EQUALIZING SAMPLING RATE CONVERTER FOR STORAGE SYSTEMS

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ABSTRACT

Data receivers for storage systems normally operate at a fixed sampling rate $1/T_s$ that is asynchronous to the baud rate $1/T$. A sampling-rate converter (SRC) serves to convert the incoming signal from the asynchronous to the synchronous clock domain. These receivers also contain an equalizer that serves to suppress intersymbol interference and noise. To limit receiver complexity, equalization burden can be shifted towards the SRC. This possibility is not exploited in any existing SRC. This paper presents SRC design methods that combine group delay flatness and out-of-band rejection criteria with the minimum mean square error equalization criterion. Numerical examples for an idealized optical recording channel validate the design methods.

1. INTRODUCTION

Receivers for data storage systems are often realized with the aid of digital IC technology. To profit optimally from the rapid advances of this technology, analog-to-digital conversion is ideally performed early on in the receiver. A common baseband topology for existing storage systems is depicted in Fig. 1. A received signal $r(t)$ is applied to an analog low pass

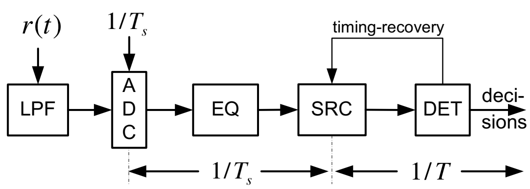

Fig. 1. Baseband receiver with asynchronous equalizer. Asynchronous and synchronous clock domains are indicated with the symbols $1/T_s$ and $1/T$, respectively.

filter (LPF) which suppresses out-of-band noise. The LPF output is digitized by an analog-to-digital converter (ADC) which operates at a crystal-controlled free-running frequency $1/T_s$ that is high enough to prevent aliasing. The ADC output is applied to an equalizer (EQ) which conditions intersymbol interference (ISI) and noise. The equalizer operates at the sampling rate $1/T_s$, i.e asynchronously to the baud rate $1/T$ [1]. It is controlled by an adaptation scheme that is not depicted for simplicity. A sampling-rate converter (SRC) [2], which forms part of a timing-recovery loop, produces an equivalent synchronous output which serves as the input of a bit detector (DET). Rather than placing the equalizer before the SRC, it would be possible to reverse their order. That would, however, cause the latency of the equalizer to contribute to the overall delay of the timing-recovery loop, thus significantly lowering its stability margin and attainable acquisition speed [3]. Also, the sampling rate $1/T_s$ can be lower than the baud rate $1/T$ whenever the channel has negative excess bandwidth. This is so, for example, in existing optical storage systems, e.g. DVD, Blu-Ray Disc. In such cases the asynchronous equalizer can have fewer taps and a lower operating speed than its synchronous counterpart, thereby lowering complexity and power dissipation.

At the heart of the SRC is an interpolation filter that mimics fractional delays, i.e. delays of a fraction μ of the sampling interval T_s. A shift register that precedes the interpolation filter produces an additional integer delay m where the overall delay $\tau = (m + \mu)T_s$ is re-determined at every symbol interval by the timing-recovery subsystem [2].

Design of the interpolator filter is a compromise between complexity and interpolation accuracy. Conventionally, this accuracy has two complementary aspects. First, the filter should introduce as little amplitude distortion as possible. This generally requires a long filter. Second, the filter should mimic a fractional delay, i.e. its group delay characteristics should be almost flat. These requirements only pertain to the passband of the recording channel, i.e. the range of frequencies in which actual data information is received. Outside that range the interpolator filter should ideally exhibit a large attenuation, and its group delay characteristics become irrelevant.

The equalizer (EQ) in Fig. 1 is complementary to the SRC in that it is conventionally meant to counteract all amplitude distortion as well as all phase distortion except for pure delays. In practice, digital recording channels have a nominal behavior that is relatively well known [2]. For this reason it is, in principle, possible to compensate for the nominal channel characteristics (amplitude as well as phase) within the SRC. This would relieve, and thereby simplify, the equalizer in that it would now only have to deal with variations of the actual channel characteristics relative to the nominal ones. Moreover, it should also simplify the interpolator filter in that it no longer requires a very flat amplitude characteristic and a steep

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transition between pass-band and stop-band. Hence, by shifting a part of the burden of the equalizer towards the SRC both blocks will be made simpler.

