DISCRETE TIME-FREQUENCY MODELS OF GENERALIZED DISPERSIVE SYSTEMS

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ABSTRACT

In this paper, we propose a discrete characterization of dispersive time-varying systems. Based on a unitary warping relation with the narrowband system model, we formulate a representation that decomposes the system output as a weighted superposition of sampled signal transformations that reflect the dispersive system characteristics. Such discrete representations can be important in designing waveforms for transmission through dispersive systems with improved processing performance. We demonstrate the usefulness of our proposed model by applying it to a shallow water acoustic environment characterized by nonlinear frequency dispersions.

1. INTRODUCTION

In nature, there exist many time-varying systems that are characterized by dispersive signal transformations where different frequencies are shifted in time by different amounts. For example, in shallow water acoustic environments, the transmitted waveform undergoes dense multipath delays and severe modal dispersions as a result of different frequency components traveling at different group velocities [1]. In such scenarios, the output signal can be modeled as a superposition of these transformations, weighted by the dispersive spreading function (DSF) [2]. This representation provides a measure in the time-frequency plane of the spread that is caused by these (often nonlinear) signal transformations.

Similar characterization models for narrowband systems have been discretized to decompose the narrowband spreading function representation into a weighted summation of sampled time-frequency shifts [3]. This signal-dependent discrete model has proven useful in many applications; it has been used to exploit an inherent joint multipath-Doppler diversity [3] and to eliminate the need for (nonlinear) estimation of actual system parameters [4]. Such a discrete model has not been developed for dispersive systems although it can be important, for example, in designing transmission waveforms to optimize processing performance.

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In this paper, we propose a discrete representation of dispersive time-varying systems in terms of sampled dispersive frequency shifts and generalized time shifts, weighted by a smoothed discrete version of the DSF. These signal transformations are specific to the dispersive nature of the system that is characterized by the nonlinear function $\xi(b)$. Our discretization model is based on exploiting a unitary warping relation between narrowband and dispersive system characterizations. The approach can be associated with the transmitted signal's finite support in transform domains matched to the dispersion characteristics that depend on $\xi(b)$. In order to demonstrate the importance of the discrete generalized model, we consider the dispersive environment for shallow water acoustic transmission.

The paper is organized as follows. In Section 2, we review the discrete narrowband system characterization. We then discuss dispersive systems and their warping relation to narrowband systems in Section 3. In Section 4, we present our discretization approach with a specific example in Section 5.

2. NARROWBAND SYSTEM CHARACTERIZATION

In this section, we review the discrete representation of narrowband linear time-varying (LTV) systems as it provides the underlying framework for deriving the discrete representation for dispersive systems.

A narrowband LTV system $\mathcal{L}$ can be characterized using the spreading function (SF). Specifically, the system output $(\mathcal{L}x)(t)$ can be interpreted as a weighted superposition of time-frequency shifted versions of the input signal $x(t)$, i.e.,

$$(\mathcal{L}x)(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{SF}_{\mathcal{L}}(\tau,\nu)\mathrm{e}^{-j\pi\tau\nu}(\mathcal{M}_\nu \mathcal{S}_\tau x)(t)d\nu d\tau\ . \quad (1)$$

The weighting function $\mathrm{SF}_{\mathcal{L}}(\tau,\nu)$ quantifies the scattering strength at time shift τ and frequency shift ν. Here, the time-shift and frequency-shift operations are defined as $(\mathcal{S}_\tau x)(t) = x(t-\tau)$ and $(\mathcal{M}_\nu x)(t) = x(t)\,\mathrm{e}^{j2\pi\nu t}$, respectively.

Based on signal time and frequency support constraints, a discrete form of the narrowband LTV system in (1) can be obtained in terms of sampled time and frequency shifts. Specifically, if $x(t)$ is bandlimited to $[f_0, f_1]$, with bandwidth $W = f_1 - f_0$, and $(\mathcal{L}x)(t)$ is time-limited to $[t_0, t_1]$, with duration $T = t_1 - t_0$, then (1) can be written as [3],

$$(\mathcal{L}x)(t) \quad = \quad \sum_{l\in\mathbb{Z}}\sum_{k\in\mathbb{Z}} \widehat{\mathrm{SF}}_{\mathcal{L}}(\frac{l}{W},\frac{k}{T})\,(\mathcal{M}_{\frac{k}{T}}\mathcal{S}_{\frac{l}{W}}x)(t) \qquad (2)$$

