$\mathcal{E}$-OPTIMAL ANOMALY DETECTION IN PARAMETRIC TOMOGRAPHY

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ABSTRACT

The paper concerns the radiographic non-destructive testing of well-manufactured objects. The detection of anomalies is addressed from the statistical point of view as a binary hypothesis testing problem with nonlinear nuisance parameters. A new detection scheme is proposed as an alternative to the classical GLR test. It is shown that this original decision rule detects anomalies with a loss of a negligible (ε) part of optimality with respect to an optimal invariant test designed for the "closest" hypothesis testing problem with linear nuisance parameters.

1. INTRODUCTION

For radiographic inspection of industrial objects (nuclear fuel rods, for example), it is desirable to detect defects, inclusions or any unexpected cavities in order to assure the safety and reliability of installations. Often, the number of projections and/or view angles available for inspection is very limited and the pixel-by-pixel reconstruction is impossible.

The defect detection problem is based on the assumption that the imaged medium is composed of an (partially) unknown background with a possibly hidden anomaly. It is considered as a parametric hypotheses testing problem between two composite alternatives with nonlinear nuisance parameters. A key assumption is the existence of a nonlinear parametric parsimonious model of the non-anomalous background to counterbalance the lack of observations. The Generalized Likelihood Ratio (GLR) test [1, 2], which is usually used to solve this kind of problem, has three major drawbacks: 1) this tool is relevant when the number of observations is very large but it is often suboptimal for a limited number of observations; 2) the GLR test requires to estimate the unknown parameters before taking a decision, which is difficult in a nonlinear case and 3) the GLR test makes no distinction between the nuisance parameters with respect to their impact on the nonlinearity of the model, which is not relevant from the practical point of view. A new detection scheme is proposed as an alternative to the GLR test: it detects anomalies with a loss of a negligible (ε) part of optimality with respect to an optimal invariant test designed for the "closest" hypothesis testing problem with linear nuisance parameters. This paper is organized as follows. First, a parametric-based approach which includes the nonlinear parsimonious parametric model of the inspected object and radiographic process is presented in section 2. Secondly, in section 3, an ε-optimal test is designed to detect anomalies in the presence of nonlinear nuisance parameters. Finally, some experimental results with real radiographies show the relevance of the theoretical developments in section 4.

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2. PROBLEM STATEMENT: ANOMALY DETECTION IN PARAMETRIC TOMOGRAPHY

2.1. Nuclear fuel rod inspection

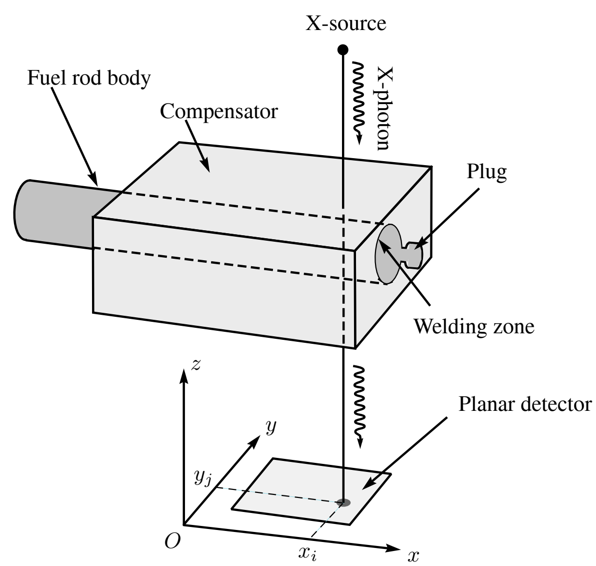

Fig. 1. Geometry of the nuclear fuel rod inspection system.

