ENERGY SPECTRUM RECONSTRUCTION FOR HPGE DETECTORS USING ANALYTICAL PILE-UP CORRECTION

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ABSTRACT

We consider the problem of pile-ups occurring in Gamma spectrometry signals for HPGe detectors. The temporal signal is transformed in a sequence of busy and idle periods, each busy period being characterized by its duration and its associated energy. We present an estimator to correct the pile-up phenomenon, based on an analytical formula. Applications on simulations and real spectrometrical signals are presented, which show good adequation between what we wish to retrieve and the estimation.

1. INTRODUCTION

A mixture of unknown radionucleides in unknown proportion can be analysed by means of γ spectrometry. In the case of HPGe detectors, photonic energy is converted into a pulse of current, whose energy is recorded and put in an histogram which characterizes radioactive elements. The phenomenon of pile-ups of electrical pulses is essentially a consequence of the random incoming of the photons. For a source with high-activity, the inter-arrival time of consecutive photons might be shorter than the typical pulse duration, thus creating clusters of pulses. Figure 1 illustrates the pile-up phenomenon : assume that T_n is the arrival time of the n-th photon, X_n the length of its associated electrical pulse and Y_n its energy. When the n-th photon arrives, its energy is recorded, and thus we observe (X_n, Y_n). On the other hand, the $(n+2)$-th photon is detected during the $(n+1)$-th busy period ; we observe in that case neither Y_{n+1} nor Y_{n+2}, but $Y'_{n+1} = Y_{n+1} + Y_{n+2}$. The duration of the cluster of pulses is $X'_{n+1} = (T_{n+2} + X_{n+2} - T_{n+1},)$. Since the energy spectrum is obtained by making an histogram, multiple fake spikes can appear that can cripple the identification of the radionucleides.

Recently, a relation that can be used for analytical pile-up correction was proposed in [1], using an similarity shared by this problem with the inference of the service time in M/G/∞ queue from the duration of the busy periods. However, the underlying algorithm proposed in [1] was sensitive to numerical differentiation, and therefore does not achieve optimal rates

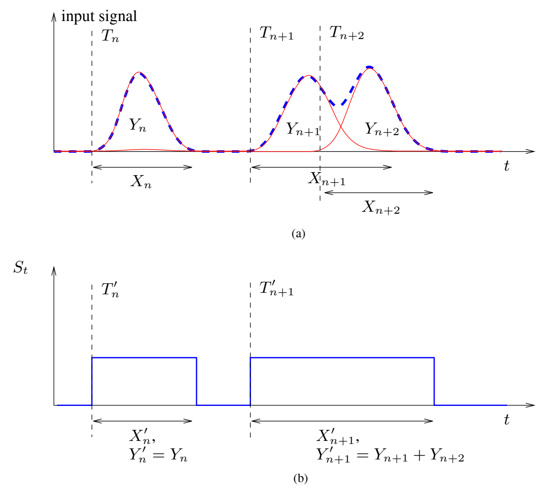


Fig. 1. Illustration of Type II Counter Problem. (a) : input signal with arrival times T_k, lengths X_k and energies Y_k, $k = n, \dots, n+2$; (b) : associated on-off observed process S_x.

of convergence. We propose in this paper an alternate estimator to bypass the step of numerical differentiation, which show good performances in both simulations and real data. We introduce in Section 2 the estimator used for pile-up correction, and the underlying algorithm. Some applications and examples are also shown on both generated densities and real data in Section 3.

2. METHODOLOGY

In this section we recall the main theorem introduced for the pile-up correction of nuclear spectrospcopy, as well as the estimator obtained from this theorem. Proof of Theorem 2.1 can be found in [2], and will be omitted here for convenience.

