EFFICIENT ALGORITHM FOR MODIFIED LOCAL POLYNOMIAL TIME FREQUENCY TRANSFORM

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ABSTRACT

This paper presents efficient algorithms for the analysis of non-stationary multi-component signals based on modified local polynomial time frequency transform. The signals to be analyzed are divided into a number of segments and the desired parameters are estimated in each segment for computing modified local polynomial time frequency transform. Compared to other reported algorithms, the length of overlap between consecutive segments is reduced to minimize the overall computational complexity. The concept of adaptive window lengths is also employed to achieve a better time-frequency resolution for each component.

1. INTRODUCTION

Due to its superior performance in dealing with non-stationary signals, time frequency transforms (TFTs) have found various applications in many areas including communications, multi-media, mechanics and biology [1]. The most popular and simplest TFT is short time Fourier transform ($STFT$) that has been widely used for many practical applications [1, 2]. Nevertheless, the $STFT$ suffers from low resolution when the analyzed signal is highly non-stationary. Local polynomial time frequency transform ($LPTFT$), referred as the generalization of $STFT$, was reported to provide high resolution for non-stationary signals [3, 4] with a local polynomial function approximating to the frequency characteristics. Unfortunately, the estimation of a number of extra parameters required by $LPTFT$ computation results in a heavy computational load.

This paper presents analysis algorithms for time varying multi-component signals containing white Gaussian. Different from previously reported algorithms, the proposed modified local polynomial time frequency transform ($MLPTFT$) reduces the overlap length between consecutive segments to minimize the number of segments to be processed. Effective methods of estimating the $MLPTFT$ parameters from each signal segment are presented. Deterioration of resolution due to the reduction of overlap length is avoided by using adaptive window lengths.

2. MODIFIED LPTFT

2.1. Segmentation

The signal with noise to be analyzed is defined as

$$x(t) = s(t) + w(t), \quad 0 \le t \le N-1 \tag{1}$$

where $w(t)$ represents white Gaussian noise and $s(t)$ contains mono- or multi- nonstationary components in the time frequency domain. It is assumed that the sampling frequency of the discrete data is normalized to be one Hz and parameter t takes integer values. The input signal $x(t)$ is divided into many small segments with a window function $h(\tau)$ in the time domain. The jth signal segment is defined as

$$x_j = x[j(Q-\alpha)+\tau]\, h(\tau),$$

where $0 \le j \le \lfloor N/(Q-\alpha)\rfloor - 1, 0 \le \alpha \le Q-1$, $-(Q-1)/2 \le \tau \le (Q-1)/2$, $\lfloor x \rfloor$ is the function to return the largest integer that is equal to or smaller than x, N is the length of signal $x(t)$, Q, which is assumed to be an odd number without loss of generality, is the length of the window $h(\tau)$ or equivalently the length of the signal segment and α represents the length of the overlap between the consecutive signal segments. Fig 1 shows examples for α =0, $Q-1$ and $(Q-1)/2$ with $Q = 5$. Heavy computational complexity is needed for estimating the extra parameters required by $LPTFT$ computation if the overlap length is large because the number of signal segments to be processed is accordingly increased.

2.2. The $MLPTFT$

The local polynomial time-frequency transform ($LPTFT$) of $x(t)$ is defined as [3]

$$LPTFT(t,f) = \sum_{\tau=-\infty}^{\infty} x(t+\tau)h(\tau) e^{-j2\pi[\sum_{m=2}^{M} l_{m-1}(t)\frac{\tau^m}{m!}+f\tau]}, \tag{2}$$

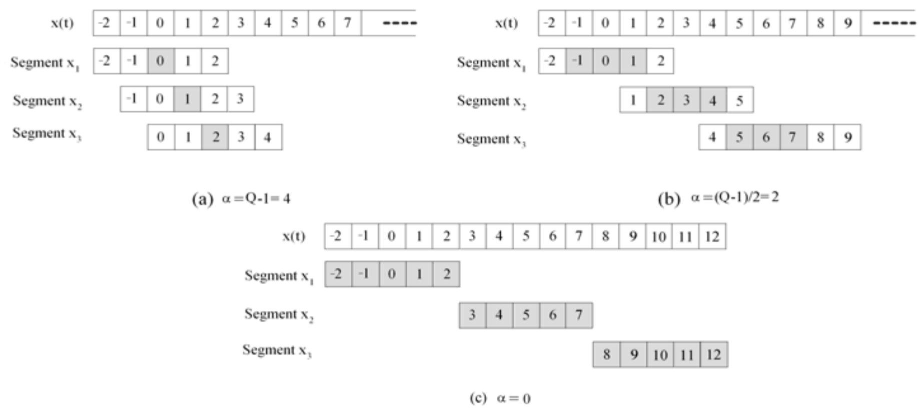


Fig. 1. Segmentation examples for overlap length $\alpha = 4$, 2 and 0 and window length $Q = 5$