where $\widehat{\mathrm{SF}}_{\mathcal{L}}(\frac{l}{W},\frac{k}{T})$ are two-dimensional (2-D) samples of the smoothed SF

$$\widehat{\mathrm{SF}}_{\mathcal{L}}(\tau,\nu) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{SF}_{\mathcal{L}}(\tilde{\tau},\tilde{\nu})\mathrm{e}^{-j\pi\tilde{\nu}\tilde{\tau}}\mathrm{e}^{-j\pi(\nu-\tilde{\nu})(t_0+t_1)}$$
$$\cdot\, \mathrm{e}^{j\pi(\tau-\tilde{\tau})(f_0+f_1)}\mathrm{sinc}((\nu-\tilde{\nu})T)\,\mathrm{sinc}((\tau-\tilde{\tau})W)d\tilde{\nu}d\tilde{\tau}, \quad (3)$$

sampled at the uniform grid $\tau = \frac{l}{W}$ and $\nu = \frac{k}{T}$. Here, $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. Furthermore, if $\mathrm{SF}_{\mathcal{L}}(\tau,\nu)$ is nonzero only when $(\tau,\nu) \in [\tau_0,\tau_1]\times[\nu_0,\nu_1]$, the summation in (2) can be truncated by choosing[1] $\lfloor\tau_0 W\rfloor \le l \le \lceil\tau_1 W\rceil$ and $\lfloor\nu_0 T\rfloor \le k \le \lceil\nu_1 T\rceil$.

3. DISPERSIVE SYSTEM CHARACTERIZATION

3.1. Dispersive Spreading Function

When a signal undergoes dispersive transformations while propagating through a medium, the SF will not, in general, be the appropriate analysis tool. To accurately characterize a dispersive system, it is important to incorporate its dispersive characteristics into the system representation.

For a dispersive system $\mathcal{Z}$ that modulates the input signal $x(t)$ with a nonlinear monotonic phase function $\xi(t/t_r)$, leading to a dispersive frequency shift $\nu(t) = (d/dt)\xi(t/t_r)$, a dispersive version of the SF was proposed to match the system dynamics [2]. Here, $t_r > 0$ is a normalization time reference. Specifically, the DSF can be used to interpret the system output $(\mathcal{Z}x)(t)$ as a weighted superposition of (possibly) dispersive transformations on $x(t)$, i.e.,

$$(\mathcal{Z}x)(t){=}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{DSF}_{\mathcal{Z}}(\zeta,\beta)\,\mathrm{e}^{-j\pi\zeta\beta}(\mathcal{D}_{\beta}^{(\xi)}\mathcal{G}_{\zeta}^{(\xi)}x)(t)d\zeta d\beta. \quad (4)$$

These transformations correspond to a dispersive frequency shift,

$$(\mathcal{D}_{\beta}^{(\xi)}x)(t) = (\mathcal{U}_{\xi}^{-1}\mathcal{M}_{\beta/t_r}\mathcal{U}_{\xi}x)(t) = \mathrm{e}^{j2\pi\beta\xi(t/t_r)}x(t)\;, \qquad (5)$$

and a generalized time shift,

$$(\mathcal{G}_{\zeta}^{(\xi)}x)(t) = (\mathcal{U}_{\xi}^{-1}\mathcal{S}_{t_r\zeta}\mathcal{U}_{\xi}x)(t)\;. \qquad (6)$$

The transformation parameters ζ and β are dimensionless.

The time domain unitary warping operator $\mathcal{U}_{\xi}$ in (5) and (6) is defined as,

$$(\mathcal{U}_{\xi}x)(t) = \frac{1}{\sqrt{|\xi'(\xi^{-1}(\frac{t}{t_r}))|}}\,x\left(t_r\xi^{-1}\left(\frac{t}{t_r}\right)\right) \qquad (7)$$

and its inverse operator $\mathcal{U}_{\xi}^{-1}$ satisfies $(\mathcal{U}_{\xi}^{-1}\mathcal{U}_{\xi}x)(t) = x(t)$. Here, $\xi'(b) = (d/db)\xi(b)$ and $\xi^{-1}(\xi(b)) = b$.