Because interpolation filters and anti-aliasing filters constitute the heart of any practical SRC, the remainder of this paper is divided into two main sections. Section 2 describes the design of equalizing interpolators. Section 3 treats the problem of equalizing anti-aliasing filters.

2. EQUALIZING INTERPOLATOR

In order to explain the principle and the design of an equalizing interpolation, let us consider, in this section, the case where the ADC frequency $1/T_s$ is equal to the baud rate, i.e. $R = T/T_s = 1$. The received signal, in Fig. 1, can be written as

$$r(t) = \sum_i a_i h(t - iT) + u(t),$$

where a_i denotes channel data, $h(t)$ is the continuous-time channel symbol response and $u(t)$ is additive noise. We denote by r_k the ADC output at the sampling instant $t_k^s = (k + \mu)T$ where μT denotes the sampling phase, i.e.

$$r_k = (h^\mu * a)_k + n_k,$$

where $h_k^\mu = h((k + \mu)T)$ and n_k is pre-filtered and sampled noise. The signal r_k is applied to a FIR filter c^μ of length L_c that is principally meant to compensate for the sampling phase μT and secondarily to equalize the channel impulse response h_k^μ towards a target response g_k of length L_g, see Fig. 2. The group delay of the interpolation filter c_k^μ must be

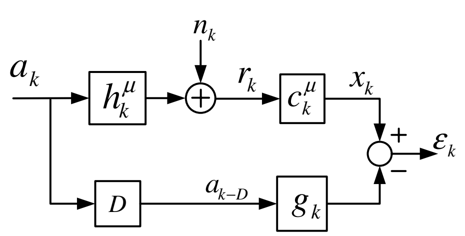

Fig. 2. Discrete-time system model for $1/T_s = 1/T$.

as close as possible to $(\frac{L_c-1}{2} + \mu)T$ at all frequencies inside the channel pass-band, i.e. $|f| < f_c$ where f_c is the channel cut-off frequency. Upon writing the frequency response of c_k^μ as $C^\mu(e^{j2\pi Tf}) = A(f)e^{-j2\pi\varphi(f)}$ where $A(f)$ and $\varphi(f)$ denote the amplitude and phase response of c_k^μ respectively, it can be easily shown that the group delay of c_k^μ satisfies

$$\varphi'(f) = -\frac{1}{2\pi}\frac{\mathrm{Im}(C^{\mu\prime}C^{\mu *})}{|C^\mu|^2},$$

where $C^{\mu\prime}$ and $C^{\mu *}$ denote the derivative and the conjugate of $C^\mu(e^{j2\pi Tf})$ respectively and $\mathrm{Im}(.)$ is the imaginary part. This expression can be simplified into

$$\varphi'(f) = \frac{\underline{c}^{\mu\mathrm{T}} G(f)\Lambda\underline{c}^\mu}{\underline{c}^{\mu\mathrm{T}} G(f)\underline{c}^\mu}, \tag{1}$$

where $G(f) = v_c v_c^\mathrm{T} + v_s v_s^\mathrm{T}$, given by $v_c = [1, \cos(2\pi fT), \ldots, \cos(2\pi(L_c - 1)fT)]^\mathrm{T}$ and $v_s = [0, \sin(2\pi fT), \ldots, \sin(2\pi(L_c - 1)fT)]^\mathrm{T}$. The matrix Λ is diagonal and its diagonal is equal to $[0, 1, \ldots, L_c - 1]$.