where $h(\tau)$ is the window function with length Q, and $\mathrm{l}(t) = [l_1(t), \cdots, l_{M-1}(t)]$ are the parameters related to the derivatives of the instantaneous frequency of $x(t)$ [3]. The $LPTFT$ is based on the idea of fitting an $(M-1)$th order polynomial function approximation of the frequency of x_j, defined in (2) with $\alpha = Q-1$, to determine the nonparametric characteristic of the signal [3]. In addition to the calculation of (2), other processing costs for the $LPTFT$ are for the estimation of both the time-varying parameter $\mathrm{l}(t)$ and window length Q.

The $LPTFT$ cannot be directly used for signals containing multiple components because individual signal components have their own parameter $\mathrm{l}(t)$ and window length Q. Let us define the $MLPTFT_p$ for signals containing p components with sets of parameters $\mathrm{L}(t) : \{\mathrm{l}_i(t);\ 1 \le i \le p\}$ and window length $Q : \{Q_i;\ 1 \le i \le p\}$ as

$$MLPTFT_p(t,f) = \sum_{\tau=-\infty}^{\infty} \frac{1}{a} x(t+\tau) e^{-j2\pi f\tau} \sum_{i=1}^{p} h_i(\tau) e^{-j2\pi \sum_{m=2}^{M} l_{i,m-1}(t)\frac{\tau^m}{m!}}, \quad (3)$$

where $a = ||\sum_{i=1}^{p} h_i(\tau) e^{-j2\pi \sum_{m=2}^{M} l_{i,m-1}(t)\frac{\tau^m}{m!}}||_2$ is the scaling factor keeping the signal energy unchanged and $||\cdot||_2$ is the 2-norm operation in terms of τ.

3. ESTIMATION OF L(T)

In the previously reported methods, the overlap factor α equals $Q-1$ which means that there are N segments of length Q for an N-point input sequence. In general, several $MLPTFT_p$s with different sets of parameters are computed for each signal segment. For segment x_j, for example, the $\mathrm{L}(j)$ that yields the maximum value [2] or values larger than a threshold [4] is selected. Because two consecutive signal segments overlap heavily, i.e., two adjacent signal segments differ by only one data point, this method requires a large computational load [4].

To reduce the overall computational load, it is necessary to minimize the length of overlap between consecutive segments, such as $\alpha < Q-1$. For segment x_j, the set of parameters $\mathrm{L}(j(Q-\alpha+\tau))$ within the duration $\lfloor -(Q-\alpha-1)/2 \rfloor \le \tau \le \lfloor (Q-\alpha-1)/2 \rfloor$ is estimated simultaneously. As shown in Fig. 1 that the parameters for the shaded data intervals are estimated from the corresponding signal segment. For example, L(2), L(3) and L(4) are estimated from segment x_2 when $\alpha = 2$ in Fig. 1(b). In this way, only $\lfloor N/(Q-\alpha) \rfloor$, instead of N signal segments, are processed to acquire $\mathrm{L}(t)$ at all time instants. Generally, α controls the tradeoff between the computational load and the smoothness of the spectrum. When $\alpha = 0$, there is no overlap and only N/Q segments are processed, which reduces the computational load Q times compared with that with $\alpha = Q-1$ in the previously reported method. In general, the $MLPTFT_p$ with $\mathrm{L}(t)$ estimated with $\alpha = 0$ yields satisfactory performance to achieve a good polynomial function approximation to the frequency components if the window length Q is small enough, which is further illustrated in the first experiment of Section 5.

The coefficients of the polynomial function model used to achieve $\mathrm{L}(t)$ [3] are estimated by searching the peak locations of the polynomial Fourier transform (PFT) of the signal segment. The PFT of x_j is defined as

$$PFT(x_j, \mathrm{a}) = \sum_{t=-\infty}^{\infty} x_j e^{-j2\pi(\sum_{m=1}^{M} a_m t^m)} \quad (4)$$

where $\mathrm{a} = \{a_1, \cdots, a_M\}$. It is assumed that p peaks are found in the PFT indicating the p components and are located at positions $\mathrm{a}_i = \{a_{i,1}, \cdots, a_{i,M}\}$, $1 \le i \le p$.