2.1. Notations and main theorem

Let $\mathcal{N}$ be an homogeneous Poisson process, with intensity λ and associated points $\{T_n\}_{n \geq 1}$. At each point T_n is associated a mark (X_n, Y_n) where X_n represents the pulse duration and Y_n the energy of the n-th photon. Suppose that $\{(X_n, Y_n)\}_{n\geq 1}$ is *i.i.d.*, with common distribution f and that $\{\{(X_n, Y_n)\}_{n\geq 1}, \{T_m\}_{m\geq 1}\}$ are mutually independent. It is assumed that for all $x \geq 0$ and $y \geq 0$, $f(x,y) \geq 0$. As mentioned in Section 1, the marks of $\mathcal{N}$ are not directly observable. By analogy with queueing theory, a maximal restriction of the signal to a segment where it is positive (respectively 0) is referred to as a *busy* (respectively *idle*) *period*. It is assumed that the only available data are the durations of the busy and idle periods and the total energy collected on a busy period. We denote by $\{T'_n\}_{n\geq 1}$ the starting point of each busy period, by X'_n the duration of the n-th busy period and by Y'_n its energy. We denote by P the probability measure associated to the observed samples $\{(X'_k, Y'_k)\}_{1\leq k\leq n}$. We also naturally define the length of the n-th idle period Z_n as $Z_n \stackrel{\text{def}}{=} T'_{n+1} - (T'_n + X'_n)$. By the lack of memory property of the exponential law, the idle periods have the same distribution (exponential with parameter λ) as the inter-arrival time law. Denote, at last, by m the marginal probability density function following the second dimension associated to f, that is for all nonnegative y :

$$m(y) \stackrel{\text{def}}{=} \int_{x=0}^{+\infty} f(x,y)\, dx \, .$$

Our objective is therefore to estimate m, given a sample of density P. The following result is stated in [1], and its demonstration is discussed in [2] :

Theorem 2.1 *We have for all complex couple (s,p) in the set $\{(z_1,z_2) \in \mathbb{C}^2 \, ; \, \mathrm{Re}(z_1) > 0, \, \mathrm{Re}(z_2) \geq 0\}$:*

$$\int_0^{+\infty} \mathrm{e}^{-(s+\lambda)\tau} \left[a(\tau,\varepsilon) - 1\right] d\tau = \frac{\lambda \mathcal{L}P(s,p)}{s+\lambda} \frac{1}{s+\lambda-\lambda\mathcal{L}P(s,p)} \, , \quad (1)$$

where

$$a(x,p) \stackrel{\text{def}}{=} \exp\left(\lambda \int_0^{\infty} \mathrm{e}^{-p\varepsilon} \left\{ \int_0^x (x-\tau) f(\tau,\varepsilon)\, d\tau \right\} d\varepsilon \right) . \quad (2)$$

and

$$\mathcal{L}P(s,p) = \iint_{\mathbb{R}_+\times\mathbb{R}_+} \mathrm{e}^{-s\tau}\mathrm{e}^{-p\varepsilon} P(d\tau, d\varepsilon) \, .$$

Equation 2 is of importance, because a can be expressed both in terms of the density of (X,Y) by definition, but also using Theorem 2.1 in terms of the observed samples of (X',Y'). Therefore we get a relation between a functional of P, which can be easily estimated from the data, and a functional of the density of interest f.

2.2. Computation of the estimator

As mentioned in the introduction, we focus in this contribution on the probability density function of m rather than on the joint density f. Our estimator is based on Fourier deconvolution and kernel estimation methods. Let $c > 0$ and $T > 0$ be arbitrary constants. Let $y \to K(y)$ be a kernel function and denote by $K^* : \nu \mapsto K^*(\nu) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} K(y)\mathrm{e}^{-\mathrm{i}\nu y} dy$ its Fourier transform. Let h be a bandwidth parameter. We have by differentiating (2) with respect to x :

$$\int_0^{+\infty} \mathrm{e}^{-p\varepsilon} \left(\int_0^x f(\tau,\varepsilon)\, d\tau \right) d\varepsilon = \frac{1}{\lambda} \frac{\partial \ln a}{\partial x}(x,p) \, . \quad (3)$$

Therefore, a standard nonparametric estimator can be obtained taking the limits $x \to \infty$ and $h \to 0$ and taking $p = \mathrm{i}\nu$ in (3) using an explicit Fourier inversion :

$$m(y) = \lim_{h,x} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\lambda} \frac{\partial \ln a}{\partial x}(x, \mathrm{i}\nu) K^*(h\nu) \mathrm{e}^{\mathrm{i}\nu y}\, d\nu \, . \quad (4)$$

