TIME-FREQUENCY REPRESENTATIONS MATCHED TO GUIDED WAVES

G.Le Touzé, B.Nicolas, J.I.Mars and J-L.Lacoume

Laboratoire des Images et des Signaux
INPG/ENSIEG
Grenoble, France

ABSTRACT

This study proposes a matched time-frequency representation construction methodology based on physical model of propagation on underwater environment. The main objective is to filter propagation modes in the time-frequency plane. Optimal representations are then built and test on real dataset for two classical waveguide model.

1. INTRODUCTION

Recent progress on time-frequency representation (TFR) are due to developments of matched methods to processed signals. The Matching Pursuit Algorithm developed by S. Mallat *et al.* [1] finds the fittest linear decomposition base for a given signal by successive iterations. The atoms dictionary is complete when the family covers all the space from which the signal results, it then constitutes the smallest most adapted time-frequency base of the signal. More recently Papandreou-Suppappola *et al.* [2] using Matching Pursuit Algorithm propose an extended dicitionary to integrate inhomogenous non linear time-frequency structures. We focus here on pressure signal of propagation in shallow water. This type of signal is composed by non-linear time-frequency modal structure. The objective is to separate these strucures to make a mode filtering in the time-frequency plane. We develop here an original TFR construction methodology in which the TFR is supervised by *a priori* knowledges (not in an adaptive way) resulting from the physics of propagation for Underwater Acoustics. We first present the general formalism of matched TFR applicable to the signals propagated in a waveguide. We build then TFR matched to the perfect waveguide (waveguide without loss) and to the Pekeris waveguide which describe both the wave propagation on Ultra Low frequency in Underwater Acoustics. We expose real signals processing examples. We finally show a mode filtering example with a watershed algorithm.

2. CONTEXT AND OBJECTIVE

In Underwater Acoustics, a small depth oceanic medium is modelised by a waveguide. Starting from a sufficiently large distance, pressure signals (resulting from an impulse source) break up into modes. It is the consequence of a dispersive propagation in the waveguide. Geoacoustic parameters (number of layers, depths, propagation velocities, densities) associated with the modes theory establish relationship between the group velocity and frequency for each whole mode m. In addition, with the knowledge of the source-sensor distance R, the distribution of energy by mode in the time-frequency plane can be deduced [3]. This energy follows for each mode a non linear curve $\nu_m = u_m(\tau)$ connecting the frequency ν to the time delay τ.

The objective is to build matched TFR to the propagated signals in a waveguide starting from *a priori* known theoretical curves. These TFR must :
- separate the modes in the time-frequency plane,
- be inversible for mode filtering.

3. TIME-FREQUENCY PROBLEM

Any TFR is subjected to the time-frequency uncertainty principle (Heisenberg-Gabor inequality) which prevents a precise localization simultaneously in time and frequency. This limitation causes an inevitable spreading out of the spectral element in time around the theoretical curves. The spectro-temporal structures for the various curves overlap even with super-resolvent methods such as the Lagunas Representation [4].

4. GENERAL METHODOLOGY

We present here a methodology and conditions to be respected by the waveguide to allow the matched TFR construction. We are starting from the dispersion relation for a mode : $\nu_m = u_m(\tau)$. Whatever the guide configuration (several layers, gradient velocitiy...), this relation can be established if propagation parameters (layers, heights, velocities and R) are known.

4.1. Principles of construction

We apply here an "atomic" TFR construction methodology consisting in projecting the signal on an atoms dictionary

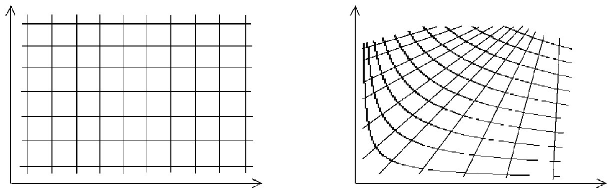

Fig. 1. Paving of time-frequency plane by projection atoms for STFT & proposed matched method