4. WINDOW LENGTH ESTIMATION

In the previous section, L(t) is estimated based on the idea of modelling each segment as an Mth-order polynomial phase signal. Therefore, the window length used in the $MLPTFT_p$ or $RMLPTFT_p$ is the same as the length of the segment. It is known that there is a trade-off between the window length and the resolution of the $MLPTFT_p$ [5]. In general, approximation errors increase with the window length if the order of the $MLPTFT_p$ is lower than that of the phase of the signal segment. For polynomial phase component whose order is not higher than that of $MLPTFT_p$, on the other hand, the $MLPTFT_p$ gives a better resolution if longer window (or segment) is used. For a good compromise, it is always desired that the length of the segment is adaptively matched to the characteristics of the signal components. In our analysis, the initial window length is selected to be small enough to provide acceptable accuracy of the approximation and the actual length of the window is increased according to the properties of consecutive signal segments.

Since we intend to increase the window length if consecutive segments have the same polyphase model, let us assume that two consecutive segments, the jth and $(j+1)$th segments, belong to the same polynomial phase model. If the jth segment has the phase $2\pi \sum_{m=0}^{M} k_{i,m} t^m$, the phase of the $(j+1)$th segment should be $2\pi \sum_{m=0}^{M} k_{i,m} (t+(Q-\alpha))^m$ because the $(j+1)$th segment is delayed by a time interval of the segment overlap compared with that of the jth segment. The difference between the coefficients of the consecutive segments is calculated by

$$\begin{aligned} &\left[\sum_{m=0}^{M} k_{i,m}(t+(Q-\alpha))^m\right] - \left[\sum_{m=0}^{M} k_{i,m} t^m\right] \\ &= \sum_{m=0}^{M} k_{i,m} \sum_{s=0}^{m} C_m^s t^s (Q-\alpha)^{m-s} - \sum_{m=0}^{M} k_{i,m} t^m \\ &= \sum_{m=0}^{M} \sum_{s=0}^{m-1} t^s (Q-\alpha)^{m-s} \end{aligned} \tag{5}$$

where $C_m^s = s!/(m!(m-s)!)$. For clarity of presentation, we define

$$\sum_{m=0}^{M} b_m t^m = \sum_{m=0}^{M} \sum_{s=0}^{m-1} t^s (Q-\alpha)^{m-s} \tag{6}$$

where b_m is the constant coefficient associated with t^m term on the right side of (6). Let us represent the coefficients of the polynomial function estimated from the jth and $(j+1)$th segments with $a_{j,m}$ and $a_{j+1,m}$, respectively, where $1 \leq m \leq M$. The difference $(a_{j+1,m} - a_{j,m})$ is compared with b_m. If each $|a_{j+1,m} - a_{j,m} - b_m|$ is smaller than a predefined threshold T_m, these two segments have the same polyphase model and the length of the window increases by $Q-\alpha$. The final window length is the total length of the consecutive segments that have the same polynomial function model.

It can be easily seen that compared with algorithm reported in [3], the computational complexity for L(t) estimation is significantly reduced. This is because, with the segmentation method shown in Fig. 1, the number of segments for an N-point sequence is reduced to be $\lfloor N/(Q-\alpha) \rfloor$ in comparison with N segments needed in [3]. The estimation of window lengths requires overheads for computation of (5) and the costs of comparison with the given threshold is trivial.

5. EXPERIMENTAL RESULTS

Two types of signals, which contain mono- and multi-component, respectively, are used to test the performance of the proposed algorithms. For simplicity, the 2nd-order $MLPTFT_p$ is used in all experiments dealing with the input sequence $x(t)$ with $N = 512$.

The first type of signal contains mono-component defined as:

$$x_1(t) = exp(-j(256/\pi)cos(\sqrt{2}\pi t/256)) + w(t). \tag{7}$$

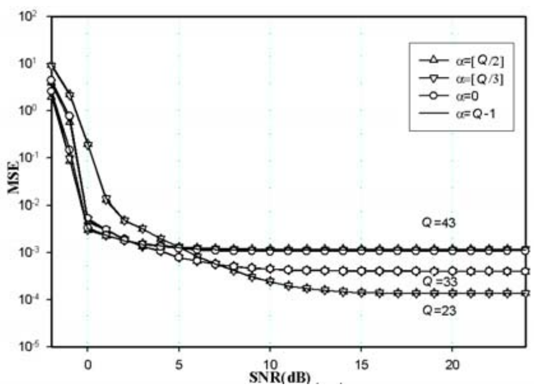


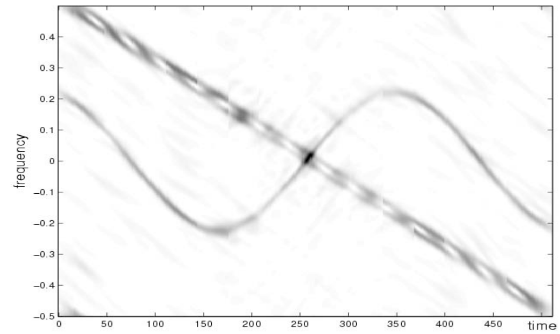
Fig. 2. The comparisons between MSEs from $MLPTFT_p$ with different α and Q.

