

# A MAXIMUM ENTROPY APPROACH TO ULTRA-WIDEBAND CHANNEL MODELING

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## ABSTRACT

In this paper, a unified framework for Ultra-Wideband channel (UWB channel) modeling based on the maximum entropy approach is provided. For a given set of constraints, a consistent model which takes into account the information at hand is obtained. Two cases are considered: channel power knowledge and knowledge of the covariance matrix. The channel power delay spectrum is also derived and the impact of the channel bandwidth is assessed through information theoretic considerations. The information variation slope with respect to the bandwidth is also studied, based on wideband measurements performed at Institut Eurecom<sup>1</sup>.

## 1. INTRODUCTION

UWB systems radiate waveforms that are formed by a sequence of “very short” pulses, i.e., pulses with durations about few hundred picoseconds. Such signals are free of sine-wave carriers and do not require IF processing because they can operate at baseband. Moreover, the system uses very low power spectral density, below the thermal noise of the receivers, which is inherently difficult to be detected and does not cause significant interference with other systems. These basic properties of the radio UWB system make it an ideal candidate for commercial, short-range, low power, low cost indoor communication systems such as Wireless Local Area Network (WLAN) and Personal Area Network (WPAN).

Although appealing, the efficiency of UWB systems is still questionable. Indeed, for large bandwidths, channel uncertainty limits the achievable rates of power constrained systems and therefore the capacity depends crucially on the model at hand. In fact, recent results [1] show that the capacity is a function of how the number of channel paths scale with the bandwidth (linear, sub-linear,...). Furthermore, although the large bandwidth is supposed to increase the rate, channel estimation of all the channel paths becomes a real bottleneck as one may dedicate a large fraction of the rate to estimate more and more channel coefficients.

<sup>1</sup>This work is part of the European Network of Excellence NEWCOM, Contract IST NoE 507325.

This contribution aims at analyzing how channel uncertainty scales with bandwidth in UWB systems. Equivalently, the number of parameters necessary to predict (or also represent) the wideband channel is determined. In this respect, a sound framework for UWB channel modelling [2] based on information theoretic tools is provided. Based on a given set of constraints (measurements, power), a channel model is derived based on the maximum entropy approach and validated through measurements performed at Institut Eurecom. Within this setting, channel uncertainty has a straightforward meaning through the notion of entropy that is analyzed with respect to the bandwidth.

Previous studies [3] have already analyzed channel uncertainty scaling through the number of significant paths. However, in many cases, additional criteria (such as AIC, MDL...) have to be considered as, for noisy measurements, the notion of significant paths is subjective. Note finally that previous contributions have also focused on characterizing the wideband channel through a limited number of parameters (AR models with few coefficients [4]). The benefit of such characterizations rely on the fact that it is possible to reproduce the channel frequency behavior using only a small number of parameters.

## 2. MAXIMUM ENTROPY MODELING (MEM)

The problem of modelling wideband channels is crucial for the efficient design of wireless systems. Indeed, the wireless channel suffers from constructive/destructive interference signaling and therefore yields a randomized channel for which one has to attribute a joint probability distribution for the channel frequency response. In this contribution, we would like to provide some theoretical grounds to model the wideband channel based on a given state of knowledge. In other words, knowing only certain things related to channel (power, measurements), how to translate that information into a model for the channel? This question can be answered in light of the Bayesian probability theory [5] and the principle of maximum entropy. The principle of maximum entropy is at present the clearest theoretical justification in conducting scientific inference: we do not need a model, entropy maximization creates

a model for us out of the information available. Choosing the distribution with greatest entropy avoids the arbitrary introduction or assumption of information that is not available. Note that this approach has been successfully used in spectrum analysis [6] and signal interpolation problems [7] [8].

## 2.1. MEM for channel power knowledge

Suppose in the following that the modeler has no knowledge about the frequency response of the wideband channel and would like, with this limited knowledge, to determine the number of clusters in the environment. The modeler knows however that the channel carries some power  $P$  and is stationary during the channel modelling phase.

Let  $\{h_i\}_{i \in \mathbb{Z}}$  be the sequence of samples at frequencies  $i\delta_f$  ( $\delta_f$  is the frequency resolution) of the channel frequency response. In this case, the spectral autocorrelation function is defined as:

$$R(k) = E[h_i h_{i+k}^*] \quad (1)$$

and the power delay spectrum is defined as

$$P(\tau) = \sum_{k=-\infty}^{\infty} R(k) e^{-j2\pi\tau k}, \quad (2)$$

where  $\tau = \frac{\hat{\tau}}{T_s}$  is the normalized delay and  $\hat{\tau}$  is the delay in seconds.

