SENSITIVITY OF HYBRID FILTER BANKS A/D CONVERTERS TO ANALOG REALIZATION ERRORS AND FINITE WORD LENGTH

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ABSTRACT

This paper studies the sensitivity of hybrid filter banks (HFB) to analog inaccuracies and finite word implementation. It is shown that very small errors affecting very simple analog structures have a dramatic influence on the performances of the HFB. The influence of the quantization of digital filter coefficients is also studied. A theoretical limit for the error introduced by the quantization of digital filter coefficients is derived.

1. INTRODUCTION

In modern communications a trend towards larger and larger bandwidths together with higher and higher working frequencies is obvious. But analog to digital (A/D) conversion of high frequency, wide band signals still creates bottlenecks. In the last 20 years many appropriate architectures for fast A/D conversion have been proposed. Among them, time interleaved structures were mostly studied and the most widely used by different A/D converters manufacturers. Unfortunately, the resolution of time-interleaved A/D converters is limited by the mismatches between converters and clock timing errors. Hybrid Filter Banks converters (as shown in Figure 1) are another option for fast A/D conversion. Such an option overcomes some of the drawbacks of the time interleaved architectures. Velazquez [1] has shown that HFB attenuate branch ADC mismatches and timing errors. HFB A/D converter architecture has been proposed by Petraglia and Velazquez [2], [3]. An intermediary architecture between hybrid filter banks and all-digital filter banks has analog filters using a switched-capacitor technique (e.g. [2]). This combines the advantages of using the well-established digital filter bank perfect reconstruction theory and the existence of analog analysis filters. However, the switched-capacitor A/D converters introduce severe speed limitations. The latter can be overcome by HFB converters.

Different methods of designing HFB have been proposed [3], [4], [5], [6], [7], [8] or [9]. However, a problem that has not been sufficiently examined is the HFB sensitivity to analog filter realization errors and to digital filters coefficient quantization. It is well known that large tolerances in the manufacturing process of analog devices are a serious problem in analog systems design. This paper studies the deterioration of HFB performances in the presence of analog and digital errors. Section 2 briefly describes the HFB to be studied. In section 3.1 performance deterioration due to analog errors is investigated and the effects of quantizing the coefficients of the digital synthesis filters are studied in section 3.2. Finally, conclusions are presented in 4. Examples and simulation results are provided all along the paper.

2. PERFECT RECONSTRUCTION CONDITIONS

A block diagram of the HFB to be analyzed is shown in Figure 1. $x(t)$ represents the input analog signal. It is supposed to be band limited to π/T by an external low-pass filter. $H_m(s)$, $m \in \{1, 2, ..., M\}$ are the continuous-time analysis filters and $F_m(z)$, $m \in \{1, 2, ..., M\}$ are the discrete-time synthesis filters. The blocks $q_1, ..., q_M$ are the branch quantizers. In this paper, quantization noise will not be considered. After the analog filtering, the branch signals $x_m(t), m \in \{1, 2, ..., M\}$ are sampled at the rate of $2\pi/MT$. The Fourier transform of $y(n)$ is [3], [10]:

$$Y(e^{j\omega}) = \frac{1}{T} \sum_{p=-\infty}^{\infty} X(j\Omega - j\frac{2\pi p}{MT}) T_p(e^{j\omega}), \qquad (1)$$

where

$$T_p(e^{j\omega}) = \frac{1}{M} \sum_{m=1}^{M} F_m(e^{j\omega}) H_m(j\Omega - j\frac{2\pi p}{MT}), \qquad (2)$$

and $\omega = \Omega T$. Since $x(t)$ is bandlimited to π/T, it can be shown that the sum in (1) has only $2M - 1$ non zero terms when $-\pi < \omega \leq \pi$ [10]. T_p for $p \in \{-(M - 1), ..., -1, 1, ..., M - 1\}$ are the aliasing functions . T_0 gives the filter bank frequency response if T_p for $p \neq 0$ are can-

