcal{G}}$.*

Step 4: *Compute the inverse filter $\hat{\mathcal{G}}$ using (8)-(9) for FIR filter or (10)-(14) for IIR filter.*

We have compared the computational workload of the proposed algorithm with the existing methods in terms of multiplication times. The comparison is given in the following table which shows that our algorithm uses considerably less computational workload in computing the inverse.

Approaches	No.of Multiplications
Approach in [4]	$n(N-1)(\frac{5n^2}{3} + 2nN + \frac{N^2}{3})$
Approach in [3]	larger than that of [4]
Our approach	$N(n^2 + n(3+2N) + \frac{3N^2-N+2}{2})$

4. EXAMPLE

Consider an example of a fifth order three-periodic FIR filter with coefficients:

	$b_{0,k}$	$b_{1,k}$	$b_{2,k}$	$b_{3,k}$	$b_{4,k}$	$b_{5,k}$
k=0,3,6,...	0.239	0.6655	0.6655	0.239	0	0
k=1,4,7,...	0	-0.5189	0	0.6793	0	-0.5189
k=2,5,8,...	0.239	-0.6655	0.6655	-0.239	0	0

The performance curve for the three periodic time varying filter is shown in Fig.1. When k=0,3,6,..., the filter follows the performance curve in solid line; when k=1,4,7,..., the filter follows the performance curve in dotted line; when k=2,5,8,..., the filter follows the performance curve in dashed line.

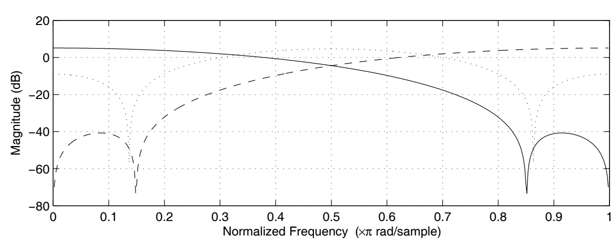

Fig. 1. Performance Curve of LPTV filter

Follow the procedure given in section 3.1, we can find coefficients of the LPTV FIR $\mathcal{T}$ as:

	$h_{0,k}$	$h_{1,k}$	$h_{2,k}$
k=0,3,6,...	0	0	1
k=1,4,7,...	1	2.1711	-6.0456
k=2,5,8,...	0	1	2.1711

And the equivalent filter $\tilde{\mathcal{G}}$ parameters are:

	$\tilde{b}_{0,k}$	$\tilde{b}_{1,k}$	$\tilde{b}_{2,k}$	$\tilde{b}_{3,k}$	$\tilde{b}_{4,k}$
k=0,3,6,...	-0.5189	0	0.6793	0	-0.5189
k=1,4,7,...	6.1475	-3.5045	1.9638	0	0
k=2,5,8,...	1.4449	2.1242	0.5189	-0.5189	0

The minimum delay of the inversion is $d = 3$.

The inverse of original filter is the cascaded system formed by the filter $\mathcal{T}$ and the inverse filter of $\tilde{\mathcal{G}}$. The parameters for the inverse filter are:

	$\hat{b}_{0,k}$	$\hat{b}_{1,k}$	$\hat{b}_{2,k}$	$\hat{b}_{3,k}$	$\hat{b}_{4,k}$
k=0,3,6,...	0	0	-1.9272	0	0
k=1,4,7,...	0	0.6921	1.5026	0	0
k=2,5,8,...	0.1627	0.3532	-0.9834	0	0

	$\hat{a}_{1,k}$	$\hat{a}_{2,k}$	$\hat{a}_{3,k}$	$\hat{a}_{4,k}$
k=0,3,6,...	0	-1.3091	0	1
k=1,4,7,...	1.4701	0.3591	-0.3591	0
k=2,5,8,...	-0.5701	-0.3194	0	0

The state-space matrices $\{\hat{A}_k, \hat{B}_k, \hat{C}_k, \hat{D}_k\}$ are obtained by equation (10), and (11) with the calculated parameters.

5. CONCLUSION

In this paper, an algorithm is presented for computing the inverse of an LPTV IIR filter using a state space approach. The inverse of some filters, which have $D_k = 0$ for some k value, can be computed by first applying an LPTV FIR filter followed by an inverse filter of the cascaded filter. Our propose algorithm has advantages in its reduced computational workload in comparison with the existing methods. An example is given to demonstrate the proposed algorithm. Because of the page limit, we are not able to present minimum realization and stability part of the inverse filter which will be reported shortly elsewhere.

6. APPENDIX

We can use the filter coefficients $b_{i,k}$ for $0 \leq n$ to form a matrix $B_{N,k} \in \mathcal{R}^{N\times(N+n)}$ as:

$$B_{N,k} = \begin{bmatrix} b_{0,k} & b_{1,k} & \cdots & \cdots & b_{n-1,k} & b_{n,k} & & & \\ & b_{0,k-1} & b_{1,k-1} & \cdots & \cdots & b_{n-1,k-1} & b_{n,k-1} & \mathbf{0} & \\ & \mathbf{0} & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots & \\ & & & b_{0,k-N+1} & b_{1,k-N+1} & \cdots & \cdots & b_{n-1,k-N+1} & b_{n,k-N+1} \end{bmatrix}$$

There always exists one matrix $T_k \in \mathcal{R}^{N\times N}$ making the first d_0 ccoefficients to be zeros. That is,

$$T_k B_{N,k} = \begin{bmatrix} \mathbf{0} & \beta_{N,k} \end{bmatrix}$$

Where $\beta_{N,k}$ has the similar form with $B_{N,k}$, and the main diagonal entries are non-zero values. The entries of $\beta_{N,k}$ are determined by the multiplications of $T_k B_{N,k}$.

We define a row vector as

$$Z = \begin{bmatrix} \mathbf{0} & 1 \end{bmatrix}$$

where $\mathbf{0} \in \mathcal{R}^{1\times d_1}$. The minimal value for d_1 is determined by matrix T_k. T_k is the matrix having the representation of:

$$T_k = \begin{bmatrix} T_{1,1} & 0 & \cdots & 0 \\ T_{2,1} & T_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \cdots \\ T_{N,1} & T_{N,2} & \cdots & T_{N,N} \end{bmatrix}$$

The main diagonal consists of $\{T_{1,1}, T_{2,2}, \cdots, T_{i,i}, \cdots T_{N,N}\}$. Let's say that diagonal 1 consists of $\{T_{2,1}, T_{3,2}, \cdots, T_{i,i-1}, \cdots, T_{N,N-1}\}$, and so on. We can determined d_1 value when diagonal $(d_1 + 1)$ has all zero entries. The minimum system delay $d = d_0 + d_1$.

For simplicity, let the order of filter $\mathcal{T}$ to be $N - 1$, i.e. $m = N - 1$. We can find the coefficients $h_{i,k}$ by:

$$\begin{bmatrix} h_{0,k} & h_{1,k} & \cdots & h_{N-1,k} \end{bmatrix} = ZT_k$$

The multiplication process ZT_k is equivalent to select the dth row of T_k.

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