In the following, we suppose that  $\int_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}} P(\tau) d\tau$  is the power of the channel equal to  $P$  and  $\tau_{max}$  is the maximum delay. The modeler would like to take into account the power constraint without additional information that is not available. Maximizing the entropy of the process guarantees such a setting as one finds the sequence of autocorrelations that make the impulse response as white as possible. In other words, such an extrapolation places the least amount of structure in the channel.

For a Gaussian random process, with power delay spectrum  $P(\tau)$ , the entropy  $H$  is given by

$$H = \log(\pi e) + \int_{-\frac{1}{2}}^{\frac{1}{2}} \log(P(\tau) + \epsilon) d\tau, \quad (3)$$

where  $\epsilon$  is an arbitrary small constant ( $\epsilon \geq 0$ ) used to regularize the non-regular Gaussian process ( $h_i$ ).

The modeler would like to maximize  $H$  under the constraint that the received power in a given delay interval is known. This statement can simply be expressed if one tries to maximize the following expression using Lagrange multipliers with respect to  $R(k)$ :

$$C = H - \mu_0 \left( \int_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}} P(\tau) d\tau - P \right) \quad (4)$$

$$\begin{aligned} \frac{\partial C}{\partial R(k)} &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{P(\tau) + \epsilon} \frac{\partial P(\tau)}{\partial R(k)} d\tau - \mu_0 \int_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}} e^{-j2\pi\tau k} d\tau, \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{P(\tau) + \epsilon} e^{-j2\pi\tau k} d\tau + \frac{\mu_0}{j2\pi k} [e^{-j2\pi\tau k}]_{-\frac{\tau_{max}}{2}}^{\frac{\tau_{max}}{2}}, \\ &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{P(\tau) + \epsilon} e^{-j2\pi\tau k} d\tau - \mu_0 \tau_{max} \text{sinc}(k\pi\tau_{max}), \\ &= 0. \end{aligned}$$

Let  $Q(\tau) = \frac{1}{P(\tau) + \epsilon}$  and  $q_k = \int_{-1/2}^{1/2} Q(\tau) e^{+j2\pi k\tau} d\tau$ . We have

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} Q(\tau) e^{j2\pi\tau k} d\tau = \mu_0 \tau_{max} \text{sinc}(k\pi\tau_{max}) \quad (5)$$

$$\Leftrightarrow q_k = \mu_0 \tau_{max} \text{sinc}(k\pi\tau_{max}). \quad (6)$$

Thus,  $Q(\tau) = \sum_{k=-\infty}^{\infty} \mu_0 \tau_{max} \text{sinc}(k\pi\tau_{max}) e^{-j2\pi k\tau}$ , which is constant for all  $-\frac{\tau_{max}}{2} \leq \tau \leq \frac{\tau_{max}}{2}$ . As a consequence,

$$P(\tau) = \begin{cases} \frac{P}{\tau_{max}}; & -\frac{\tau_{max}}{2} \leq \tau \leq \frac{\tau_{max}}{2} \\ 0; & \text{elsewhere.} \end{cases}$$

and  $R(k) = P \text{sinc}(k\pi\tau_{max})$  for all  $k$ .  $P(\tau)$  is constant and vanishes on  $[-\frac{1}{2}, +\frac{1}{2}] / [-\frac{\tau_{max}}{2}, +\frac{\tau_{max}}{2}]$ . In other words if there is no knowledge except the maximum delay, the model gives an infinite number of clusters and the power is equally split across the different clusters. The methodology can be easily extended if the modeler has knowledge of the bandwidth (which determines the number of correlation coefficients  $R(k)$ ) used.

## 2.2. MEM for covariance channel knowledge

In the following, we suppose that the modeler has knowledge (through measurements) of a finite number of frequency autocorrelation coefficients  $R(k)$ . The number of coefficients is determined by the measured bandwidth as well as the measurement resolution. Based on this knowledge, the modeler would like to derive a wideband model taking into account account the previous constraints and not more, i.e., the modeler would like to extrapolate the missing autocorrelation coefficients for deriving the power delay spectrum function. Using the same methodology as Section 2.1, the following theorem due to Burg [9] can be obtained:

**Theorem 1** *The maximum entropy rate stochastic process  $\{h_i\}_{i \in \mathbb{Z}}$  that satisfies the constraints*

$$E[h_i h_{i+k}] = R(k), \quad k = 0, 1, \dots, N, \quad \text{for all } i, \quad (7)$$

is the  $N$ -th order Gauss-Markov process of the form

$$h_i = - \sum_{k=1}^N a_k h_{i-k} + Z_i, \quad (8)$$

where the  $Z_i$  are i.i.d.  $\sim N(0, \sigma^2)$  and  $a_1, a_2, \dots, a_N, \sigma^2$  are chosen to satisfy Equation (7).