A nuclear fuel rod is composed of a body and a plug as shown in Fig. 1. The body is manufactured separately from the plug and, before its use, the plug is welded with the body. The goal of the nuclear fuel rod inspection is to detect defects (anomalies) in the welding zone which corresponds to a tangential part of the fuel rod (see Fig. 1). During the monitoring process, the nuclear fuel rod is imaged with a tomographic system composed of a X-source and a planar detector. The fuel rod is put into a compensator which is made of the same material to avoid the high contrast of radiography near the edges of the fuel rod. The goal is to decide between the two possible situations: $\mathcal{H}_0 = \{\text{there is no anomaly}\}$ and $\mathcal{H}_1 = \{\text{there is at least one anomaly}\}$.

2.2. Physical background

To simplify the problem, the parallel-beam geometry is used in the paper and the X-rays are all oriented along the z-axis (see Fig. 1). The planar detector coincides with the xOy-plane. The measurements $\zeta(x, y)$ at different points (x, y) of the detector are modeled as independently distributed random variables [3] such that:

$$\zeta(x, y) \sim \Pi(m(x, y)) = \Pi(\mu(x, y) + \omega(x, y)), \quad (1)$$

where $\Pi(m)$ denotes the Poisson law with parameter $m > 0$. The *unknown* quantity $\mu(x, y)$ (resp. $\omega(x, y)$) represents the mean number of photons passing through the media (resp. the mean number of extra photons, caused primarily by scattered radiations) at the position (x, y).

Let r be the radius of the fuel rod and $l(x, y; r)$ be the material (the fuel rod together with the compensator) thickness corresponding to the location (x, y) on the detector (see Fig. 1). It is assumed that an unknown value of r belongs to the interval $I = [r_0 - \varrho; r_0 + \varrho]$, where ϱ is a small positive constant and r_0 is exactly known. It is assumed that the quantity $\mu(x, y)$ can be well approximated [4] by the polynomial function:

$$\mu(x, y) \approx \widehat{\mu}(x, y; r, a_0, \mathbf{a}) = a_0 + \sum_{k=1}^{n_{\mathbf{a}}-1} a_k\, l^k(x, y; r), \quad (2)$$

where $\mathbf{a} = (a_1\, a_1\, a_2\, \ldots\, a_{n_{\mathbf{a}}-1})^T$ is the vector of coefficients, and the impact of scattered radiations can be approximated by a bivariate polynomial function:

$$\omega(x, y) \approx \widehat{\omega}(x, y; \mathbf{b}) = \sum_{u=0}^{n_x} \sum_{v=0}^{n_y} b_{u,v} x^u y^u, \quad (3)$$

where $\mathbf{b} = (b_{0,0}\, b_{1,0}\, \ldots\, b_{n_x,n_y})^T$. To avoid the redundancy with the term $b_{0,0}$ in (3), the term a_0 from equation (2) is omitted in the rest of the paper. It is assumed that the vector $\mathbf{a}$ belongs to a compact set $K_{\mathbf{a}} \subset \mathbb{R}^{n_{\mathbf{a}}-1}$ and the vector $\mathbf{b}$ belongs to a compact set $K_{\mathbf{b}} \subset \mathbb{R}^{n_{\mathbf{b}}}$ with $n_{\mathbf{b}} = (n_x + 1)(n_y + 1)$ to warrant the validity of the approximation given by equations (2) and (3).

2.3. Measurement model

By considering equations (2) and (3), equation (1) can be rewritten as:

$$\zeta(x, y) \sim \begin{cases} \Pi(m(x, y)) & \text{under} \quad \mathcal{H}_0 \\ \Pi(\theta(x, y) + m(x, y)) & \text{under} \quad \mathcal{H}_1 \end{cases} \quad (4)$$

where $\theta(x, y)$ represents the local (at (x, y)) variation of the mean number of X-photons arrived on the planar detector due to the anomaly at the position (x, y). For the considered problem, the exposure time and the X-flux intensity are high enough to warrant a good signal-to-noise ratio. Consequently, the Gaussian approximation of the Poisson distribution is relevant, which leads to a more tractable detection problem when anomalies are unspecified. Hence, the measurement model (4) is approximated by the following one:

$$\zeta(x, y) = \begin{cases} \widehat{m}(x, y; \mathbf{c}) + \xi(x, y) & \text{under} \quad \mathcal{H}_0 \\ \theta(x, y) + \widehat{m}(x, y; \mathbf{c}) + \xi(x, y) & \text{under} \quad \mathcal{H}_1 \end{cases},$$

with $\widehat{m}(x, y; \mathbf{c}) = \widehat{\mu}(x, y; r, \mathbf{a}) + \widehat{\omega}(x, y; \mathbf{b})$, $\mathbf{c} = (r, \mathbf{a}, \mathbf{b}) \in K$, $K = I \times K_{\mathbf{a}} \times K_{\mathbf{b}} \subset \mathbb{R}^{n_{\mathbf{c}}+1}$, $n_{\mathbf{c}} = n_{\mathbf{a}} + n_{\mathbf{b}}$ and $\xi(x, y) \sim \mathcal{N}(0, \sigma^2(x, y))$. The standard deviation $\sigma(x, y)$ is defined by $\sigma(x, y) = \nu(\bar{m}(x, y))^{\frac{1}{2}}$ where $0 \leq \nu \leq 1$ is a known experimental coefficient independent of (x, y) and $\bar{m}(x, y)$ is an experimental mean value for $m(x, y)$.

The planar detector, which is composed of $n = n_x\, n_y$ discrete sensors, can be viewed as a $n_x \times n_y$ matrix. Let us note $\zeta_{i,j}$ the sensor measurement at the row i and the column j. By denoting $\mathrm{vec}(\{\zeta_{i,j}\})$ the lexicographical ordering of measurements $\zeta_{i,j}$, the above approximated measurement model can be rewritten:

$$\Xi = \mathrm{vec}(\{\zeta_{i,j}\}) = \begin{cases} M(\mathbf{c}) + \boldsymbol{\xi} & \text{under} \quad \mathcal{H}_0 \\ \boldsymbol{\theta} + M(\mathbf{c}) + \boldsymbol{\xi} & \text{under} \quad \mathcal{H}_1 \end{cases}, \quad (5)$$

where $\boldsymbol{\theta} = \mathrm{vec}(\{\theta_{i,j}\})$, $M(\mathbf{c}) = \mathrm{vec}(\{\widehat{m}_{i,j}(\mathbf{c})\})$ and $\boldsymbol{\xi} = \mathrm{vec}(\{\xi_{i,j}\})$. The random vector $\boldsymbol{\xi} \sim \mathcal{N}(0, \Sigma)$ follows the n-dimensional Gaussian law with a zero mean and a known diagonal positive definite covariance matrix Σ.

3. ANOMALY DETECTION: HYPOTHESES TESTING WITH NUISANCE PARAMETERS

3.1. Hypotheses testing: problem statement

Since the matrix Σ is known, the testing problem (5) consists of choosing between the two alternatives:

$$\mathcal{H}_0 = \{\mathbf{y} \sim \mathcal{N}(\boldsymbol{\theta} + H(\mathbf{c}), I_n);\ \boldsymbol{\theta} = 0,\ \mathbf{c} \in K\} \quad (6)$$

$$\mathcal{H}_1 = \{\mathbf{y} \sim \mathcal{N}(\boldsymbol{\theta} + H(\mathbf{c}), I_n);\ \boldsymbol{\theta} \neq 0,\ \mathbf{c} \in K\}, \quad (7)$$

with $\mathbf{y} = \Sigma^{-\frac{1}{2}}\Xi$, $H(\mathbf{c}) = \Sigma^{-\frac{1}{2}}M(\mathbf{c})$ and $\Sigma^{-\frac{1}{2}}$ is the square-root matrix of Σ^{-1} such that $\Sigma^{-\frac{1}{2}}\Sigma^{-\frac{1}{2}} = \Sigma^{-1}$.