The estimation of instantaneous frequency of $x_1(t)$ is conducted with Gaussian noise $w(t)$ of different variances. Monte Carlo simulations are performed to obtain the mean square error (MSE) for each estimator. The MSE is defined by $\frac{1}{N}\sum_{t=0}^{N-1}(\tilde{f}(t) - f(t))^2)$, where $f(t)$ is the true instantaneous frequency and $\tilde{f}(t)$ is the estimation of $f(t)$ according to the curve peak positions in the $MLPTFT_p$ of $x_1(t)$. The MSEs are compared for different overlap lengths $\alpha = 0$, $\lfloor Q/2 \rfloor$, $\lfloor Q/3 \rfloor$ and $Q-1$, as shown in Fig. 2. A large range of window lengths ($6 < Q < 80$) have been tried. Only the results using $Q = 23$, $Q = 33$ and $Q = 43$ are shown because the MSEs achieved with these windows are mostly below 10^{-2}. It is observed that for high SNRs, smaller window length generally gives smaller MSEs, for example, the curve for $Q = 23$ gives the best performance when $SNR > 7$ dB. With low SNRs, i.e., $SNR \leq 7$ dB, larger window length yields lower MSEs, as seen from Fig. 2. This is because that MSEs are mainly influenced by the bias and the variance of the input sequence [3]. When SNR is high, the MSE is mainly affected by the bias which increases with the increase of window length. When SNR is low, the variance of the signal is the dominant factor affecting the MSE. The variances decrease with the increase of window length so that the MSE becomes relatively small. It is worth mentioning that, When SNR is extremely low, e.g., below 0 dB, MSEs deteriorate significantly. This is because the windows used in our proposed $MLPTFT_p$ are generally with smaller length. The use of narrow window leads to the increasing influence of noise especially for low SNR. The most important observation made in Fig. 2 is that the MSE performances for different overlap lengths are very close to each other regardless of the window lengths, which leads to the conclusion that the decrease of overlap length between segments does not deteriorate the performance of $MLPTFT_p$.

Let us consider the signal contains multiple components, which is defined as:

$$x_2(t) = exp(-j(256/\pi)cos(\sqrt{2}\pi t/256)) + exp(-j.002\pi t^2) + exp(-j.002\pi t^2 + 0.06\pi t) + w(t) \quad (8)$$

where $w(t)$ is the Gaussian noise with $SNR = 0$ dB. This signal is used to test the performance of using adaptive window lengths. Fig. 3 shows the $MLPTFT_p$s of $x_2(t)$ that are computed with and without using adaptive window lengths. By comparing the difference between the estimated parameters in the consecutive segments with b_m in (6), the window length of the algorithm is enlarged to N to obtain a high resolution. It is shown that the resolution in Fig. 3(b) for the linear component of $x_2(t)$ is improved significantly and the two parallel chirp components are clearly distinguished in comparison with Fig. 3(a) in which the window length is fixed.



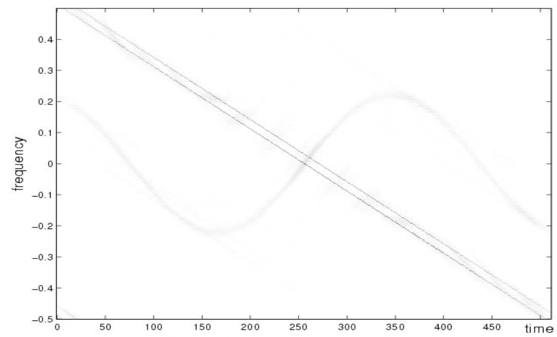


Fig. 3. Performance comparisons

6. CONCLUSION

This paper presents analysis algorithms effectively dealing with time varying multi-component signals. In particular, these algorithms allow the reduction of computational complexity by minimizing the length of overlap between consecutive signal segments. Experiments show that by using the proposed algorithms of parameters estimation and adaptive window length, the signals containing both single and multiple components with Gaussian and impulse noises can be more accurately represented in the time-frequency domain.

7. REFERENCES

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