On the other hand, a can be expressed with respect to P using Theorem 2.1. For convenience, we denote by Ψ the complex-valued function defined for all (ω, p) in $\mathbb{R}\times\{z \in \mathbb{C} \, ; \, \mathrm{Re}(z) \geq 0\}$ as

$$\Psi(\omega,p) \stackrel{\text{def}}{=} \frac{\lambda \mathcal{L}P(c+\mathrm{i}\omega, p)}{(c+\mathrm{i}\omega+\lambda)(c+\mathrm{i}\omega+\lambda-\lambda\mathcal{L}P(c+\mathrm{i}\omega,p))} \, .$$

Using this definition and Theorem 2.1, we get for all (T,p) in $\mathbb{R}_+^* \times \{z \in \mathbb{C} \, ; \, \mathrm{Re}(z) \geq 0\}$:

$$a(T,p) = 1 + \frac{\mathrm{e}^{(c+\lambda)T}}{2\pi} \int_{-\infty}^{+\infty} \Psi(\omega,p) \mathrm{e}^{\mathrm{i}\omega T}\, d\omega \, . \quad (5)$$

We now use (4) and (5) to construct an estimator of m. A natural estimate of λ is given by

$$\hat{\lambda}_n = \left(\frac{1}{n} \sum_{k=1}^n Z_k \right)^{-1} \, , \quad (6)$$

and the Laplace transform $\mathcal{L}P$ can be estimated given a sample $\{(X'_k, Y'_k)\}_{1\leq i\leq n}$ as follows :

$$\widehat{\mathcal{L}P}_n(c+\mathrm{i}\omega, \mathrm{i}\nu) = \frac{1}{n} \sum_{k=1}^n \mathrm{e}^{-(c+\mathrm{i}\omega)X'_k - \mathrm{i}\nu Y'_k} \, . \quad (7)$$

We now need to estimate the partial derivative $\frac{\partial \ln a}{\partial x}$. Adapting the approach of [3] in a related problem, we get from the power series expansion of Ψ

$$\Psi(\omega, \mathrm{i}\nu) = A_1(\omega,\mathrm{i}\nu) + A_2(\omega,\mathrm{i}\nu) \, , \quad (8)$$

where we define

$$A_1(\omega,\mathrm{i}\nu) \stackrel{\text{def}}{=} \frac{\lambda \mathcal{L}P(c+\mathrm{i}\omega, \mathrm{i}\nu)}{(c+\mathrm{i}\omega+\lambda)^2}$$

$$A_2(\omega,\mathrm{i}\nu) \stackrel{\text{def}}{=} \frac{\lambda \mathcal{L}P(c+\mathrm{i}\omega, \mathrm{i}\nu)}{c+\mathrm{i}\omega+\lambda} \Psi(\omega,\mathrm{i}\nu)$$

Therefore, using (5) and (8), we obtain :

$$a(x, \mathrm{i}\nu) = 1 + a_1(x, \mathrm{i}\nu) + a_2(x, \mathrm{i}\nu) \tag{9}$$

where we get from explicit Fourier inversion

$$a_1(x, \mathrm{i}\nu) = \int_0^x \lambda(x-\tau)\mathrm{e}^{\lambda\tau} \int_0^{+\infty} \mathrm{e}^{-\mathrm{i}\nu\varepsilon} P(d\tau, d\varepsilon)$$

and

$$a_2(x, \mathrm{i}\nu) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} A_2(\omega, \mathrm{i}\nu)\mathrm{e}^{(c+\lambda+\mathrm{i}\omega)x}\, d\omega\ .$$

Moreover we have :

$$\frac{\partial a_2}{\partial x}(x, \mathrm{i}\nu) = \frac{\mathrm{e}^{(c+\lambda)x}}{2\pi} \times \int_{-\infty}^{+\infty} \lambda \mathcal{L}P(c+\mathrm{i}\omega, \mathrm{i}\nu)\Psi(\omega, p)\mathrm{e}^{\mathrm{i}\omega x}\, d\omega\ . \tag{10}$$

and

$$\frac{\partial a_1}{\partial x}(T, \mathrm{i}\nu) = \iint_{\mathbb{R}_+^2} \mathbb{1}_{\{\tau \le T\}} \mathrm{e}^{\lambda\tau - \mathrm{i}\nu\varepsilon} P(d\tau, d\varepsilon)\ . \tag{11}$$

and we have using (4) and (9) :

$$\frac{\partial \ln a}{\partial x}(x, \mathrm{i}\nu) = \frac{1}{a(x, \mathrm{i}\nu)} \left[\frac{\partial a_1}{\partial x}(x, \mathrm{i}\nu) + \frac{\partial a_2}{\partial x}(x, \mathrm{i}\nu) \right] \tag{12}$$