Let $\mathcal{K}_\alpha = \{\delta : \sup_{\mathbf{c} \in K} \Pr_{\boldsymbol{\theta}=0,\mathbf{c}}(\delta(\mathbf{y}) = \mathcal{H}_1) \leq \alpha\}$ be the class of tests $\delta : \mathbb{R}^n \mapsto \{\mathcal{H}_0, \mathcal{H}_1\}$ with upper-bounded maximum false alarm probability, where the probability $\Pr_{\boldsymbol{\theta},\mathbf{c}}$ stands for the vector of observations $\mathbf{y}$ being generated by the distribution $\mathcal{N}(\boldsymbol{\theta} + H(\mathbf{c}), I_n)$ and α is the prescribed probability of false alarm. The power function β is defined with the probability of detection: $\beta(\boldsymbol{\theta}; \mathbf{c}) = \Pr_{\boldsymbol{\theta} \neq 0,\mathbf{c}}(\delta = \mathcal{H}_1)$. The subtlety of the above mentioned hypotheses testing problem consists of choosing between $\mathcal{H}_0$ and $\mathcal{H}_1$ with the best possible performance indexes (α, β) while considering $\mathbf{c}$ as a nonlinear nuisance parameter.

3.2. Hypotheses testing: optimal and suboptimal tests

In the case of a vector parameter $\boldsymbol{\theta}$, the crucial issue is to find an optimal solution over a set of alternatives which is rich enough. Unfortunately, Uniformly Most Powerful (UMP) tests scarcely exist, except when the parameter $\boldsymbol{\theta}$ is scalar, the family of distributions has a monotone likelihood ratio, and the test is one-sided [5]. Moreover, due to the fact that the vector-function $\mathbf{c} \mapsto H(\mathbf{c})$ is nonlinear and the nuisance parameter vector $\mathbf{c}$ belongs to a compact K, the direct application of the theory of invariant tests to the problem given by equations (6)-(7) is also compromised.

To provide a remedy for this situation, the following approach is developed in the paper. First, since the measurement model nonlinearity is related to the geometrical imperfections of the fuel roads, it is proposed to design a linear parametric model of the exactly-shaped fuel rod. Secondly, an ε-optimal test is designed with respect to an optimal invariant test based on the "closest" linear model [1, 6, 7, 8].

Definition 1 *A test $\delta \in \mathcal{K}_\alpha$ is called ε-optimal on the region Θ with respect to an optimal one $\delta^* \in \mathcal{K}_\alpha$ if there exists a (small) positive constant $\varepsilon > 0$ such that*

$$\sup_{\boldsymbol{\theta}\in\Theta,\mathbf{c}\in K} \mid \beta_\delta(\boldsymbol{\theta};\mathbf{c}) - \beta_{\delta^*}(\boldsymbol{\theta};\mathbf{c}) \mid \leq \varepsilon. \qquad (8)$$

3.3. Optimal invariant test: linear nuisance parameters

Let us consider the hypotheses testing problem given by equations (6)-(7) in the linear case: $H(\mathbf{c}) = H\mathbf{c}$, where H is a known full column rank matrix of size $n \times n_{\mathbf{c}}$. In the experimental context this means the radius r is known and only beam hardening and X-scattering parameters are unknown: $\mathbf{c} = (\mathbf{a}^T, \mathbf{b}^T)^T$.

Let us note $P_H^\perp = I_n - H(H^TH)^{-1}H^T$ the orthogonal projection on the null space of the matrix H and let $\mathcal{S}$ be the family of surfaces $\mathcal{S} = \{S_c \ : \ c > 0\}$ with

$$S_c = \{\boldsymbol{\theta} \ : \ \|P_H^\perp\boldsymbol{\theta}\|_2^2 = c^2\}. \qquad (9)$$