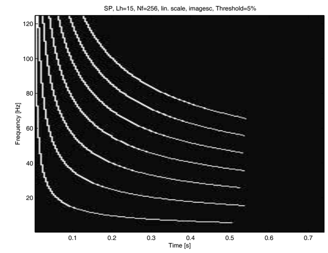

Fig. 2. Theoretical layouts for modal curves for R=3500m (with North Sea survey configuration)

which is paving the time-frequency plane. Classically, atoms are built starting from a Gaussian window $h(t)$ by translation around time τ, frequential modulation $\xi(t)$ and possibly weighting so that the representation be inversible (as we will see it thereafter). Atoms thus obey the following general equation :

$$h_{\tau,\xi}(t) = h_\tau(t)e^{j\xi(t)} \tag{1}$$

with $\tau \in D_f$ the time domain of signal and $\nu \in \mathbb{R}$ because physic of propagation says that the signal is limited in time by the arrival time and the cuting time. The originality is to start from the physic of propagation in projecting the signal on the theoretical curves of modes : $\nu_m = u_m(\tau)$. With this purpose and in order to adapt as well as possible to the signal, we project it on atoms having the theoretical modes instantaneous phase $\phi_m(t)$. The instantaneous frequency is the derivative of the instantaneous phase (phase is thus : $\phi_m(t) = 2\pi \int \nu_m dt$). Projection on atoms of modulation $\xi(t) = \phi_m(t)$ gives :

$$\Psi_h(\tau, m) = \int x(t)h_\tau(t)exp\left(j\phi_m(t)\right)\, dt \tag{2}$$

To pass from the time-mode plane to the time-frequency one we substitute m by m' resulting from the inversion of the relation of dispersion $m' = v(\nu, \tau)$. Finally :

$$TFR_h(\tau, \nu) = |\Psi_h(\tau, m)|^2_{m=m'} \tag{3}$$

This technique can be seen as the projection of the signal on dispersive modal curves. The relation $m' = v(\nu, \tau)$ allows to pass from the theoretical curves (for m whole) to all time-frequency space (m no more necessarily whole). The originality of this method can be seen in layouts of the paving of the time-frequency plane by the atoms. Indeed, the atoms of projection differ from a place to another in the time-frequency plane (figure 1).

4.2. Conditions of TFR construction

In order to allow the matched TFR construction, layouts of theoretical curves which determines the projection respects the following conditions :
- Existence of the $m = v(\nu, \tau)$ relation meaning that to each useful point of the time-frequency plane, a projection curve corresponds.
- Univocity of the $m = v(\nu, \tau)$ relation. Indeed, to a time-frequency point only one curve of projection should correspond. In the contrary case, the atoms dictionary would give place to several overlaped pavings on the time-frequency plane and that would mean that modal curves are likely to cross in the time-frequency plane.

Lastly, to be inversible, atoms of TFR must constitute a base (and not a frame) and thus respect the "closing" condition defined in [5] :

$$\int_{\mathbb{R}} \int_{D_f} h_{\tau,\nu}(t) h^*_{\tau,\nu}(t')\, d\tau\, d\nu = \delta(t - t') \tag{4}$$

This relation must be valid on the spectro-temporal domain of the signal. Application of this condition involves the creation of a base $\{h_{\tau,\nu}(t); \tau \in D_f\ et\ \nu \in \mathbb{R}\}$ as well as a redefinition of the basic projection window $h_\tau(t)$ which is not necessarily anymore a delayed version of basic window $h(t)$.

5. TFR MATCHED TO WELL KNOWN MODELS

A method according to this methodology was exposed in [6] for the perfect waveguide (the simplest guide with two layers and perfect reflexion). In this guide, the relation $\nu_m = u_m(\tau)$ is for the m^{th} mode [3] :

$$\nu_m = \frac{(2m-1)C_1^2\, \tau}{4D\sqrt{(C_1\tau)^2 - R^2}} \tag{5}$$

where C_1 and C_2 are the velocity in water and in the sediment layer, D the waveguide depth and R the source-sensor distance. The time domain is : $D_f = \left]\frac{R}{C_1}, \frac{RC_2}{C_1^2}\right[$.

No precision is given in [6] concerning the inversion possibility. We thus initially made this TFR inversible by defining atoms equations.

