# A NEW TVAR MODELING IN CASCADED FORM FOR NONSTATIONARY SIGNALS

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## ABSTRACT

**In the nonstationary process identification, a time varying autoregressive (TVAR) model may possess temporal model instability in the conventional direct form. In this study, we propose a new TVAR cascaded form to overcome model instability. The model stability in TVAR the cascaded form is accomplished through the parameterization of time varying poles using pole tracking and monitoring methods. Our simulation on synthetic data demonstrates that the TVAR cascaded form stability can easily be achieved, monitored, and controlled. The performance evaluation of TVAR cascaded model has shown that the Cartesian coordinate with orthogonal representation performs better than other pole representation.**

## 1. INTRODUCTION

Most temporal signals such as speech and biomedical signals have time varying statistics and thus are nonstationary. Nonstationary signal analysis methods are categorized into nonparametric and parametric approaches. The nonparametric approaches are based on time-dependent spectral representations that include the short-time Fourier transform, and the time-frequency distribution [1,2]. The parametric approaches are based on time-varying linear predictive models, such as autoregressive models. The nonstationary process is represented using an autoregressive model with its parameters changing with time. It is often used to track fast time-varying signal dynamics, which is not possible with nonparametric approaches. TVAR model parameters can be estimated using gradient based adaptive algorithm and basis function methods [3]. The basis function method assumes that the parameter variations can be approximated by a linear combination of known basis functions, which allows relatively fast parameter evolution. The model parameters can be obtained either by the blockwise cascaded processing of all the data at one time or the recursive processing of each datum sequentially. Power time and Legendre polynomials basis function are considered in this study [4,5].

Because TVAR models may have temporal instability, the time-varying poles of the estimated model are not guaranteed to remain inside the unit circle in the $z$-plane. The root-finding algorithm factorizes the transfer function of the TVAR filter in the direct form to solve the current instability problem. Because it is computationally demanding it is practically inappropriate for real time processing. Moreover, TVAR direct form may not provide the most convenient information for some applications, while the poles of the system transfer function usually contain the physical information of the underlying process [7,8]. Possible temporal instability in the TVAR direct form is a major limitation and clipping technique or other methods [4,6] cannot completely eliminate these limitations of instability. These constrains are computationally expensive due to high nonlinear mapping of these constraints. To overcome these issues, we have explored time-invariant filters in cascaded form and the parameterization of the time-invariant model in terms of poles and zeros. We found that by using cascaded form, poles and zeros can be estimated directly from the data.

## 2. TVAR MODEL IN CASCADED FORM

The nonstationary process can be represented by a TVAR model [3] (Figure 1) using the following expression:

$$x(n) = -\sum_{i=1}^{p} c_i(n)x(n-i) + v(n) \tag{1}$$

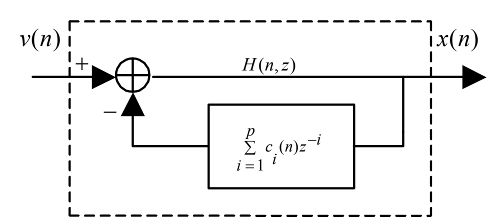


***Figure 1**: Time-varying AR model in direct form*

Where $c_i(n)$ are the TVAR model parameters, $p$ is the model order, $x(n)$ is the model output, and $v(n)$ is the input white Gaussian noise with zero mean and variance $\sigma_v^2$. The signal generating system is considered as a linear all-pole time varying filter with the transfer function in direct form expressed as follows:

$$H(n,z)=\frac{1}{1+\sum_{i=1}^{p}c_i(n)z^{-i}}=\frac{1}{P(n,z)} \quad (2)$$

Where $H(n,z)$ has all the poles within the unit circle on the $z$-plane to guarantee its stability. To allow the pole locations to be readily estimated and constrained, the TVAR model is formulated in cascaded form (Figure 2). The time-varying transfer function $H(n,z)$ is the product of cascaded sections as follows:

$$H(n,z)=\frac{1}{\prod_{k=1}^{p/2}(1+p_k^1(n)z^{-1}+p_k^2(n)z^{-2})}=\frac{1}{1+\sum_{i=1}^{p}c_i(n)z^{-i}} \quad (3)$$