Then, it is shown [8, 9] that the test

$$\delta^*(\mathbf{y}) = \begin{cases} \mathcal{H}_0 & \text{if } \ \Lambda(\mathbf{y}) = \|P_H^\perp\mathbf{y}\|_2^2 < \gamma_\alpha \\ \mathcal{H}_1 & \text{else} \end{cases}, \qquad (10)$$

where the threshold γ_α is chosen to satisfy the false alarm bound α, $\Pr_{\boldsymbol{\theta}=0,\mathbf{c}}(\Lambda(\mathbf{y}) \geq \gamma_\alpha) = \alpha$, is Uniformly Best Constantly Powerful (UBCP)[1] in the class $\mathcal{K}_\alpha$ over the family of surfaces $\mathcal{S}$ (9). The statistics Λ is distributed according to the χ^2 law with $n - n_{\mathbf{c}} - 1$ degrees of freedom. This law is central under $\mathcal{H}_0$ and non-central under $\mathcal{H}_1$ with the non-centrality parameter $\lambda^2(\boldsymbol{\theta}) = \boldsymbol{\theta}^T P_H^\perp \boldsymbol{\theta}$.

[1] A test $\bar{\delta} \in \mathcal{K}_\alpha$ is UBCP on $\mathcal{S}$ if 1) $\beta_{\bar{\delta}}(\boldsymbol{\theta}') = \beta_{\bar{\delta}}(\boldsymbol{\theta}'')$, $\forall \boldsymbol{\theta}', \boldsymbol{\theta}'' \in S_c$; 2) $\beta_{\bar{\delta}}(\boldsymbol{\theta}) \geq \beta_\delta(\boldsymbol{\theta})$, $\forall \boldsymbol{\theta} \in S_c$, $\forall c > 0$ for any test $\delta \in \mathcal{K}_\alpha$ which satisfies 1).

3.4. $\mathcal{E}$-optimal test: nonlinear nuisance parameters

A bit of algebra shows that:

$$H(\mathbf{c}) = \Sigma^{-\frac{1}{2}} F(r)\mathbf{a} + \Sigma^{-\frac{1}{2}} G\mathbf{b}, \qquad (11)$$

where $F(r) = (F_1(r) \ \dots \ F_{n_\mathbf{a}}(r))$ is an $n \times n_\mathbf{a}$ matrix, $G = (G_1 \ \dots \ G_{n_\mathbf{b}})$ is an $n \times n_\mathbf{b}$ matrix, $F_k(r) = \mathrm{vec}\big(\{l_{i,j}^k(r)\}\big)$, $G_k = \mathrm{vec}\big(\{x_i^u y_j^v\}\big)$ such as $k = u(n_y + 1) + v + 1$. The second-order approximation of the function $F(r)$ around r_0 leads to:

$$F(r) = F(r_0) + \ell_0 \dot{F}(r_0) + \frac{1}{2}\ell_0^2 \ddot{F}(r_0) + \ell_0^2 \boldsymbol{\kappa}(\ell_0), \qquad (12)$$

where $\ell_0 = r - r_0$, $\lim_{r\to 0} \boldsymbol{\kappa}(r) = 0$ and $\dot{F}(r_0)$ (resp. $\ddot{F}(r_0)$) is the $n \times n_\mathbf{a}$ matrix of first order (resp. second-order) derivatives of F at r_0. From (11) and (12), it follows that:

$$H(\mathbf{c}) = H_1(\ell_0)\mathbf{x} + H_2(r_0)\ell_0^2\mathbf{a} + \ell_0^2\boldsymbol{\kappa}(\ell_0),$$

where $H_1(\ell_0) = \Sigma^{-\frac{1}{2}}\Big(F(r_0) + \ell_0\dot{F}(r_0) \ \ G\Big)$ is an unknown $n \times (n_\mathbf{a} + n_\mathbf{b})$ matrix, $\mathbf{x} = (\mathbf{a}^T, \mathbf{b}^T)^T$ and $H_2(r_0) = \Sigma^{-\frac{1}{2}}\ddot{F}(r_0)$. Hence, it appears that the non-linear function $H(\mathbf{c})$ can be rewritten as a sum of two *a priori* non-negligible linear terms: the nominal part $H_1(\ell_0)\mathbf{x}$ representing nuisances and the residual part $H_2(r_0)\ell_0^2\mathbf{a}$ representing model errors due to the linear approximation, and a negligible term $\ell_0^2\boldsymbol{\kappa}(\ell_0)$. Let us define now the following approximation to the initial hypotheses testing problem (6)-(7):