The mismatch in group delay of c_k^μ with respect to its ideal value, i.e. $(\frac{L_c-1}{2} + \mu)T$, needs in practice to stays below a predefined margin $\delta_g T$ where δ_g depends on system sensitivity to phase errors. In other words, the interpolation filter needs to meet

$$\tau_1 \le \varphi'(f) \le \tau_2, \quad \forall |f| < f_c \tag{2}$$

where $\tau_1 = (\frac{L_c-1}{2} + \mu)T - \delta_g T$ and $\tau_2 = (\frac{L_c-1}{2} + \mu)T + \delta_g T$. We introduce a finite frequency grid $f_i \in [0, f_c]$, $i = 1...N_c$, and define the corresponding constraints sets as

$$\mathcal{S}_i = \{\underline{c} : \tau_1 \le \frac{\underline{c}^\mathrm{T} G(f_i)\Lambda\underline{c}}{\underline{c}^\mathrm{T} G(f_i)\underline{c}} \le \tau_2\}. \tag{3}$$

The problem of equalizing interpolation boils down to designing the filter $c_k^\mu \in \mathcal{S}_i, \forall i$, while achieving amplitude equalization. In this paper we consider minimum mean square error (MMSE) equalization that seeks to minimize

$$J(\underline{c}^\mu) = E[\epsilon_k^2] = \underline{c}^{\mu\mathrm{T}}\mathbf{Q}\underline{c}^\mu - 2\underline{c}^{\mu\mathrm{T}}\underline{v} + \underline{g}^\mathrm{T}\mathbf{R}_a\underline{g}, \tag{4}$$

where $\underline{c}^\mu = [c_0^\mu, \ldots, c_{L_c-1}^\mu]^\mathrm{T}$, $\underline{g} = [g_0, \ldots, g_{L_g-1}]^\mathrm{T}$, $\mathbf{Q} = \mathbf{H}\mathbf{R}_a\mathbf{H}^\mathrm{T} + \mathbf{R}_n$ and $\underline{v} = \mathbf{H}\mathbf{R}_a\underline{g}$ where the matrix $\mathbf{H}$ has entries $H_{p,q} = h((q - p + \mu)T)$ and $\mathbf{R}_a$ and $\mathbf{R}_n$ denote the autocorrelation matrices of the input data and noise respectively. The design of the equalizing interpolator can now be formulated as

$$\underline{c}^\mu = \arg \min_{\underline{c} \in \bigcap_{i=1}^{N_c} \mathcal{S}_i} J(\underline{c}). \tag{5}$$

It should be noted that the group delay of (1) is related to the filter coefficients in a nonconvex rational manner, hence the constraints sets $\mathcal{S}_i$ are nonconvex in general. It follows that standard optimization techniques that hold for convex constraints sets do not apply to our problem (5). However, we will show that, modulo a linear transformation, (5) is equivalent to finding the orthogonal projection of the MMSE solution, i.e. the minima of (4), over an intersection of nonconvex sets. The vector space projection method (VSPM) [4] extended to nonconvex sets [5] can then be applied. In [5] a parallel projection algorithm, also known in literature as the Parallel Generalized Projection Algorithm (PGPA), was shown to ensure weak convergence even if the constraint sets are non-intersecting. As an example, this theorem was used in [6] to design allpass filters under group delay constraints.

Via decomposing the positive definite matrix $\mathbf{Q}$ into $\mathbf{\Gamma}^\mathrm{T}\mathbf{\Gamma}$, where $\mathbf{\Gamma}$ is positive definite, one can show that (5) yields

$$\underline{c}^{\mu\prime} = \Gamma\underline{c}^\mu = \arg \min_{\underline{c}' \in \bigcap_{i=1}^{N_c} \mathcal{S}_i'} \|\underline{c}' - \Gamma\underline{c}_0\|^2, \tag{6}$$

where $\|.\|$ denotes the L_2-norm and $\underline{c}_0 = \mathbf{Q}^{-1}\underline{v}$ is the MMSE solution. The definition of the new constraints sets $\mathcal{S}_i'$ is similar to (3) by replacing $G(f_i)$ with $G'(f_i) = \Gamma^{\mathrm{T}^{-1}} G(f_i)\Gamma^{-1}$