The perfect model is very approximate. We thus want, following the same principle, to create an matched representation to the Pekeris waveguide [7]. It is a 2 layers model which integer the reflexion coefficient according to the incidence angle. It is much more realistic than the perfect model. In this

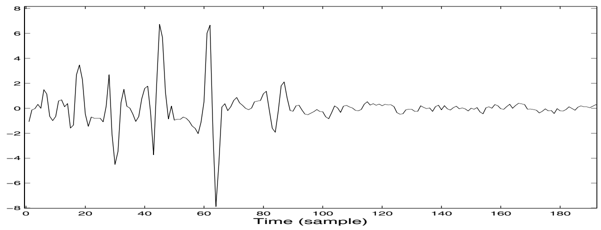

Fig. 3. Time pressure signal for R=3500m (North Sea)

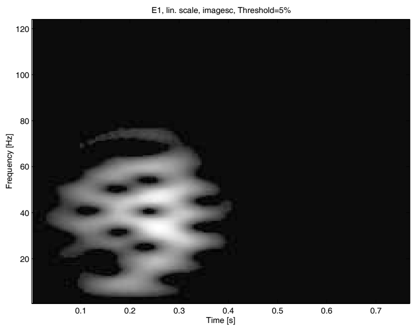

Fig. 4. Spectrogram [8] for R=3500m (North Sea)

model, dispersion equation $\nu_m = u_m(\tau)$ of modes connecting frequency to time doesn't have an analytical solution. We obtain an analytical approximation of this relation giving an approximate Pekeris model. The matched TFR to the Pekeris guide can thus be built and closing conditions established on atoms family so as to make the TFR inversible.

6. RESULTS AND APPLICATIONS

6.1. Results on real data

We apply these two TFR to a real case resulting from a survey in North Sea for which the source-sensor distance $R = 3500m$ is relatively small. The theoretical curves of the first 7 modes are presented figure 2. Time version of the pressure signal is shown figure 3 and its TFR figures 4 to 7. Benefit given by matched TFR with respect to the traditional methods (spectrogram, reassigned spectrogram and Lagunas method) can clearly be seen : for the classical methods, time-frequency compromise doesn't make possible to distinguish modes which is the case for matched methods. Mode filtering is then possible. In addition, the localization is lightly more precise in the Pekeris case than in the perfect case (less interferences between modes).

We present another example coming from a survey in the Lions Gulf for which $R = 14000m$. Because of the longer distance R, a better mode separation given by the Pekeris model with respect to the perfect model (figure 8 and 9) can be seen. It shows the interest we have to choose the fittest model.

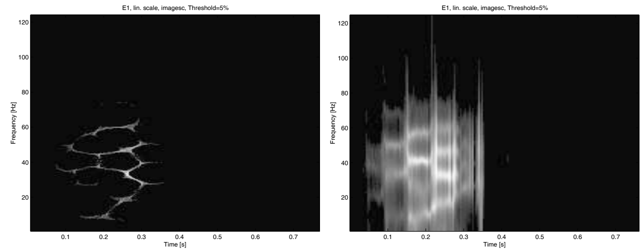

Fig. 5. Reassigned Spectrogram & Lagunas Representation for R=3500m (North Sea)

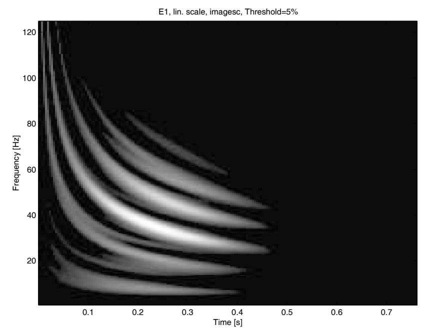

Fig. 6. Squared version of perfect model matched representation for R=3500m (North Sea)