$$\overline{\mathcal{H}}_0 = \{\mathbf{y} \sim \mathcal{N}(\boldsymbol{\theta} + H_1(\ell_0)\mathbf{x} + H_2(r_0)\ell_0^2\mathbf{a}, I_n);\ \boldsymbol{\theta} = 0\} \qquad (13)$$

$$\overline{\mathcal{H}}_1 = \{\mathbf{y} \sim \mathcal{N}(\boldsymbol{\theta} + H_1(\ell_0)\mathbf{x} + H_2(r_0)\ell_0^2\mathbf{a}, I_n);\ \boldsymbol{\theta} \neq 0\}, \qquad (14)$$

where $\mid \ell_0 \mid \leq \varrho$ and $\mathbf{x} \in K_\mathbf{a} \times K_\mathbf{b}$, by omitting the negligible term $\ell_0^2\boldsymbol{\kappa}(\ell_0)$.

Since the measurement model is linear according to the nuisance parameter $\mathbf{x}$, it is necessary to reject it by using the orthogonal projection $P_{H_1(\ell_0)}^\perp$. Unfortunately, the projection matrix depends on the unknown difference $\ell_0 = r - r_0$ and, hence, the computation of $P_{H_1(\ell_0)}^\perp$ is impossible. For this reason it is proposed to reject the whole vector space $R(H_0)$ spanned by the columns of $H_0 = \Sigma^{-\frac{1}{2}}(\dot{F}(r_0) \ \ F(r_0) \ \ G)$. Indeed, it is straightforward to verify that $R(H_1(\ell_0)) \subset R(H_0)$ for all $\mid \ell_0 \mid \leq \varrho$ and the space $R(H_0)$ is the minimal space (in the inclusion sense) which contains all subspaces $R(H_1(\ell_0))$. We finally obtain the test $\overline{\delta}$ defined by:

$$\overline{\delta}(\mathbf{y}) = \begin{cases} \overline{\mathcal{H}}_0 & \text{if } \ \overline{\Lambda}(\mathbf{y}) = \|\overline{\mathbf{y}}\|_2^2 = \|P_{H_0}^\perp\mathbf{y}\|_2^2 < \gamma_\alpha \\ \overline{\mathcal{H}}_1 & \text{else} \end{cases}, \qquad (15)$$

where the threshold γ_α is chosen to satisfy the false alarm bound α: $\Pr_{\boldsymbol{\theta}=0,\mathbf{c}}(\overline{\Lambda}(\mathbf{y}) \geq \gamma_\alpha) = \alpha$. Then, it is shown (the proof is omitted) that there exists a small constant $\varepsilon > 0$ such that

$$\sup_{\boldsymbol{\theta}\in\boldsymbol{\Theta}_m} \sup_{\mathbf{c}\in K} \mid \beta_{\delta^*}(\boldsymbol{\theta};\mathbf{c}) - \beta_{\overline{\delta}}(\boldsymbol{\theta};\mathbf{c}) \mid \leq \varepsilon. \qquad (16)$$

Hence, the test $\overline{\delta}$ is ε-optimal on $\Theta_m = R\,(H_0)^{\perp}$ with respect to the optimal one δ^* when the radius r_0 is known. Here, the subspace Θ_m corresponds to detectable anomalies (see [9] for more details).