[1]Note that $\lfloor x\rfloor$ ($\lceil x\rceil$) rounds x to the integer nearest to zero (infinity).

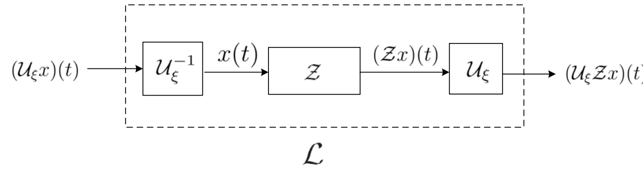

Fig. 1. The warping relation between the dispersive system $\mathcal{Z}$ and the unitary equivalent narrowband system $\mathcal{L} = \mathcal{U}_{\xi}\mathcal{Z}\mathcal{U}_{\xi}^{-1}$.

Depending on the function $\xi(b)$, the formulation in (4) can simplify to a specific interpretation. For example, when $\xi(b) = \ln b$, (4) describes the system output as a weighted superposition of hyperbolic frequency shifts and scale changes [2]. The latter follows since (6) simplifies to dilation/compression signal transformations. When $\xi(b) = \sqrt{b^2-\alpha^2}$ and $b \gg \alpha$, then the system in (4) corresponds to a shallow water acoustic environment causing dispersive frequency shifts in (5) and approximate time shifts in (6) on the transmitted waveform (see Section 5).

3.2. Unitary Warping Relations

Unitary warping methods have played an important role in matching signal or system dispersive characteristics [2, 5]. Indeed, the DSF can be obtained as the narrowband SF of the warped system $\mathcal{U}_{\xi}\mathcal{Z}\mathcal{U}_{\xi}^{-1}$ [2],

$$\mathrm{DSF}_{\mathcal{Z}}(\zeta,\beta) = \mathrm{SF}_{\mathcal{U}_{\xi}\mathcal{Z}\mathcal{U}_{\xi}^{-1}}\left(t_r\zeta,\frac{\beta}{t_r}\right)\;. \qquad (8)$$

This warping relation is depicted in Fig. 1, where the input and output of the dispersive system $\mathcal{Z}$ are $x(t)$ and $(\mathcal{Z}x)(t)$, respectively. $\mathcal{L} = \mathcal{U}_{\xi}\mathcal{Z}\mathcal{U}_{\xi}^{-1}$ is a unitary equivalent narrowband system [2], for which the input and output are the time warped signals, $(\mathcal{U}_{\xi}x)(t)$ and $(\mathcal{U}_{\xi}\mathcal{Z}x)(t)$, respectively. Following (1), this equivalent narrowband system $\mathcal{U}_{\xi}\mathcal{Z}\mathcal{U}_{\xi}^{-1}$ can be characterized by it SF,

$$(\mathcal{U}_{\xi}\mathcal{Z}x)(t) \quad = \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{SF}_{\mathcal{U}_{\xi}\mathcal{Z}\mathcal{U}_{\xi}^{-1}}(\tau,\nu)\mathrm{e}^{-j\pi\tau\nu}$$
$$\cdot\,(\mathcal{M}_{\nu}\mathcal{S}_{\tau}\mathcal{U}_{\xi}x)(t)d\nu d\tau\;. \qquad (9)$$

3.3. Matched Signal Transforms

As we have seen, the dispersive environment $\mathcal{Z}$ is characterized by the change in phase $\xi(t/t_r)$ of the transmitted waveform in (5). The matched signal transform (MST) is a linear transform that highly localizes signals with phase $\xi(t/t_r)$. For a given $\xi(t/t_r)$, the MST of $x(t)$ is defined as [5]

$$\aleph_x^{(\xi)}(c) = \int_{t\in\wp}\sqrt{|\xi'(t/t_r)|}\,x(t)\,\mathrm{e}^{-j2\pi c\xi(\frac{t}{t_r})}dt\;, \qquad (10)$$

where $c\in\mathbb{R}$ and $\wp$ is the domain of $\xi(\frac{t}{t_r})$.