and Λ with $\Lambda' = \Gamma\Lambda\Gamma^{-1}$. According to (6), $\Gamma \underline{c}^{\mu}$ can be interpreted as the orthogonal projection of $\Gamma \underline{c}_0$ over $\bigcap_{i=1}^{N_c} \mathcal{S}'_i$. The sets $\mathcal{S}'_i$ can be written as $\mathcal{S}'_i = {\mathcal{S}'}_i^1 \cap {\mathcal{S}'}_i^2$ where ${\mathcal{S}'}_i^1 = \{\underline{c}' : \tau_1 \leq \frac{\underline{c}'^{\mathrm{T}} G(f_i)' \Lambda' \underline{c}'}{\underline{c}'^{\mathrm{T}} G(f_i)' \underline{c}'}\}$ and ${\mathcal{S}'}_i^2 = \{\underline{c}' : \frac{\underline{c}'^{\mathrm{T}} G(f_i)' \Lambda' \underline{c}'}{\underline{c}'^{\mathrm{T}} G(f_i)' \underline{c}'} \leq \tau_2\}$. The solution of (6) can then be based on the PGPA theorem which consists of iteratively applying a weighted sum of the orthogonal projections $P_i^{1,2}$ over ${\mathcal{S}'}_i^{1,2}$. For the conciseness of the paper, we refer to [6] where a very similar derivation of $P_i^{1,2}$ can be found. The algorithm of designing an equalizing interpolator can be summarized as follows:

step 1. we initially set $\underline{c}'_0 = \Gamma \underline{c}_0$.
step 2. $\forall n \geq 0$, $\underline{c}'_{n+1} = \sum_{j=1}^{2} \sum_{i=1}^{N_c} w_i^j P_i^j \underline{c}'_n$.
If $\underline{c}'_{n+1} \in \bigcap_{i=1}^{N_c} \mathcal{S}'_i$ then go to step 3 otherwise repeat step 2.
step 3. after convergence, $\underline{c}^{\mu} = \Gamma^{-1} \underline{c}'_{\infty}$.

The weights w_i^j must satisfy $\sum_{i,j} w_i^j = 1$. An obvious choice is $w_i^j = \frac{1}{2N_c}$, however, in our application it was observed that a much faster convergence is obtained by choosing
$w_i^j = \frac{\|\underline{c}'_n - P_i^j \underline{c}'_n\|^2}{\sum_{l,m} \|\underline{c}'_n - P_l^m \underline{c}'_n\|^2}$.

Numerical Example:
By way of illustration we consider an idealized optical storage channel according to the Braat-Hopkins model [7], where the optical channel cut-off frequency is $f_c = 1/(3T)$. Data a_k is taken to be run-length-limited with run-length parameters $(\mathrm{d}, \mathrm{k}) = (1, 7)$. The target response has 5 taps $g = [0.17, 0.5, 0.67, 0.5, 0.17]$. Fig. 3 shows the amplitude and group delay of a 7-tap equalizing interpolator for $\mu = 0.3$ and $\delta_g = 0.003$, i.e. 0.3% of the bit interval. The signal to noise ratio (SNR) is fixed to 15 dB. Compared to a 7-tap MMSE equalizer together with a 6-tap Lagrange interpolation [8], the equalizing interpolator has a negligible loss in MSE of only 0.05 dB. This means that the equalizer of Fig. 1 becomes superfluous.

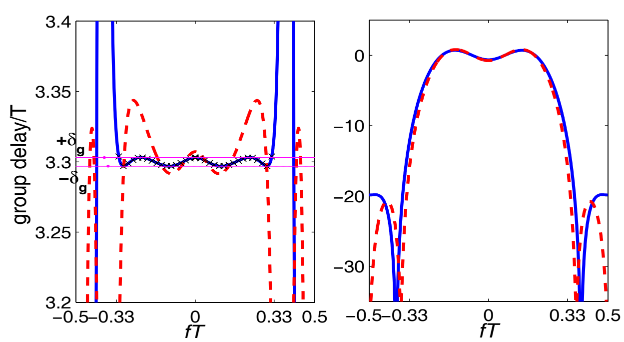

Fig. 3. left plot: group delay of the 7-tap equalizing interpolator for $\mu = 0.3$ and $\delta_g = 0.003$ (solid) and the 7-tap MMSE equalizer (dashed). The crosses denote the frequencies f_i. right plot: amplitude responses of the two equalizers.