4. EXPERIMENTAL RESULTS WITH REAL RADIOGRAPHIES

Because of the limited volume of the paper, experimental results are not described in details. The planar detector is composed of $n_x = 50 \times n_y = 100$ sensors, i.e. $n = 5\,000$, with a resolution of 0.030 mm, $\nu = 0.1749$, $\alpha = 10^{-2}$, $\varrho = 0.05$ mm, $n_{\mathbf{a}} = 2$ and $n_x = n_y = 3$. It is assumed that $\mathbf{a} = (a_1\ a_2)^T$ verifies $-2\,000 \leq a_1 \leq 2\,000$ and $-20\,000 \leq a_2 \leq 20\,000$. Radiographies have the estimated signal-to-noise ratio $\text{SNR}_{\text{dB}} = 10\log(\text{SNR}) \approx 70.1$ dB with $\text{SNR} = H(\mathbf{c})^T\Sigma^{-1}H(\mathbf{c})$.

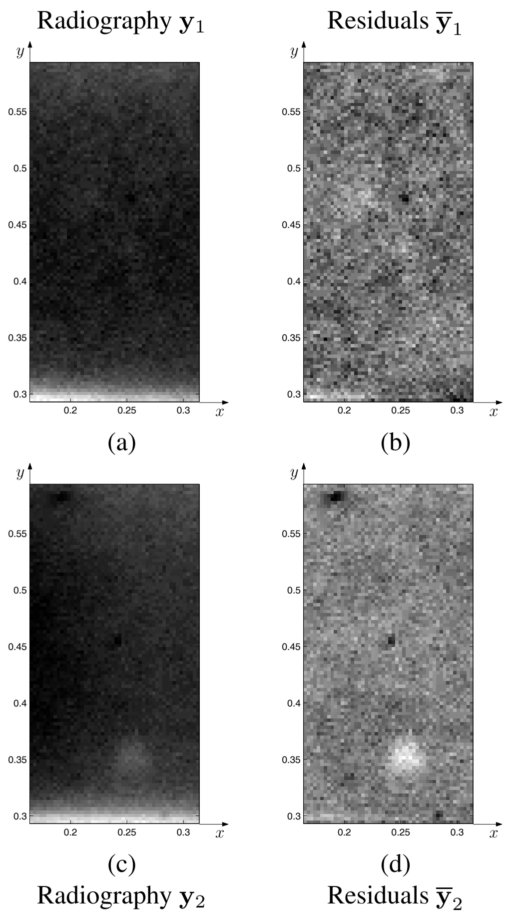

Fig. 2. (a) Radiography $\mathbf{y}_1$ of a safe fuel rod, (b) residuals $\overline{\mathbf{y}}_1 = P_{H_0}^{\perp}\mathbf{y}_1$ of the radiography (a), (c) radiography $\mathbf{y}_2$ of a fuel rod with an anomaly, (d) residuals $\overline{\mathbf{y}}_2 = P_{H_0}^{\perp}\mathbf{y}_2$ of the radiography (c).

When the inspected object is anomaly-free (see radiography $\mathbf{y}_1$ in Fig. 2(a)), the unknown background is properly rejected and the residuals are close to a stationary "white noise" (see Fig. 2(b)). Fig. 2(c) presents a radiography $\mathbf{y}_2$ with an anomaly. This leads to the residuals with an "anomaly signature" (white and black spots) as shown in Fig. 2(d). Under $\mathcal{H}_0$, the decision function is $\overline{\Lambda}(\mathbf{y}_1) = 4\,986.02 < \gamma_{0.01}$ with $\gamma_{0.01} = 5\,222.27$. Under $\mathcal{H}_1$, its value is $\overline{\Lambda}(\mathbf{y}_2) = 5\,883.24 > \gamma_{0.01}$. Since anomalies are assumed to belong to the detectable space Θ_m and the nuisance parameter space K is bounded, the upper bound $\varepsilon \approx 10^{-3}$ is estimated by sampling K to find the largest difference between the power functions β^* and $\beta_{\overline{\delta}}$. The loss of optimality is almost negligible and the false alarm probability holds an acceptable level.

5. CONCLUSION

A parsimonious nonlinear parametric model is proposed to describe radiographic non-destructive inspections (parametric tomography). A new ε-optimal statistical test is developed to solve the problem of anomaly detection. The experimental results on real radiographic data confirm the relevance and efficiency of the proposed solution.

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