Note that the Fourier transform of the warped signal $(\mathcal{U}_\xi x)(t)$ in (7) is the MST of $x(t)$ up to an axis scaling,

$$(\mathcal{F}\mathcal{U}_\xi x)(f) \triangleq \int_{-\infty}^{\infty} (\mathcal{U}_\xi x)(t)\, \mathrm{e}^{-j2\pi f t} dt = \aleph_x^{(\xi)}(f t_r)\,. \quad (11)$$

Equation (11) shows that the MST generalizes the Fourier transform as it is highly localized when analyzing the generalized chirp $g(t) = \sqrt{|\xi'(\frac{t}{t_r})|}\, \mathrm{e}^{j2\pi c_0 \xi(\frac{t}{t_r})}$, in the same way that sinusoids are localized by the Fourier transform. Specifically, $\aleph_g^{(\xi)}(c) = \delta(c - c_0)$.

4. DISCRETE DISPERSIVE SYSTEM CHARACTERIZATION

In this section, we propose a discrete DSF representation based on sampling the dispersive frequency shift and generalized time shift parameters in (5) and (6), respectively. Our proposed discretization is derived from the existing discrete SF representation by utilizing the unitary warping relation between the SF and DSF. The dispersive discrete model can be useful in many applications. For example, it can facilitate waveform design to improve reception performance by exploiting the inherent diversity in shallow water acoustic transmissions [6].

4.1. Discrete DSF Representation

We consider a dispersive time-varying system $\mathcal{Z}$ with characteristic function $\xi(t/t_r)$, input $x(t)$ and output $(\mathcal{Z}x)(t)$ as in (4). We assume that the MST of $x(t)$ in (10), $\aleph_x^{(\xi)}(c)$, has a bounded support $c \in [c_0, c_1]$, with an effective width $\mathcal{C} = c_1 - c_0$. This is a reasonable assumption as the MST is well-matched to changes in phase by $\xi(t/t_r)$. We also assume that the time warped signal $(\mathcal{U}_\xi \mathcal{Z}x)(t)$ has a bounded support $t \in [t_r\gamma_0, t_r\gamma_1]$, with a normalized duration $\Gamma = \gamma_1 - \gamma_0$. Then, the output $(\mathcal{Z}x)(t)$ in (4) can be decomposed as a weighted summation,

$$(\mathcal{Z}x)(t) = \sum_{l\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} \widehat{\mathrm{DSF}}_{\mathcal{Z}}(\frac{l}{\mathcal{C}}, \frac{k}{\Gamma})(\mathcal{D}^{(\xi)}_{\frac{k}{\Gamma}} \mathcal{G}^{(\xi)}_{\frac{l}{\mathcal{C}}} x)(t)\,. \quad (12)$$

Here, $(\mathcal{D}^{(\xi)}_{\frac{k}{\Gamma}} \mathcal{G}^{(\xi)}_{\frac{l}{\mathcal{C}}} x)(t)$ are discrete dispersive frequency and generalized time shifted versions of the input signal $x(t)$, and the weighting coefficients $\widehat{\mathrm{DSF}}_{\mathcal{Z}}(\frac{l}{\mathcal{C}}, \frac{k}{\Gamma})$ are 2-D samples of a smoothed version of the DSF given by

$$\widehat{\mathrm{DSF}}_{\mathcal{Z}}(\zeta,\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{DSF}_{\mathcal{Z}}(\tilde{\zeta},\tilde{\beta}) \mathrm{e}^{-j\pi(\beta-\tilde{\beta})(\gamma_0+\gamma_1)}$$
$$\cdot\, \mathrm{e}^{j\pi(\zeta-\tilde{\zeta})(c_0+c_1)} \mathrm{sinc}((\beta-\tilde{\beta})\Gamma)\, \mathrm{sinc}((\zeta-\tilde{\zeta})\mathcal{C}) d\tilde{\zeta} d\tilde{\beta}\,. \quad (13)$$

The summations in (12) also admit a finite approximation due to physical constraints of the dispersive system. Specifically, if the DSF is nonzero only when $(\zeta, \beta) \in [\zeta_0, \zeta_1] \times [\beta_0, \beta_1]$, the weighting coefficients $\widehat{\mathrm{DSF}}_{\mathcal{Z}}(\frac{l}{\mathcal{C}}, \frac{k}{\Gamma})$ in (13) are significantly nonzero only when the smoothing function's mainlobe supports, $[(l-1)/\mathcal{C}, (l+1)/\mathcal{C}]$ and $[(k-1)/\Gamma, (k+1)/\Gamma]$, are effectively overlapped with $[\zeta_0, \zeta_1]$ and $[\beta_0, \beta_1]$, respectively. As a result, the summation limits are determined as $\lfloor \zeta_0 \mathcal{C} \rfloor \le l \le \lceil \zeta_1 \mathcal{C} \rceil$ and $\lfloor \beta_0 \Gamma \rfloor \le k \le \lceil \beta_1 \Gamma \rceil$.