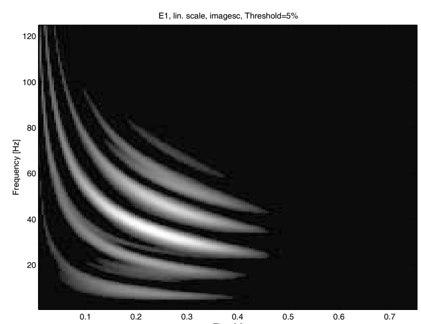

Fig. 7. Squared version of Pekeris model matched representation for R=3500m (North Sea)

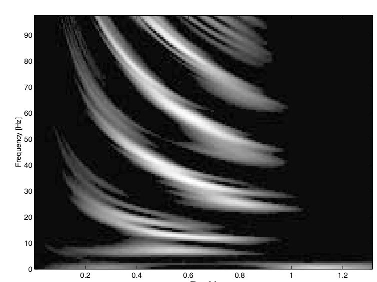

Fig. 8. Squared version of perfect model matched representation for R=14000m (Lion Gulf)

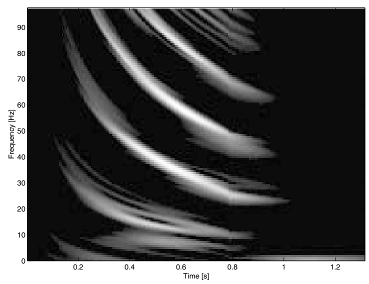

Fig. 9. Squared version of Pekeris model matched representation for R=14000m (Lion Gulf)

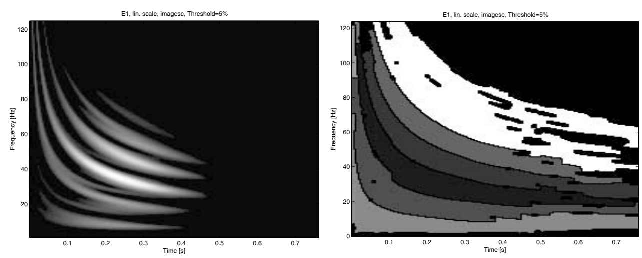

Fig. 10. Adapted representation of North Sea data & Watershed algorithm result on this representation

6.2. Applications

In Underwater Acoustics, modes give a crucial information for source localisation : modes amplitude give informations about source depth, modes phase and modal curve shape about source-sensor distance. To reach those informations, a mode filtering is necessary. However mode filtering is allowed and facilitated by matched (and inversible) TFR thanks to the better separation of modes in the time-frequency plane. To make this mode filtering possible, we are using the classical image segmentation Watershed algorithm [9]. To filter mode, we thus apply the following process :
- Adapted inversible TFR construction of the signal
- Watershed on this TFR
- Selection of the wanted mode
- Inversion of the selected mode

We have the time version of modes, we can thus reach phase amplitude, phase infomation and more precise shape in the time-frequency plane no more interferences with others modes are present. An example of mode filtering is shown on the North Sea data figures 10 and 11. For those data, it is impossible to filter with classical method as we can see on figure 4.

7. CONCLUSION

In this study, we propose a general methodology allowing to build TFR whatever the model under certain conditions. We apply this methodology to two traditional models of propagation in Underwater Acoustics on real data. In a signal processing context, we apply the algorithm philosophy, namely to project the signal on fittest time-frequency atoms, but the correspondence being done here starting from *a priori* knowledge and not in an adaptive way. It results from this that time-frequency uncertainty is "channeled" thanks to the atoms following the theoretical curves. We finally test these methods on real signals and showed the benefit which they bring with respect to the traditional methods. This approach makes possible to combine the physics of propagation with the signal processing.

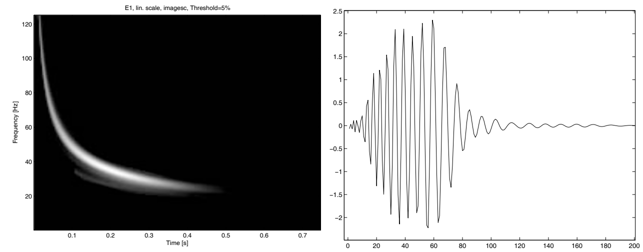

Fig. 11. Selected mode (mode 3) on the time-frequency plane & Time representation of this mode after inversion

8. REFERENCES

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