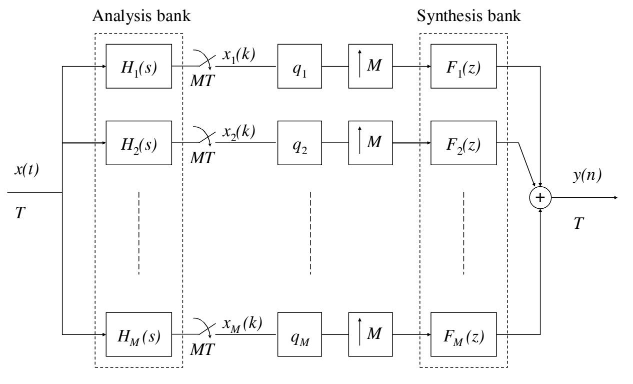

Fig. 1. Hybrid filter bank

celled and is called the distortion function. Perfect reconstruction means the output is only a scaled, delayed and sampled version of the input:

$$T_p(e^{j\omega}) = \begin{cases} ce^{-j\omega d}, & p = 0 \\ 0, & p \in \{-(M-1), ... -1, 1, ..., (M-1)\}. \end{cases} \quad (3)$$

$d \in \mathbb{R}, d > 0$ is the filter bank's delay and $c \in \mathbb{R}$ is a scale factor.

It must be emphasized that the aliasing errors which are predominant, limit the spurious free dynamic range (SFDR) of the ADC converter built in HFB configuration. Assuming, for instance, the classical 6 dB/bit scheme, the maximum expected resolution cannot be higher than $\max_{p,\omega} |T_p(e^{j\omega})|_{\text{dB}}/6$.

3. SENSITIVITY

3.1. Effects of analog components realization errors

This section studies one of the most important drawbacks of implementing A/D converters in HFB structure: the degradation of the HFB performances in the presence of analog components realization errors. Indeed, analog tolerances being non zero, the implemented transfer functions of the analog filters may be quite different from the theoretical, wanted transfer functions. As a result, the perfect reconstruction conditions that were well approximated by the desired transfer functions are no longer satisfied by those obtained in the manufacturing process.

In order to study the influence of realization errors, a realistic example has been chosen. The range of possible choices for the analog filters is small [7], [8] since only simple structures may be implemented in high frequency analog electronics. For example, basic resonators were chosen as analysis filters:

$$H_m(s) = \frac{\frac{\Omega_m}{Q_m}s}{s^2 + \frac{\Omega_m}{Q_m}s + \Omega_m^2}, m \in \{2, ..., M\}. \quad (4)$$

This corresponds to a parallel LC structure with Ω_m being the natural frequency of the resonator and Q_m the quality factor. For $H_1(s)$, low pass filters are usually considered. Simple RC circuits were used:

$$H_1(s) = \frac{\Omega_1}{s + \Omega_1},\ \Omega_1 = \frac{1}{R_1 C_1}, \quad (5)$$

where Ω_1 is the cut-off frequency of the filter. Taking into account realization errors in the previous case, (4) and (5) become:

$$H_m(s) = \frac{\frac{\Omega_m}{Q_m}(1+\Delta_{1m})s}{s^2 + \frac{\Omega_m}{Q_m}(1+\Delta_{1m})s + \Omega_m^2(1+\Delta_{2m})}, \quad (6)$$
$$m \in \{2, ..., M\},$$

$$H_1(s) = \frac{\Omega_1(1+\Delta_{11})}{s + \Omega_1(1+\Delta_{11})}. \quad (7)$$

In (6) and (7) Δ_{1m} and Δ_{11} are the relative errors of the analog filters coefficients:

$$\Delta_{1m} = \frac{\varepsilon_{R_m}\varepsilon_{C_m} + \varepsilon_{C_m} + \varepsilon_{R_m}}{1 + \varepsilon_{R_m}\varepsilon_{C_m} + \varepsilon_{C_m} + \varepsilon_{R_m}}, \quad (8)$$

$$\Delta_{2m} = \frac{\varepsilon_{L_m}\varepsilon_{C_m} + \varepsilon_{C_m} + \varepsilon_{L_m}}{1 + \varepsilon_{L_m}\varepsilon_{C_m} + \varepsilon_{C_m} + \varepsilon_{L_m}}, \quad (9)$$

where $m \in \{1, ..., M\}$ and $\varepsilon_{R_m}, \varepsilon_{C_m}, \varepsilon_{L_m}$ are the relative realization errors of the analog components of the resonators. In order to evaluate the error effects on the distortion and aliasing functions, Monte Carlo simulation with 1000 trials was performed using (8) and (9) to compute analog filters coefficients errors. Different error values for resistors and capacitors were taken. A four channel hybrid filter bank was considered with 128-length FIR synthesis filters. The results are summarized in Table 1. Figure 2 presents a comparison between the magnitudes of the distortion and aliasing functions for the three cases considered: in Case 1 there are no analog errors. In Case 2 a 1 percent error for the resistors and 0.5 percent error for the capacitors were considered. In Case 3, the resistors were affected by a 5 percent error whereas the capacitors were affected by a 1 percent error.