Estimators $\widehat{a}_n(T, \mathrm{i}\nu)$ (respectively $\hat{I}_{2,n}(T, \mathrm{i}\nu)$) of a (respectively $\frac{\partial a_2}{\partial x}(T, \mathrm{i}\nu)$) can be obtained by plug-in directly estimators (6) and (7) in (5) (respectively (10)). Moreover, the estimator $\hat{I}_{1,n}(T, \mathrm{i}\nu)$ of $\frac{\partial a_1}{\partial x}(T, \mathrm{i}\nu)$ can be obtained using (11) as follows :

$$\hat{I}_{1,n}(T, \mathrm{i}\nu) \overset{\text{def}}{=} \frac{1}{n} \sum_{k=1}^{n} \mathbb{1}_{\{X'_k \le T\}} \mathrm{e}^{\widehat{\lambda}_n X'_k - \mathrm{i}\nu Y'_k}$$

From (4) and and (12), we finally deduce the following estimator for the marginal energy density function,

$$\widehat{m}(y; c, T, h, n) = \frac{1}{2\pi} \times \int_{-\infty}^{+\infty} \frac{\hat{I}_{1,n}(T, \mathrm{i}\nu) + \hat{I}_{2,n}(T, \mathrm{i}\nu)}{\widehat{a}_n(T, \mathrm{i}\nu)} K^*(h\nu)\mathrm{e}^{\mathrm{i}\nu y}\, d\nu \tag{13}$$

Remark that, contrary to the algorithms proposed in [1] there is no differentiation involved in the final estimator.

3. APPLICATIONS AND DISCUSSION

We present in this section results on simulated and real data. For the real data, we dispose of $N = 10^6$ *busy* and *idle* sequences measured using the ADONIS system, which is based on the principles described in [4], from a Cesium source, and we wish to retrieve its associated energy spectrum. Given these observations, we apply the algorithm described in Section 2.

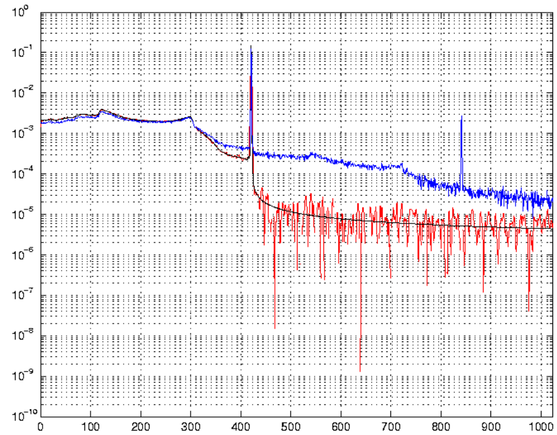


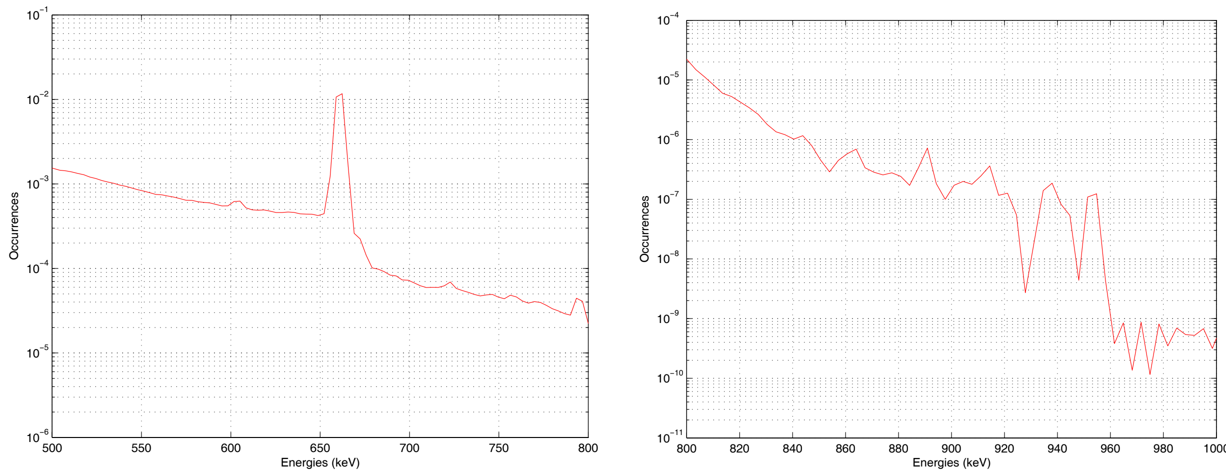
Fig. 2. Results on simulations : ideal (black), observed (blue) and corrected energy spectrum (red).