**Remark:** The theorem does not assume the  $h_i$  to be wide-sense stationary.

A process satisfying (8) is also called autoregressive of order  $N$  ( $AR(N)$ ). The coefficients  $a_1, a_2, \dots, a_N, \sigma^2$  are obtained by solving the Yule-Walker equations

$$R(0) = - \sum_{k=1}^N a_k R(-k) + \sigma^2, \quad (9)$$

$$R(l) = - \sum_{k=1}^N a_k R(l-k), \quad l = 1, 2, \dots, N. \quad (10)$$

Fast algorithms such as the Levinson and Durbin algorithm have been devised which exploit the special structure of these equations to efficiently calculate the coefficients  $a_1, a_2, \dots, a_N$  from the covariance samples  $R(0), \dots, R(N)$ . The power delay spectrum of the  $N$ th order Gauss-Markov process (8) is given by

$$P(\tau) = \frac{\sigma^2}{\left| 1 + \sum_{k=1}^N a_k e^{-i2\pi k\tau} \right|^2}. \quad (11)$$

In general, from a finite set of  $N$  frequency measurements  $[h_1^l, \dots, h_N^l]$  over a bandwidth of  $N\delta_f$  ( $l$  is the  $l$ th channel realization), there are many ways to estimate the spectral autocorrelation coefficients. In the following, the sample autocorrelation function is defined as:

$$\hat{R}^N(k) = \frac{1}{L} \frac{1}{N-k} \sum_{l=1}^L \sum_{i=1}^{N-k} h_i^l (h_{i+k}^l)^*, \quad k \geq 0.$$

For a given  $N$ , we estimate and determine the autocorrelation function  $\hat{R}^N(k)$ , the autocorrelation coefficients  $\hat{a}_k^{(N)}$  and the power delay spectrum  $\hat{P}^N(\tau)$ . As a consequence, the estimated entropy is given by:

$$\hat{H}^N = \log(\pi e) + \int_{-\frac{1}{2}}^{\frac{1}{2}} \log \frac{\sigma^2}{\left| 1 + \sum_{k=1}^N \hat{a}_k^{(N)} e^{-i2\pi k\tau} \right|^2} d\tau. \quad (12)$$

The roots of the power delay spectrum (11) determine the number of significant clusters. Practically, although the roots may exist, some may be not significant and therefore unnecessary to model. In order to assess the number of significant modelling coefficients, we will analyze in Section 3 the information  $\hat{H}^N$  contained in the process with respect to  $N$ .

### 3. MEASUREMENT DESCRIPTION

#### 3.1. Measurement setup

The measurements were carried out in the Mobile Communications Laboratory of Institut Eurecom, which is a typical lab environment, rich in reflective and diffractive objects. The measurement device used is a wideband Vector Network Analyzer (VNA) which allows complex transfer function parameter measurements in the frequency domain, extending from 10MHz to 20GHz. This instrument has low inherent noise, less than -110dBm for a measurement bandwidth of 10Hz, and high measurement speed, less than 0.5ms/point. The maximum number of equally-spaced frequency samples (amplitudes and phase) per measurement is 2001. The measurement data were acquired and controlled remotely using RSIB protocol over Ethernet permitting off-line signal processing and instrument control in MATLAB.

In order to perform truly wideband measurements with sufficient resolution, several bands were concatenated by using consecutive measurements. In this contribution, measurements were performed from 3GHz to 9GHz by concatenating three groups of 2001 frequency samples per 2GHz sub-bands (3-5GHz, 5-7GHz, 7-9GHz). This yields a 1MHz spacing between the frequency samples. Systematic and frequent calibration (remotely controlled) was employed to compensate the undesirable frequency-dependent attenuation factors that might affect the collected data. The wideband antennas employed in the measurements are omnidirectional in the vertical plane and have an approximate bandwidth of 7.5GHz (varying from 3.1GHz to 10 GHz). They were not perfectly matched across the entire band, with a Voltage Standing Wave Ratio (VSWR) varying from 2 to 5.