Other filter structures were studied. Butterworth filters were also considered since they are used as analysis filters in some previous HFB designs [4]. Third order Butterworth filters were taken into account for this study. Again, a Monte Carlo simulation with 1000 trials was performed for a four channel hybrid filter bank with 128-length FIR synthesis filters. The results are shown in table 2.

Only very small errors on analog components generate very significant errors on the distortion and aliasing functions. Such errors could mean that HFB might be impossible to implement in the real world. It is obvious that some kind of calibration of the synthesis filters taking into account the real values of the analysis filters (as obtained after physical implementation) must be done. This will make the object of future publications.

Table 1. HFB performances in the presence of analog realization errors for an analysis bank using resonators

Analog errors	Mean aliasing (dB)	Peak aliasing (dB)	Mean distortion (dB)	Peak distortion (dB)
No analog errors	-151	-126	$1.7 \cdot 10^{-9}$	$1.2 \cdot 10^{-6}$
$\varepsilon_{R_m} = 0.01$ $\varepsilon_{L_m} = 0.01$ $\varepsilon_{C_m} = 0.005$	-48	-41	0.0004	0.04
$\varepsilon_{R_m} = 0.01$ $\varepsilon_{L_m} = 0.01$ $\varepsilon_{C_m} = 0.01$	-45	-38	0.0004	0.05
$\varepsilon_{R_m} = 0.05$ $\varepsilon_{L_m} = 0.05$ $\varepsilon_{C_m} = 0.01$	-35	-29	0.02	0.2

Table 2. HFB performances in the presence of analog realization errors for Butterworth analysis filters

Analog errors	Mean aliasing (dB)	Peak aliasing (dB)	Mean distortion (dB)	Peak distortion (dB)
No analog errors	-102	-80	$4.6 \cdot 10^{-7}$	$3.3 \cdot 10^{-4}$
$\varepsilon_{R_m} = 0.01$ $\varepsilon_{L_m} = 0.01$ $\varepsilon_{C_m} = 0.005$	-53	-37	0.0005	0.05
$\varepsilon_{R_m} = 0.01$ $\varepsilon_{L_m} = 0.01$ $\varepsilon_{C_m} = 0.01$	-51	-35	0.001	0.07
$\varepsilon_{R_m} = 0.05$ $\varepsilon_{L_m} = 0.05$ $\varepsilon_{C_m} = 0.01$	-40	-24	0.01	0.2

3.2. Effects of digital filters coefficient quantization

In this section it is assumed that the FIR filters are implemented in direct form structure. It is also assumed that filter multiplications are done with higher internal precision and that only the output of each digital filter is quantized. A fixed arithmetic representation for filter coefficients is supposed to be used and no analog realization errors are taken into account. The error which the quantization of the coefficients produces in the transfer function can be easily evaluated. If

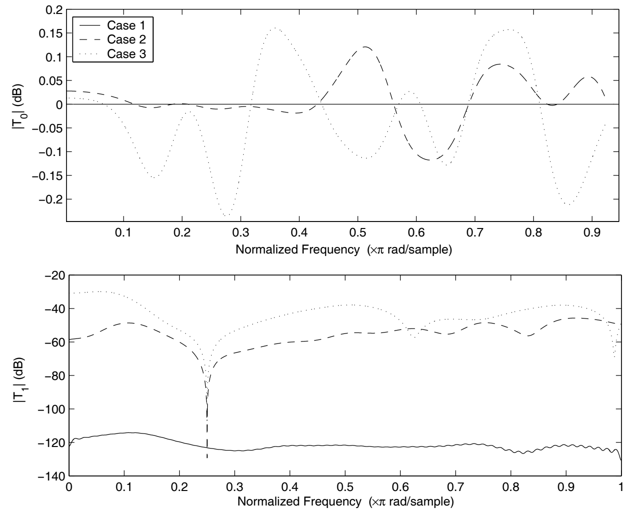

Fig. 2. Comparison between the magnitudes of the distortion (high) and aliasing (low) functions in three cases: Case 1 - No analog errors; Case 2 - $\varepsilon_{R_m} = \varepsilon_{L_m} = 0.01, \varepsilon_{C_m} = 0.005$; Case 3 - $\varepsilon_{R_m} = \varepsilon_{L_m} = 0.05, \varepsilon_{C_m} = 0.01$