The direct form parameters can be calculated through a multiple convolution of cascaded form parameters by the following equation:

$$\{P(n)\}=\underset{k=1}{\overset{p/2}{Conv}}\{P_k(n)\} \quad (4)$$

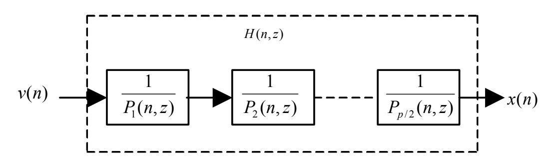


*Figure 2: Time-varying AR model in cascaded form.*

When each cascaded section consists of a time-varying conjugate complex pole pair $\{z_k(n),z_k^*(n)\}$, the transfer function denominator in each section becomes:

$$P_k(n,z)=1-2\,\mathrm{Re}[z_k(n)]z^{-1}+|z_k(n)|^2\,z^{-2}$$
$$1\le k\le p/2, n=1,\ldots,N \quad (5)$$

In the cascaded process, each pole pair may be represented in form of second-order section. When the time variation of $\omega_k(n)$ is the time-varying pole in the $k^{th}$ cascaded section, then it is the linear combination of time functions as follows:

$$\omega_k(n)=\sum_{j=0}^{q}\omega_{kj}f_j(n) \quad (6)$$

Where $\{f_j(n), 0\le j\le q\}$ is a set of basis functions and $\{\omega_{kj}\}$ is a set of basis coefficients. The second-order section coefficients are related to each time-varying complex pole pair as follows:

$$\tilde{c}1_k(n)=-2\,\mathrm{Re}(z_k(n)),\qquad \tilde{c}2_k(n)=|z_k(n)|^2$$
$$1\le k\le p/2, n=1,\ldots,N \quad (7)$$

Where $\tilde{c}1_k(n)=\sum_{j=0}^{q}\tilde{c}1_{kj}f_j(n)$ and $\tilde{c}2_k(n)=\sum_{j=0}^{q}\tilde{c}2_{kj}f_j(n)$.

In the Cartesian coordinates the time-varying complex pole pair can be represented as follows:

$$\{z_k(n),z_k^*(n)\}=dx_k(n)\pm jdy_k(n),\ 1\le k\le p/2, n=1,\ldots,N \quad (8)$$

Where $dx_k(n)=\sum_{j=0}^{q}dx_{kj}f_j(n)$ and $dy_k(n)=\sum_{j=0}^{q}dy_{kj}f_j(n)$.

In polar coordinates, each time-varying complex pole pair is represented as follows:

$$\{z_k(n),z_k^*(n)\}=r_k(n)(\cos\theta_k(n)\pm j\sin\theta_k(n)),$$
$$1\le k\le p/2, n=1,\ldots,N \quad (9)$$

Where $r_k(n)=\sum_{j=0}^{q}r_{kj}f_j(n)$ and $\theta_k(n)=\sum_{j=0}^{q}\theta_{kj}f_j(n)$.

Using the TVAR cascaded form, the time-varying autoregressive process can be considered as the time-varying linear prediction in cascaded form (Figure3). The prediction error $\varepsilon(n)$ can be expressed in $z$-domain as follows [8]:

$$E(n,z)=X(z)\prod_{k=1}^{p/2}P_k(n,z) \quad (10)$$

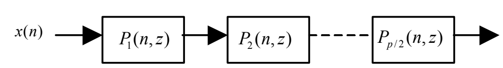


***Figure 3**: Time-varying linear prediction in cascaded form*

The system equations for parameter estimation are obtained by minimizing the squared prediction error $\xi=\sum_n\varepsilon^2(n)$ with respect to each basis coefficient $\omega_{kj}$ as:

$$\frac{\partial\xi}{\partial\omega_{kj}}=2\sum_n\varepsilon(n)\frac{\partial\varepsilon(n)}{\partial\omega_{kj}}=0,\quad 1\le k\le p/2, 0\le j\le q \quad (11)$$

The error gradient component and its corresponding $z$-domain representation are denoted as follows:

$$g_{\omega_{kj}}(n)=\frac{\partial\varepsilon(n)}{\partial\omega_{kj}},\ G_{\omega_{kj}}(n,z)=\frac{\partial E(n,z)}{\partial\omega_{kj}} \quad (12)$$

The gradient component can be calculated from the partial derivative of prediction error with respect to $\omega_{kj}$ as follows:

$$G_{\omega_{kj}}(n,z)=X(z)\prod_{\substack{m=1\\m\ne k}}^{p/2}P_m(n,z)\cdot\frac{\partial P_k(n,z)}{\partial\omega_{kj}} \quad (13a)$$

It can be represented by a linear combination of the input samples $[x(n-1),\ldots,x(n-p)]$ as:

$$g_{\omega_{kj}}(n)=\sum_{i=1}^{p}w_{i,\omega_{kj}}(n)x(n-i) \quad (13b)$$

Where $\{w_{i,\omega_{kj}}(n)\}=\frac{\partial\{P_k(n)\}}{\partial\omega_{kj}}*\underset{m=1,m\ne k}{\overset{p/2}{Conv}}\{P_m(n)\}$.