3. EQUALIZING ANTI-ALIASING FILTERS

In many practical systems, the SRC filters are split into two filters, see Fig. 4. A first anti-aliasing filter p_n, of length L_p, rejects a specific frequency band in order to prevent noise and data aliasing. A second filter $c^{t_k^r}$ that depends on t_k^r resamples the filtered signal at the sampling instants t_k^r, provided by the timing-recovery subsystem. Such structure allows a relaxation on the stop-band constraints of $c^{t_k^r}$. This simplifies greatly the SRC. It is important to mention here that depending on the channel cut-off frequency and the oversampling rate $R = T/T_s$, $c^{t_k^r}$ can precede the filter p_n. The results of this section can be easily rewritten in this case. The SRC filter $c^{t_k^r}$ is implemented via a sample selector and an interpolation filter [2]. The interpolation filter can be designed as explained in the previous section. The filter p_n should then tackle all phase distortions and the remaining amplitude distortions, left by the equalizing interpolator, while providing enough attenuation at the stop-band. The MSE of a system

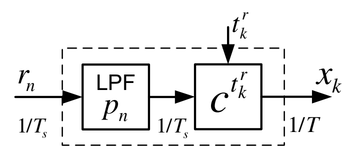

Fig. 4. practical implementation of SRC.

employing the SRC architecture of Fig. 4 can be found in [9]. This is given by

$$J(\underline{p}) = \underline{p}^T \mathbf{Q}_R \underline{p} - 2\underline{p}^T \underline{v}_R + \underline{g}^T \mathbf{R_a} \underline{g}, \qquad (7)$$

where $\underline{p} = [p_0, ..., p_{L_p-1}]^T$, the matrix $\mathbf{Q}_R$ and the vector $\underline{v}_R$ depend on the oversampling rate R and are given by

$$\mathbf{Q}_R = \mathbf{F}\mathbf{R}_a\mathbf{F}^{\mathrm{T}} + \mathbf{C}\mathbf{R}_u\mathbf{C}^{\mathrm{T}}; \quad \underline{v}_R = \mathbf{F}\mathbf{R}_a\mathbf{g},$$

where the matrix $\mathbf{F}$ has entries $\mathbf{F}_{p,q} = \sum_n c(nT_s - pT_s)h(qT - nT_s)$, $\mathbf{C}$ is given by $\mathbf{C}_{p,q} = c(-pT_s - qT_s)$ and the autocorrelation matrices of the input data and noise are denoted by $\mathbf{R}_a$ and $\mathbf{R}_u$ respectively. $c(t)$ is the symbol response of the SRC interpolation filter. Similarly to Section 2, we introduce a finite frequency grid f_i, $i = 1...N_c$ in the stop-band, and constraint the filter p_n to meet

$$|P(e^{j2\pi f_i T_s})|^2 \leq \delta_a, \quad i = 1...N_c$$

where $P(e^{j2\pi f T_s}) = \sum_{n=0}^{L_p-1} p_n e^{-j2\pi n f T_s}$ and $10\log(\delta_a)$ is the desired stop-band attenuation. This amplitude constraint can be written as $\underline{p}^{\mathrm{T}} \underline{m}_i \underline{m}_i^{\mathrm{H}} \underline{p} \leq \delta_a$ where
$\underline{m}_i = [1, e^{-j2\pi f_i T_s}, \ldots, e^{-j2\pi(L_p-1)f_i T_s}]^{\mathrm{T}}$ and $[.]^{\mathrm{H}}$ denotes transpose conjugate. The optimization problem related to $\underline{p}$ can be formulated as

$$\underline{p} = \arg \min_{\forall i \;\; F_i(\underline{p}) \leq 0} J(\underline{p}), \qquad (8)$$