4.2. Discretization Procedure

In essence, the discretization procedure leading to (12) is a generalization of the discrete narrowband SF representation in (2) via the unitary warping relation in (8). The basic idea is to unwarp the output signal from the warped narrowband system $\mathcal{L} = \mathcal{U}_\xi^{-1} \mathcal{Z} \mathcal{U}_\xi$ in Fig. 1.

Considering the composite narrowband system $\mathcal{U}_\xi \mathcal{Z} \mathcal{U}_\xi^{-1}$ in Fig. 1, based on the assumption that $x(t)$ is bounded in the MST domain and using (11), we conclude that the input signal $(\mathcal{U}_\xi x)(t)$ is frequency-limited within $[\frac{c_0}{t_r}, \frac{c_1}{t_r}]$, with bandwidth $W = \frac{\mathcal{C}}{t_r}$. Also, we assumed that the time-warped output signal $(\mathcal{U}_\xi \mathcal{Z} x)(t)$ is time-limited within $[t_r\gamma_0, t_r\gamma_1,]$, with time duration $T = t_r\Gamma$. Thus, applying the discrete model in (2), we can write (9) as

$$(\mathcal{U}_\xi \mathcal{Z}x)(t) = \sum_{l\in\mathbb{Z}}\sum_{k\in\mathbb{Z}} \widehat{\mathrm{SF}}_{\mathcal{U}_\xi\mathcal{Z}\mathcal{U}_\xi^{-1}}(\frac{l}{W}, \frac{k}{T})(\mathcal{M}_{\frac{k}{T}} \mathcal{S}_{\frac{l}{W}} \mathcal{U}_\xi x)(t), \quad (14)$$

where $\widehat{\mathrm{SF}}_{\mathcal{U}_\xi\mathcal{Z}\mathcal{U}_\xi^{-1}}(\frac{l}{W}, \frac{k}{T})$ can be determined according to (3). Using the warping relation (8), we obtain the smoothed DSF in (13) as,

$$\widehat{\mathrm{SF}}_{\mathcal{U}_\xi\mathcal{Z}\mathcal{U}_\xi^{-1}}(\frac{l}{W}, \frac{k}{T}) \equiv \widehat{\mathrm{DSF}}_{\mathcal{Z}}(\frac{l}{\mathcal{C}}, \frac{k}{\Gamma})\,. \quad (15)$$

Replacing (15) in (14), and applying $\mathcal{U}_\xi^{-1}$ to both sides, based on the fact that $\mathcal{U}_\xi^{-1}$ is a linear operator, we obtain

$$(\mathcal{Z}x)(t) = \sum_{l\in\mathbb{Z}}\sum_{k\in\mathbb{Z}} \widehat{\mathrm{DSF}}_{\mathcal{Z}}(\frac{l}{\mathcal{C}}, \frac{k}{\Gamma}) \left(\mathcal{U}_\xi^{-1} \mathcal{M}_{\frac{k}{T}} \mathcal{S}_{\frac{l}{W}} \mathcal{U}_\xi x\right)(t)\,.$$

This is equivalent to (12), because, using (5) and (6), we have

$$\begin{aligned} \mathcal{U}_\xi^{-1} \mathcal{M}_{\frac{k}{T}} \mathcal{S}_{\frac{l}{W}} \mathcal{U}_\xi &= (\mathcal{U}_\xi^{-1} \mathcal{M}_{\frac{k}{T}} \mathcal{U}_\xi)(\mathcal{U}_\xi^{-1} \mathcal{S}_{\frac{l}{W}} \mathcal{U}_\xi) \\ &= \mathcal{D}^{(\xi)}_{\frac{t_r k}{T}} \mathcal{G}^{(\xi)}_{\frac{l}{t_r W}} = \mathcal{D}^{(\xi)}_{\frac{k}{\Gamma}} \mathcal{G}^{(\xi)}_{\frac{l}{\mathcal{C}}}\,. \end{aligned}$$

5. SHALLOW WATER ACOUSTIC TRANSMISSION

In shallow water environments, acoustic transmission is subjected to multipath distortion as well as frequency-dependent group velocity dispersion [1]. In this section, we illustrate the use of our dispersive system modeling technique in characterizing the shallow water environment.