#### 3.2. Measurement environment

The data analyzed in Section 4 were collected in a Line of Sight (LoS) setting with measurements performed at spatially different locations. The experiment area is set by fixing the transmitting antenna on a mast at 1m above the ground and close to the VNA. The receiving antenna moves then to different locations on a horizontal linear grid of side 50 cm in steps of 5cm. The transmitter antenna's height was varied by 5cm up to 20cm after completion of the measurements at various receiver positions. The transmitter and the receiver were separated by a distance of six meters.

### 4. RESULTS

In the following, the scaling of channel uncertainty with respect to the bandwidth is analyzed. In Fig. 1, the variation of  $\hat{H}^N$  is plotted for the LoS case as well as the Gaussian case (zero mean i.i.d frequency samples are generated in this case) versus  $N\delta_f$  (the frequency band is  $[3000, 3000 + N\delta_f]$  MHz). As one can observe, the channel uncertainty decreases with

bandwidth. However, in the Gaussian case, additional information provided by the frequency samples does not lower the uncertainty as the samples are completely independent. Remarkably, the results show that in UWB settings and for a given channel representation complexity (here, the slope of the entropy), there is an optimum number of parameters to be chosen. In other words, AR modelling based on a limited number of parameters is adequate. In this respect, we have plotted in Fig. 2 and Fig. 3 the power delay spectrum based on 500MHz and 6GHz measurements. The results show that with increasing bandwidth, one is certainly able to capture the small variations but a great amount of the information is already included in the 500MHz band. Note that the delay of 20 nanoseconds corresponds to the 6 meter distance between the transmitter and the receiver. Moreover, in both cases, the AR model fits the measured power delay spectrum response.

## 5. CONCLUSION: INFORMATION VERSUS COMPLEXITY

In section 4, we have shown that although in modelling, one should take into account all the information at hand, there is a compromise to be made in terms of model complexity. Each information added will not have the same effect on the channel model and might as well more complicate the model for nothing rather than bring useful insight on the behavior of the propagation environment. In this respect, entropy is a useful measure and the slope decrease characterizes how information scales with bandwidth. In particular, in wide-band schemes, we have shown that it is possible to reproduce the channel frequency behavior with a limited number of coefficients since the channel uncertainty decreases with bandwidth. We have also provided a sound methodology to model channels when additional channel constraints are given.

## 6. REFERENCES

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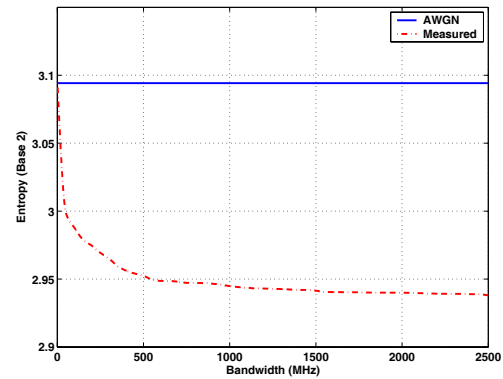


Fig. 1. Entropy variation with respect to the bandwidth.

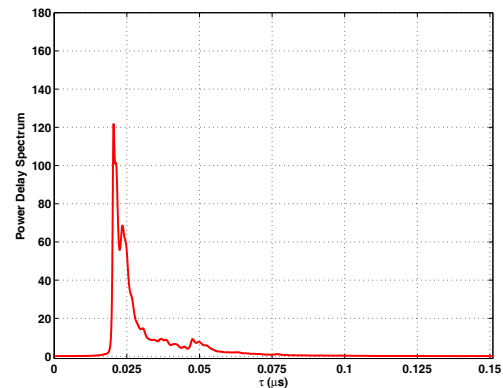


Fig. 2. Power delay spectrum with 500Mhz bandwidth.

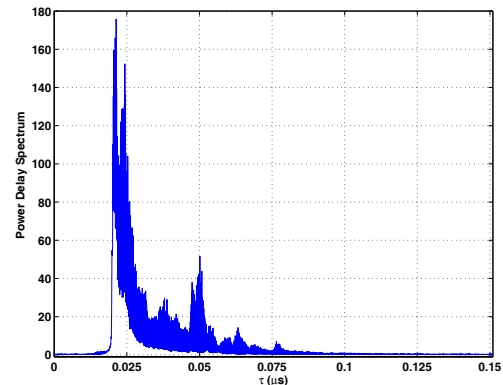


Fig. 3. Power delay spectrum with 6Ghz bandwidth.