where $F_i(\underline{p}) = \underline{p}^{\mathrm{T}} \underline{m}_i \underline{m}_i^{\mathrm{H}} \underline{p} - \delta_a$. The equalizing anti-aliasing filters problem can now be stated in terms of minimizing the quadratic function $J(\underline{p})$ subject to the inequalities constraints $F_i(\underline{p}) \leq 0$ where $F_i(\underline{p})$ are real differentiable and convex functions because the real matrices $\underline{m}_i\underline{m}_i^{\mathrm{H}}$ are positive. Because the function $J(\underline{p})$ is also convex, we know that if a solution of (8) exists than it is unique and it is characterized by the Kuhn-Tucker (KT) conditions [10]. However, solving the KT conditions can be quite complex in general. For this purpose we propose to use the Uzawa algorithm [11] which is an iterative method allowing one to solve an inequality constrained minimization problem, of a structure as in (8) by replacing it with a sequence of unconstrained minimization problems. If we denote the Lagrangian $\mathcal{L}(\underline{p}, \underline{\lambda}) = J(\underline{p}) + \sum_{i=1}^{N_c} \lambda_i F_i(\underline{p})$, the Uzawa algorithm in our context is written as:

$$\mathcal{L}(\underline{p}^{(n)}, \underline{\lambda}^{(n)}) = \min_{\underline{p}} \mathcal{L}(\underline{p}, \underline{\lambda}^{(n)}) \tag{9}$$

$$\forall i \qquad \lambda_i^{(n+1)} = \max(0, \lambda_i^{(n)} + \eta F_i(\underline{p}^{(n)})), \tag{10}$$

where $\eta > 0$ is a fixed adaptation constant and the superscript (n) indicates the n^{th} iteration. Equation (10) ensures that the Lagrange multipliers are always positive. The unconstrained minimization in (9) yields a simple linear system. In facts, it can be easily shown that (9) is equivalent to

$$\underline{p}^{(n)} = \left(\mathbf{Q}_R + \sum_{i=1}^{N_c} \lambda_i^{(n)} \underline{m}_i \underline{m}_i^{\mathrm{H}} \right)^{-1} \underline{v}_R. \tag{11}$$

Initially we set $\lambda_i^{(0)} = 0$ and $\underline{p}^{(0)} = \mathbf{Q}_R^{-1}\underline{v}_R$ (the MMSE equalizer). At every iteration, we apply (11) and (10) and check if $F_i(\underline{p}^{(n)}) \leq 0$, $\forall i$. The algorithm is stopped if this latter condition is met.

Numerical Example:
Using the same channel as in the example of Section 2 at an oversampling rate R=1.25, Fig. 5 shows the amplitude response of a 9-tap filter p_n for an attenuation of -35 dB in $[0.3/T_s, 0.5/T_s]$. The MSE difference between the filter p_n and the MMSE equalizer is less than 0.1 dB. This shows that the equalization burden can be shifted towards the SRC which means that the equalizer of Fig. 1 can be omitted.

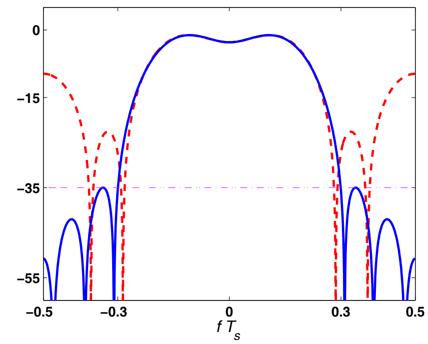

Fig. 5. The Amplitude response of the 9-tap filter p_n (solid) and the 9-tap MMSE equalizer (dashed).

4. CONCLUSIONS

In order to limit overall complexity of receivers for storage systems, sampling-rate converter filters can be designed to perform channel equalization. This paper presents design methods that combine group delay flatness and out-of-band rejection criteria, required for sampling-rate converter filters, together with minimum mean square error equalization. This approach and the corresponding design methods are validated for an idealized optical storage system.

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