The specific equations pole representations are given as follows:

$$\{w_{i,\tilde{c}1_{kj}}(n)\} = \{0,1,0\} * \underset{m=1, m\neq k}{\overset{p/2}{Conv}} \{P_m(n)\} \cdot f_j(n)$$
$$\{w_{i,\tilde{c}2_{kj}}(n)\} = \{0,0,1\} * \underset{m=1, m\neq k}{\overset{p/2}{Conv}} \{P_m(n)\} \cdot f_j(n) \quad (14)$$

If $\Omega$ is a basis coefficient vector, then the corresponding error gradient vector can be expressed as:

$$\Psi_\Omega = \frac{\partial \varepsilon(n)}{\partial \Omega} \quad (15)$$

The system equations (11) can be written in vector form as:

$$\sum_n \varepsilon(n)\Psi_\Omega = 0 \quad (16)$$

Due to the gradient generating process in cascaded form, the gradient vector $\Psi_\Omega$ depends on $\Omega$. The system equations for the time-varying autoregressive model in cascaded form are nonlinear with respect to basis coefficients. To solve the set of nonlinear system equations, the Gauss-Newton algorithm [11] is used, where the minimization of the prediction error is obtained by performing searches in the Newton direction using the error gradient and the inverse of the estimated Hessian matrix. The coefficient's estimate is updated as follows [12]:

$$\Omega_n = \Omega_{n-1} - \gamma \tilde{P}_n \Psi_{\Omega_n} \varepsilon(n) \quad (17)$$

Where $\gamma$ is the step size to control the convergence rate and $\tilde{P}_n$ is the inverse of the estimated Hessian matrix, updated using Gauss-Newton algorithm, which is expressed as [13]:

$$\tilde{P}_n = \frac{1}{1-\gamma}\left[\tilde{P}_{n-1} - \frac{\tilde{P}_{n-1}\Psi_{\Omega_n}\Psi_{\Omega_n}^T \tilde{P}_{n-1}}{\frac{1-\gamma}{\gamma} + \Psi_{\Omega_n}^T \tilde{P}_{n-1} \Psi_{\Omega_n}^T}\right] \quad (18)$$

### 3. PERFORMANCE EVALUATION

Effective model-based linear prediction, spectral analysis and frequency estimations are the goals of investigating TVAR cascaded model. Thus, a TVAR quality indicator average prediction gain can be expressed as:

$$\overline{PG} = 10\log_{10}\left(\frac{\overline{\sigma}_x^2}{\overline{\sigma}_\varepsilon^2}\right) \quad (19)$$

Where $\overline{\sigma}_x^2 = \mathrm{E}\left[\sum_{n=1}^{N} |x(n)|^2\right]$ and $\overline{\sigma}_\varepsilon^2 = \mathrm{E}\left[\sum_{n=1}^{N} |\varepsilon(n)|^2\right]$. This criterion shows the goodness of fit between the predicted and the true signal. The expected path of the frequency estimates is denoted as $\bar{f}(n) = \mathrm{E}[\hat{f}(n)]$ to give a general trend of frequency variation. In addition, the average estimated time varying pole trajectories are shown to give a qualitative view of dynamic system behavior. Only the single pole pair case is emphasized in this paper. For the single pole pair case, the synthetic data set of length $N = 256$ is generated as the output of a second-order all-pole time varying filter with the model parameter $c_1(n) = -2\cos[2\pi f(n)]$, and $c_2(n) = 1$, where the normalized frequency is as follows:

$$f(n) = \begin{cases} 0.1 + 0.2\dfrac{n}{N}, & 1 \le n \le 128 \\ 0.3 + 0.3\left(\dfrac{n-128}{N}\right)^2, & 129 \le n \le 256 \end{cases} \quad (20)$$

Figures 4 (a) and (b) show the average estimated pole trajectories of the TVAR model in direct form (blockwise) and cascaded form (Cartesian coordinate) ($q = 4$). In Figure 4 (a), the estimated pole trajectory using the direct form model matches the general trend of the true trajectory, while some estimated poles leave the unit circle at the end of analysis interval due to the estimation error. In Figure 4 (b), the estimated pole moves along the true trajectory and follows the abrupt change. Because of each basis coefficients update, all the estimated time varying poles remain inside the unit circle.