5.1. Modal Frequency Dispersion

Due to the interaction between the ocean surface and sediment, the acoustic underwater propagation can be represented in terms of normal modes [1]. At each mode, different frequency components travel at different speeds. As a result, an impulse signal transmitted at time $t = 0$ through the shallow water medium, will arrive at the mth mode asymptotically having a nonlinear time-varying phase [1]

$$\phi(t/t_r) = \phi(t) = 2\pi f_m (t^2 - \alpha^2)^{\frac{1}{2}} \,. \tag{16}$$

Here[2], $\alpha = R/c$, R is the distance between the source and the receiver, and f_m is the cutoff frequency of the mth mode. The phase in (16) was obtained using the method of stationary phase assuming $t \gg \alpha$ [1].

In Fig. 2, the characteristic dispersive instantaneous frequency $\frac{d}{dt}\phi(t) = f_m t/(t^2 - \alpha^2)^{1/2}$ is plotted for the first 6 modes with $c = 1500$ m/s and $R = 30$ km. Note that this dispersive characteristic has been tested by analyzing experimental data in [7] using time-frequency techniques.

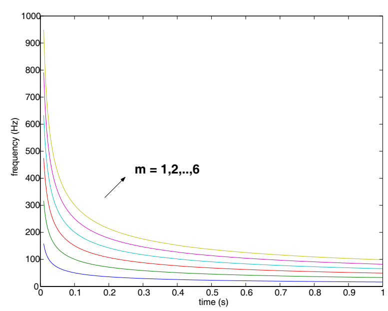

Fig. 2. Dispersive instantaneous frequency corresponding to the first 6 modes of $\phi(t)$ in (16).

5.2. Discrete Representation

Based on the aforementioned discussion, a shallow water environment with dense multipath and modal frequency dispersions can be characterized using the DSF in (4). Specifically, due to the characteristic phase in (16), the function $\xi(t)$ in (4) is chosen as $\xi(t) = (t^2 - \alpha^2)^{1/2}$, $t \gg \alpha$. The received signal $y(t)$ can be further simplified and decomposed into a weighted summation of constant time-shifted and dispersive frequency-shifted versions of the input signal $x(t)$,

$$y(t) \approx \sum_{l=1}^{L} \sum_{k=1}^{K} \hat{H}\left(\frac{l}{W}, \frac{k}{T}\right) x\left(t - \frac{l}{W}\right) \mathrm{e}^{j2\pi \frac{k}{T}(t^2-\alpha^2)^{\frac{1}{2}}} \,. \tag{17}$$

Here, T and W are the time duration and bandwidth of the warped signal $q(t) = \frac{t^{1/2}}{(t^2-\alpha^2)^{1/4}} x((t^2 - \alpha^2)^{1/2})$. The summation limits are $L = \lceil T_d W \rceil$ and $K = \lceil F_m T \rceil$, where T_d is the multipath delay spread and F_m is the cutoff frequency for the highest mode. The weighting coefficients $\hat{H}(\frac{l}{W}, \frac{k}{T})$ correspond to the sampled and smoothed DSF of the system obtained using (13).

An important implication of the model in (17) is the time-frequency diversity that is inherent to a dispersive time-varying channel. This is conceptually similar to the narrowband time-frequency model providing joint multipath-Doppler diversity [3], and to the wideband time-scale model providing joint multipath-scale diversity [8]. We are currently investigating waveform design and reception schemes matched to shallow water dispersion to effectively achieve diversity gains [6].

[2]Without loss of generality, we assume $t_r = 1$.

6. CONCLUSION

Based on the unitary warping relation between narrowband and dispersive time-varying systems, we proposed a generic approach to represent dispersive systems in terms of discrete and (possibly) nonlinear signal transformations that are weighted by samples of the smoothed DSF. The corresponding sampling intervals are related to the signal supports in matched transform domains. Furthermore, we illustrated the use of the model for characterizing shallow water acoustic environments. This model may be useful, in practice, for leveraging potential diversity gains in shallow water acoustic transmissions.

7. REFERENCES

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