3.1. Results on simulation

Figure 2 represents our results on a simulated density close to real observations with $\lambda = 0.04$, X being drawn according to a truncated Gamma distribution independently from Y. As it can be seen, the multiple spikes and the piled-up Compton continuum are both corrected.

3.2. Results on real data

We observe a mixture of Cesium 137 (which has one monoenergetic spike at 662 keV), Cesium 134 (which have some representative energy spikes at 569, 604, 795, 802 and 847 keV) and other isotopes at unknown quantities at $\lambda = 0.02$. Choosing $T = 1\,\mu s$, we keep about 90% of our observations. Figure 3 shows the spectrum obtained by our pile-up correction method, compared to the observed one. In this experiment, we can see that the Compton continuum has been corrected, as much as the multiple spike. Moreover, we retrieve most of the energy spikes of Cs 137 and Cs 134 in enlarged zones in Figures 4-(a) and 4-(b), and we now distinguish spikes of other isotopes of the Cesium. which were hidden by the piled-up Compton continuum. More precisely, using the JEF 2.2 database for identifying radionucleides, we retrieve some spikes of Cesium 136 (725 keV), Cesium 130 (671 and 894 keV) and Cesium 129 (864, 906 and 946 keV), thus our results are quite promising. Some spikes however does not correspond to radioactive elements. The most important problem comes from the additive noise, that is not taken into account in our modeling. Indeed, if we assume that the additive noise has a probability measure G, we dispose in fact of observations with associated probability measure $P * G$, thus we must correct the pile-up correction by replacing $\mathcal{L}P$ by $\mathcal{L}P/\mathcal{L}G$. This problem is nevertheless limited in the case of HPGe de-



(a) (b)

Fig. 4. (a) Enlarged zone on the range [500 keV ; 800 MeV]. (b) Enlarged zone on the range [800 keV ; 1 MeV].

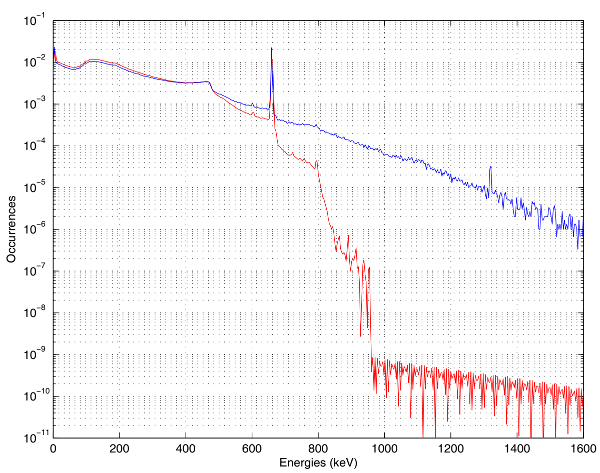


Fig. 3. Piled-up (blue) and corrected (red) energy spectrum of Cesium with real data.

tectors, where the SNR ratio is very good, so that we have $\mathcal{L}G \approx 1$, but it could explain why the quality of our estimator could be improved.

4. CONCLUSION

In this paper we investigated the problem of correcting the pileup phenomenon in γ spectrometry. Using methods close to the ones applied in the density deconvolution framework, we obtain very promising results on real data. The use of HPGe detectors which limits the problem of the additive noise is crucial in our approach. The application of a denoising technique before applying our algorithm be developed in future contributions.

5. REFERENCES

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