the desired transfer function is:

$$F_m(e^{j\omega}) = \sum_{n=0}^{N-1} f_m(n) e^{-j\omega n},\ m \in \{1, 2, ..., M\} \quad (10)$$

then the implemented one is:

$$\widetilde{F}_m(e^{j\omega}) = \sum_{n=0}^{N-1} \widetilde{f}_m(n) e^{-j\omega n} \quad (11)$$

the error introduced by coefficient quantization is:

$$E_m(e^{j\omega}) = \widetilde{F}_m(e^{j\omega}) - F_m(e^{j\omega}) = \sum_{n=0}^{N-1} \epsilon_m(n) e^{-jn\omega}, \quad (12)$$

where $\epsilon_m(n) = \widetilde{f}_m(n) - f_m(n)$. If a rounding quantization is used, then $\epsilon_m(n) \in \left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right)$, with $\Delta = 2^{-B}$ and B is the length of the digital word and:

$$\left|E_m(e^{j\omega})\right| \leq \sum_{n=0}^{N-1} |\epsilon_m(n)| = N\frac{\Delta}{2}. \quad (13)$$

Using (2) and (12), the implemented aliasing and distortion functions are:

$$\widetilde{T}_p(e^{j\omega}) = T_p(e^{j\omega}) + \frac{1}{M} \sum_{m=1}^{M} E_m(e^{j\omega}) H_m(j\Omega - j\frac{2\pi p}{MT}), \quad (14)$$

$$p \in \{-(M-1), ... -1, 0, 1, ..., (M-1)\}.$$

The error appearing in the distortion and aliasing functions can be written as:

$$E_{T_p}(e^{j\omega}) = \widetilde{T}_p(e^{j\omega}) - T_p(e^{j\omega}). \quad (15)$$

Table 3. HFB performances in the presence of coefficient quantization

	Mean aliasing (dB)	Peak aliasing (dB)	Mean distortion (dB)	Peak distortion (dB)
No quantization	-122	-112	10^{-8}	$1.3 \cdot 10^{-5}$
16 bits fixed point	-88	-78	$1.2 \cdot 10^{-5}$	$6 \cdot 10^{-4}$
32 bits fixed point	-122	-112	10^{-8}	$1.3 \cdot 10^{-5}$
32 bits floating point	-122	-112	10^{-8}	$1.3 \cdot 10^{-5}$

Let us assume $|H_m(e^{j\omega})| \leq 1$ and consider a uniform distribution of the analog filters response in the working frequency band (e.g. resonators with equally spaced natural frequencies). In the passband of each of the filters, the others present important attenuations. Hence:

$$\left|E_{T_p}(e^{j\omega})\right|_{max} \approx \frac{1}{M} \max_{m,\omega} \left|E_m(e^{j\omega})\right| = \frac{1}{M} N \frac{\Delta}{2}. \quad (16)$$

Equation (16) is useful after the design phase to decide the necessary number of bits for the FIR coefficients quantization. The error introduced by coefficients quantization (16) must be smaller than the aliasing and distortion errors obtained in the design process.

An eight-channel HFB was considered. Equally spaced, constant band resonators were used as analysis filters. Tests were made for fixed and floating point formats for coefficient quantization. No deterioration was noticed in the case of 32 bit floating point, as expected. When fixed point, 16-bit quantified coefficients are used, (16) results in a predicted error value of -78 dB in the aliasing functions. The same result was obtained in the simulation: the peak aliasing deteriorated from the initial -112 dB to -78 dB due to 16-bit quantization. The simulation results are summarized in Table 3.

4. CONCLUSIONS

In this paper, one of the most important challenges in designing HFB based ADC was identified: the dramatic performance degradation that could be seen in the presence of analog realization errors. Consequently, readjustment of synthesis filters coefficients after physical implementation of the analysis bank appears mandatory. The effects of digital filters coefficient quantization were also studied. A theoretical error as seen in the distortion and aliasing functions due to coefficient quantization was determined. The simulation confirmed the validity of this result.

5. REFERENCES

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