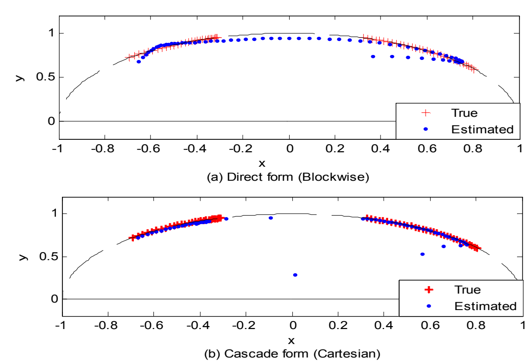


***Figure 4**: The average estimated pole trajectories of the TVAR model in direct form and cascaded form*

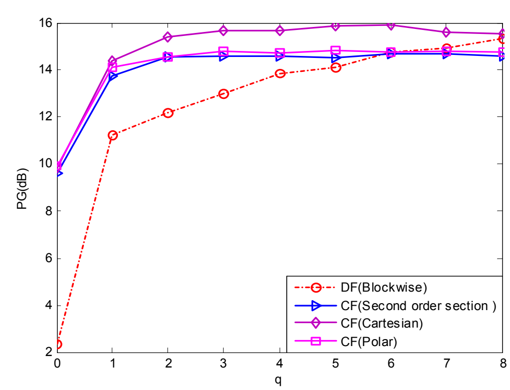


***Figure 5**: $\overline{PG}$ patterns for direct form (blockwise) and cascaded form with various basis dimensions ( q = 0 to 8).*

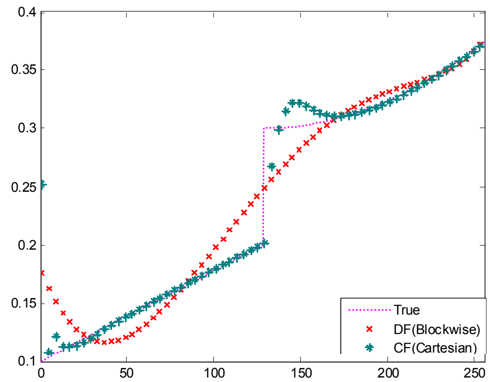


***Figure 6**: The expected path of frequency estimates $\bar{f}(n)$ via the TVAR modeling in direct form and cascaded form.*

Figure 5 compares the $\overline{PG}$ with the TVAR in direct form (blockwise) at three different forms of pole representation in cascaded form with various basis dimensions ($q = 0$ to 8). The basis dimension ($q = 4$) appears to be enough for the cascaded form to achieve suitable tracking performance, while the direct form needs a relatively large basis dimension to obtain good matching performance. The time varying pole representation in Cartesian coordinate shows its superior performance with the $\overline{PG}$ about 1dB higher than those of other pole representations. The orthogonality between the real and imaginary parts of the Cartesian pole representation makes it more robust for error estimation. When there is a small error in one direction, it will have negligible effect on the other direction, and thus good pole estimate can still be obtained. Although the radius and angle of the ($r,\theta$) pole representation are orthogonal, it is sensitive to deviations of angle when the radius is large. A small error in the angle estimate may bring the estimated pole far from its true position.

Figure 6 shows the expected path of the frequency estimate $\bar{f}(n)$ with the TVAR model in direct form (blockwise) and cascaded form (Cartesian coordinate) with a basis dimension ($q = 4$), where the true frequency trajectory is also shown for comparison. The direct form (blockwise) works as frequency matching, with the global optimization over the whole block of data. The cascaded form (Cartesian) works as frequency tracking, with the local approximation adjusted upon each data sequentially. For an abrupt parameter change, the direct form only gives a smooth approximation while the cascaded form can catch up with the new trend with a small overshoot.

## 4. CONCLUSION

The TVAR cascaded form is presented in this paper in terms of time varying pole representations. During identification process, it is shown that the estimated poles can be easily constrained in the unit circle, which guarantees the stability of the model. This new TVAR cascaded form with pole representation offers a robust approximation of time variations in frequencies and outperforms the conventional pole representation.

## 5. REFERENCES

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